Right to Repair:
Pricing, Welfare, and Environmental Implications

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The “right to repair” (RTR) movement calls for government legislation that requires manufacturers to provide repair information, tools, and parts so that consumers can independently repair their own products with more ease. The initiative has gained global traction in recent years. Repair advocates argue that such legislation would break manufacturers’ monopoly on the repair market and benefit consumers. They further contend that it would reduce the environmental impact by reducing e-waste and new production. Besides, the RTR legislation may also trigger a price response in the product market as manufacturers try to mitigate the profit loss. This paper employs an analytical model to study the pricing, welfare, and environmental implications of RTR. We find that as the RTR legislation continually lowers the independent repair cost, manufacturers may initially cut the new product price and then raise it. This non-monotone price adjustment may further induce a non-monotone change in consumer surplus, social welfare, and the environmental impact. Strikingly, the RTR legislation can potentially lead to a “lose-lose-lose” outcome that compromises manufacturer profit, reduces consumer surplus, and increases the environmental impact, despite repair being made easier and more affordable.

Key words: sustainable operations, repair, durable goods, after-sales service, pricing, extended producer responsibility, consumer surplus, environmental impact

1. Introduction
From motor vehicles to consumer electronics to farm equipment, modern technology products are getting increasingly complex. Repairing them is difficult, if not impossible, for consumers or third-party service shops without the aid of manufacturers. Yet, manufacturers often withhold repair
information, proprietary repair tools and spare parts, forcing consumers to either settle for a lofty repair price or forgo repair altogether.

The “right to repair” (RTR) movement is meant to change that. It calls for government legislation that requires manufacturers to share repair information (e.g., manuals, schematics, and documentation), provide diagnostic tools, and supply service parts to make it easy for consumers to repair their own products (either by themselves or through third-party independent service shops).

The initiative has gained global traction. In the US, the Motor Vehicle Owners’ Right to Repair Act was passed in Massachusetts in 2012 and was effectively adopted at the national level starting from automotive year 2018 (The Atlantic 2014); the state law was later amended in 2020 to cover telematics systems (The Verge 2020b). Twenty states have considered similar legislation for electronics more broadly (The Guardian 2019), but none of the technology right-to-repair bills have been passed as of 2019 (Fortune 2019). Multiple 2020 Democratic presidential candidates proposed national right-to-repair laws that ensure farmers can fix their tractors and other farm equipment (The New York Times 2019). Campaigns for the right-to-repair regulations are also seen in Canada, Australia, and the European Union (Fortune 2019). In particular, the EU has passed legislative rules regarding the supply of replacement parts (BBC 2019b) and mandatory labeling to indicate reparability (The Verge 2020a). The right-to-repair initiative has garnered support from consumer advocacy groups and environmental organizations, such as the US Public Interest Research Group (PIRG), the Repair Association, and iFixit, to name a few.

While the right-to-repair legislation is likely multifaceted, it is safe to say that once enacted, it will make independent repair easier and less costly. For example, without the right-to-repair law, phones and tablet parts are often glued together, and devices can easily break when pried apart (The Guardian 2019), which “makes repairs costly” (Vox 2019). Moreover, Apple’s proprietary “pentalobe” screws cannot be removed with common screwdrivers (The Guardian 2019). The lack of proper repair tools is not only an issue for cellphones but also tractors (Time 2017). In addition, manufacturers sometimes charge a high fee for access to repair manuals (Scher 2020) or refuse to sell replacement parts (Vox 2019). As a result, those conducting independent repair often have to rely on self-made tools, hunt for unauthorized manuals, and use refurbished or copied parts (The Economist 2017), all of which make independent repair difficult and costly (in terms of time, effort, and financial cost). By contrast, the right-to-repair law mandates an increase in the availability of repair tools, information, and parts, which may stimulate more competition that further drive down the cost of acquiring spare parts (Matchar 2016). All of these changes will make independent repair easier and less costly. Our paper focuses on this cost-reduction angle.

As a subject of constant legislative evolution and active policy debate, the right to repair is commonly believed to benefit consumers and the environment. At the heart of the advocacy for
the right to repair lies the argument that it breaks manufacturers’ monopoly on the repair market and empowers consumers with more (affordable) repair choice, which should make them better off (Gizmodo 2019, Vox 2019). Repair advocates also contend that giving consumers the right to repair is environmentally favorable (The Economist 2017, Forbes 2018, The Washington Post 2018, BBC 2019a). The rationale is that insufficient repair will cause product under-utilization and generate too much e-waste (Sabbaghi and Behdad 2018); if repair is made easier, products will last longer and consumers will not buy as many new products or prematurely throw away as many old ones, which translates into less production and less e-waste, thereby reducing the environmental impact.

However, the right-to-repair legislation is often met with considerable resistance from manufacturers, many of whom lobbied or filed suit against it (Huffington Post 2016, Time 2017, Reuters 2020). While manufacturers usually justify their pushback on privacy, cyber-security, and safety grounds (Gizmodo 2019, Forbes 2020), many believe the true motivation is an economic one. It is understandable that giving consumers the right to repair would hurt manufacturers’ bottom line by diminishing both repair profit and new product sales. However, it is less clear how the legislation would shape manufacturers’ pricing decisions in the product market as they try to mitigate the (inevitable) profit loss. Any assessment of the welfare and environmental implications is not complete without incorporating this pricing perspective, yet it is largely missing in today’s policy debate. Our paper fills this gap.

We employ an analytical model in which a profit-maximizing manufacturer who makes a finitely durable product at a cost and sells it to a heterogeneous population of consumers over an infinite horizon. Consumers can either buy new products from the manufacturer, or buy used products from a secondary market. A new product may fail after use, in which case, consumers decide whether to throw it away (and buy a new one), seek manufacturer repair, or conduct independent repair. Both the manufacturer and consumers would incur a cost for performing repair, but the manufacturer has a cost advantage. The manufacturer sets both the price of the new product it sells and the price of the repair service it provides, if at all. Our model treats the right-to-repair legislation as an external force that reduces the cost of independent repair (which cannot fall below the manufacturer’s repair cost).

In general, providing repair service is a double-edged sword for the manufacturer. On the one hand, it strengthens consumers’ valuation of the product over its entire life cycle, which enables the manufacturer to price the new product higher; on the other hand, it encourages consumers to substitute toward old products, which cannibalizes new product sales. How much repair the manufacturer would like to offer depends on how costly it is to make new products. If new products are cheap to produce, then the manufacturer can easily churn out new products in volume, and thus have no incentive to offer repair. Therefore, it will induce a “no-repair” equilibrium to the
extent possible by charging an exorbitantly high repair price. By contrast, if the production cost is high, the manufacturer would like the product to last as long as possible, and hence provides repair for free, inducing a “full-repair” equilibrium. Nevertheless, if the production cost is intermediate, the manufacturer takes a balanced approach and charges an intermediate repair price that induces some consumers to repair, resulting in a “partial-repair” equilibrium.

In light of these results, a lower independent repair cost due to the right to repair legislation will not make a difference if the production cost is high (since manufacturer repair is always offered free of charge in this case), but it will compel the manufacturer to rectify the pricing decision otherwise. Specifically, if the production cost is low, the manufacturer will adopt a volume strategy by lowering the new product price. Doing so reduces the used product price in the secondary market, which disincentivizes repair and resale that would otherwise be spawned by easier independent repair. Hence, a lower new product price allows the manufacturer to maintain the desired no-repair equilibrium and counteract cannibalization.

If the production cost is intermediate, the manufacturer still follows the volume strategy initially, but as the independent repair cost continues to decline and falls below a certain point, the manufacturer switches to a margin strategy by raising the new product price and offering free repair, which gives rise to a full-repair equilibrium. In this case, should the manufacturer keep lowering the price to guard against repair, it would render the profit margin too thin. Since products are not cheap to make in the first place and repair is so hard to eliminate (because independent repair becomes sufficiently easy), the manufacturer might as well facilitate repair and prolong product lifespan. Doing so increases the resale value of a new product, which, in turn, enables the manufacturer to charge a higher price. This result implies that the right-to-repair legislation can trigger a non-monotone, U-shaped price response in the product market.

The above price response has welfare and environmental implications. When the production cost is low, both consumer surplus and social welfare improve as a result of the right to repair legislation, thanks to the manufacturer’s price cut. However, the volume strategy also causes consumers to buy more, thereby increases the environmental impact. Altogether, the right-to-repair legislation would bring about a lose-win-lose outcome, hurting the manufacturer and the environment, but benefiting consumers. When the production cost is intermediate, as the independent repair cost falls, consumer surplus will first increase (along with a price drop) and then decrease (along with a price hike), and so will social welfare. The total environmental impact will follow a similar trend if the production and disposal phases are responsible for most of the environmental impact. However, if the use phase is the main contributor, the per-unit use impact of old products is much higher due to degraded energy efficiency, and product durability is not too high, then the total environmental impact will monotonically increase as the independent repair cost falls.
All told, our analysis suggests the right-to-repair legislation can lead to a win-lose, a lose-win, or a lose-lose outcome, for consumers and the environment, depending on the market conditions. In particular, if the production cost is intermediate and the environmental impact is use-phase-dominated, then a right-to-repair bill that results in a sharp reduction of the independent repair cost can create a lose-lose-lose situation that compromises manufacturer profit, reduces consumer surplus, and exacerbates the environmental impact.

These insights are robust to a number of model extensions we consider in the paper. Notably, when the manufacturer can also decide on product durability in conjunction with price, we numerically find that a decrease in the independent repair cost may also trigger a similar non-monotone durability response: the manufacturer initially complements the price cut with a reduction in durability to further limit cannibalization but when the independent repair cost falls below a certain point, the manufacturer raises the product price and improves durability at the same time to double down on enhancing consumer valuation.

The remainder of the paper is organized as follows. §2 provides a literature review. §3 introduces the model. §4 characterizes the equilibrium. §5 examines how the right to repair legislation affects product prices, consumer surplus, social welfare, and the environmental impact. §6 studies several model extensions. §7 concludes the paper and discusses future research directions.

2. Related Literature
To the best of our knowledge, our paper is the first to analytically study the economic and environmental implications of the right to repair. As such, our work is primarily related to two streams of literature: after-sales service and sustainable operations.

In the literature on after-sales service, Cohen and Whang (1997) consider a manufacturer who is a monopolist in the product market but competes with an independent service shop for the after-sales service. The manufacturer sets the product price in the first stage, and then in the second stage, the manufacturer and the independent service shop set their respective service qualities and service prices. Their game-theoretic analysis shows that the services of the two parties are maximally differentiated in both quality and price and that a large proportion of the manufacturer’s profit can come from service provision. Kim et al. (2007) study performance-based contracts in after-sales service supply chains using a principal-agent framework. Debo et al. (2008) examine the issue of service inducement that may arise in car repair. Guajardo et al. (2012) empirically investigate the impact of performance-based contracting on product reliability. Jain et al. (2013) derive the optimal performance-based contract for outsourcing of repair and restoration services. Guajardo et al. (2016) empirically examine the impact of service attributes such as warranty length and after-sales service quality on consumer demand in the US automobile industry. Relatedly,
various warranty issues have been studied in the literature, such as those regarding the optimal protection period (Glickman and Berger 1976), oligopolistic competition (DeCroix 1999), strategic claim behavior (Gallego et al. 2014a), dynamic reliability learning (Gallego et al. 2014b), and supply chain implications (Heese 2012).

These papers on after-sales services either do not consider the cannibalization of the repair service on new product sales, or do not study the extent to which manufacturers can control the repair market, or are silent on the environmental dimension. Our paper contributes to this stream of literature by explicitly incorporating these important elements.


Most works in the sustainable operations and durable goods literature do not explicitly consider product failure or after-sales repair services. Several papers incorporate these issues to varying degrees. Mann (1992) and Kinokuni (1999) consider a setting where consumers can conduct preventive maintenance to reduce the probability of product failure, but once a product fails, it can no longer be repaired and put back to use. Utaka (2006) allows a proprietary repair service to be offered exclusively by the manufacturer and studies the signaling role of warranties. Fu et al. (2021) study how product reliability affects a durable good manufacturer’s choice of the warranty length in the presence of secondary market interference. Their theoretical model and empirical analysis show that warranty length offering is a U-shape function of product reliability. Their model assumes that if a product is not covered by warranty, then its expected valuation is reduced, but abstracts away from modeling consumers’ response or the manufacturer’s lever in the event of a product failure outside the warranty coverage (e.g., whether consumers have a repair option, whether they choose to repair, how the manufacturer charges the repair price, etc). Our paper contributes to this stream of literature by explicitly investigating how the availability of independent repair in the repair market influences the product market, and consequently, consumer surplus, social welfare, and the environmental impact.
3. Model

We consider a discrete time, infinite-horizon, sequential game between a manufacturer and consumers. Periods are indexed by $t$. All players are forward-looking and have a common discount factor $\rho \in (0, 1]$. The manufacturer sells a finitely durable product to consumers. Following the durable goods literature (Desai and Purohit 1998, Hendel and Lizzeri 1999, Huang et al. 2001, Agrawal et al. 2012, 2016, Alev et al. 2020), we assume that the product lasts for at most two periods. A product in the first period of its useful life is referred to as a new product. After one period of use, the product fails with probability $f \in (0, 1)$, but is repairable. If a failed product is not repaired, then it has no value to consumers and will be scrapped. If it is repaired, then it can be used for a second period. A product in the second period of its useful life (either one that has not failed after one period of use or one that has failed but is repaired) is referred to as a functional used product. A product reaches end-of-life after two periods of use, at which point it has no value to consumers and will be scrapped.

The market contains a continuum of consumers and the total mass of consumers is normalized to one. Each consumer uses at most one product in any given period. Consumers are heterogeneous in their valuations of the product. A consumer of type $\theta$ derives a per-period gross utility $\theta$ from a new product, and a per-period gross utility $\delta \theta$ from a functional used product (possibly one that has failed but been repaired). We assume that $\theta$ is uniformly distributed in $[0, 1]$ and $\delta \in (0, 1)$. As in the aforementioned durable goods literature, $\delta$ is often interpreted as the built-in product durability; it captures the economic and physical depreciation of the product after one period of use, i.e., consumers obtain a lower per-period gross utility from a used product than a new one, making used products an imperfect substitute to new products.

In each period $t$, the manufacturer sets new product price $p_{tn}$ and repair price $p_{tr}$ (for consumers who seek manufacturer repair) to maximize its total discounted profit. The manufacturer’s unit production cost is $c$ and its unit repair cost is $c_M$. If consumers perform independent repair (i.e., either repair on their own or enlist the help of a third-party service shop), they incur a cost $c_I$.

Given the prices, consumers make purchasing and repair decisions to maximize their expected total discounted utility. At the end of period $t$, consumers who have a functional used product (with one period of useful life left) may choose to sell it on the secondary market at price $p_{tu}$. The existence of an active secondary market of used products is a common feature of durable goods (see, e.g., Makov et al. 2019). Consistent with the aforementioned durable goods literature, we restrict the attention of our analysis to stationary equilibria, where prices do not vary over time. Hence, we can drop the dependence of prices on the time index and let $p_{tn} = p_n$, $p_{tr} = p_r$ and $p_{tu} = p_u$. The focus on stationarity eliminates transient effects attributed to no used products being up for sale in the first period. All model primitives are common knowledge except that each consumer is privately informed of their own type $\theta$. To focus the paper, we impose the following assumptions.
Assumption 1. (i) \( c_M \leq c_I \); (ii) \( c_M < \delta \); (iii) \( c < 1 + \delta - c_M f \).

Assumption 1-(i) captures the fact that the manufacturer is likely to have a cost advantage over consumers in performing repair (with or without the right-to-repair legislation). Assumption 1-(ii) requires that the manufacturer’s repair cost be lower than the maximum value of a repaired product; Together with Assumption 1-(i), it rules out the uninteresting case where repair is so costly that it will never be provided by the manufacturer or sought independently by consumers. Assumption 1-(iii) requires the production cost be not too high and thus ensures the manufacturer can make a profit.

Since the right to repair legislation is intended to facilitate independent repair, its effect is captured in our model by a reduction of the independent repair cost \( c_I \). As such, we will assess the impact of the right to repair through the lens of investigating the comparative statics of various equilibrium metrics with respect to \( c_I \), i.e., how the equilibrium outcomes change if independent repair is made less costly (to the extent that \( c_I \) is no less than \( c_M \)). Table 1 summarizes the main notation used in the paper.

<table>
<thead>
<tr>
<th>Table 1</th>
<th>Glossary of main notation</th>
</tr>
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<tbody>
<tr>
<td>Symbol</td>
<td>Definition</td>
</tr>
<tr>
<td>( f )</td>
<td>Probability of product failure</td>
</tr>
<tr>
<td>( \delta )</td>
<td>Product durability</td>
</tr>
<tr>
<td>( \theta )</td>
<td>A consumer’s per-period gross utility of a new product</td>
</tr>
<tr>
<td>( c_M, c_I )</td>
<td>Manufacturer repair cost and independent repair cost per unit</td>
</tr>
<tr>
<td>( c )</td>
<td>Production cost per unit</td>
</tr>
<tr>
<td>( \rho )</td>
<td>Discount factor</td>
</tr>
<tr>
<td>( p_n, p_u, p_r )</td>
<td>New product price, used product price, manufacturer repair price</td>
</tr>
<tr>
<td>( \alpha )</td>
<td>Consumers’ repair probability upon a product failure</td>
</tr>
<tr>
<td>( \pi )</td>
<td>Manufacturer profit</td>
</tr>
<tr>
<td>( CS )</td>
<td>Consumer surplus</td>
</tr>
<tr>
<td>( SW )</td>
<td>Social welfare, ( SW = \pi + CS )</td>
</tr>
<tr>
<td>( \mathcal{E} )</td>
<td>Environmental impact</td>
</tr>
<tr>
<td>( Q )</td>
<td>Quantity of new products produced per period</td>
</tr>
<tr>
<td>( U )</td>
<td>Quantity of products in use per period</td>
</tr>
<tr>
<td>( U^u )</td>
<td>Quantity of old products used per period</td>
</tr>
<tr>
<td>( R )</td>
<td>Quantity of products repaired per period</td>
</tr>
<tr>
<td>( \gamma_p, \gamma_d, \gamma_r )</td>
<td>Production, disposal and repair impacts per unit</td>
</tr>
<tr>
<td>( \gamma_{u1}, \gamma_{u2} )</td>
<td>Use impacts of new and old products per unit per period</td>
</tr>
<tr>
<td>( \gamma_q )</td>
<td>( \gamma_q = \gamma_p + \gamma_{u1} + \gamma_d )</td>
</tr>
</tbody>
</table>

3.1. Demand Characterization

At the beginning of each period, a consumer may face three possible states: state 0, in which she does not have a product that can be further used; state 1, in which she has a functional used product; state \( \bar{1} \), in which she has a failed used product. Let \( \mathcal{V}(s) \) be a consumer’s value function.
(expected total discounted utility) in state $s \in \{0, \bar{1}, 1\}$. Thus, the Bellman equations for consumers’ Markov decision process are:

\[
\begin{align*}
V(0) &= \max \{ \rho V(0), \delta \theta - p_u + \rho V(0), \theta - p_u + \rho[(1 - f) V(\bar{1}) + f V(1)] \}; \\
V(\bar{1}) &= \max \{ p_u + V(0), \delta \theta + \rho V(0) \}; \\
V(1) &= \max \{ V(0), -\min\{c_I, p_r\} + V(\bar{1}) \}.
\end{align*}
\]

Equation (1a) specifies the value function in state 0, in which a consumer can choose to remain inactive, buy a used product from the secondary market, or buy a new product. Equation (1b) specifies the value function in state $\bar{1}$, in which a consumer can choose to sell the used product or keep using it. Equation (1c) specifies the value function in state 1 in which a consumer can choose to scrap the failed product or repair it. Similar to what has been established in the literature (e.g., Hendel and Lizzeri 1999), we can rigorously show in our model that holding on to a functional used product for a second period is not optimal for consumers. Lemma 1 summarizes this result and further shows that three consumer segments will arise in equilibrium.

**Lemma 1.** There exist $\theta_1, \theta_2$ with $\theta_1 \leq \theta_2$ such that consumers with $\theta \in [0, \theta_1)$ are inactive; consumers with $\theta \in [\theta_1, \theta_2)$ purchase a used product every period; consumers with $\theta \in [\theta_2, 1]$ purchase a new product every period.

In steady state, every period repeats itself. Hence, in the remainder of the paper, following the convention of the literature, we let $\rho = 1$ and take a per-period perspective. In each period, consumers with $\theta \in [0, \theta_1)$ remain inactive and have zero utility, i.e., $V_i(\theta) = 0$. Consumers with $\theta \in [\theta_1, \theta_2)$ buy a used product at the beginning of every period and have a per-period utility $V_u(\theta) = \delta \theta - p_u$. Consumers with $\theta \in [\theta_2, 1]$ buy a new product at the beginning of every period and have a per-period expected utility

\[
V_n(\theta) = \theta - p_n + (1 - f)p_u + f[p_u - \min\{c_I, p_r\}]^+.
\]

We explain the above expected utility as follows. At the end of each period, if the product consumers buy at the beginning of the period does not fail (which occurs with probability $1 - f$), then consumers sell it in the secondary market at price $p_u$; if the product fails (which occurs with probability $f$), then they decide on their repair probability $\alpha \in [0, 1]$ as follows.

(i) if $p_u > \min\{c_I, p_r\}$ (i.e., the gain from selling a functional used product exceeds the cost of repair), consumers repair the failed product and sell it on the secondary market with probability $\alpha = 1$ for a net profit $p_u - \min\{c_I, p_r\}$. Since all consumers who experience a product failure perform repair in this case, we refer to it as full repair.
(ii) if $p_u < \min\{c_I, p_r\}$, they choose not to repair (i.e., $\alpha = 0$). Since no consumers who experience a product failure perform repair in this case, we refer to it as no repair.

(iii) if $p_u = \min\{c_I, p_r\}$, then consumers are indifferent to repair and play a mixed strategy by choosing to repair and resell with probability $\alpha \in [0, 1]$. In particular, if $\alpha = 0$ ($\alpha = 1$), then there is no repair (full repair); if $\alpha \in (0, 1)$, then only a fraction of consumers who experience a product failure perform repair and we refer to such a case as partial repair.

In the model, thresholds $\theta_1$ and $\theta_2$ satisfy the indifference conditions $V_u(\theta_1) = V_i(\theta_1)$ and $V_u(\theta_2) = V_u(\theta_2)$. The used product price $p_u$ clears the secondary market and is determined by equating demand and supply:

$$\theta_2 - \theta_1 = (1 - \theta_2)(1 - f + f\alpha).$$

Note that $\theta_1$, $\theta_2$, and $\alpha$ are all functions of $p_u$; we suppress this dependency for succinctness. Further, recognize that $\theta_2 - \theta_1 \leq 1 - \theta_2$, i.e., the “buy used” segment is weakly smaller than the “buy new” segment; this is because those who buy new may not always sell their products after one period of use on the secondary market (as they may forgo repair in the event of a product failure).

### 3.2. The Manufacturer’s Pricing Problem

The manufacturer sets the product and repair prices to maximize its per-period profit:

$$\pi \triangleq \max_{p_n, p_r} (p_n - c)(1 - \theta_2) + (p_r - c_M)(1 - \theta_2)f\alpha \cdot 1_{\{p_r \leq p_u \text{ and } p_r \leq c_I\}}.$$

The first term in the manufacturer’s objective function corresponds to profit from new product sales, and the second term, the repair profit. The indicator function implies the repair profit is nonzero only if the repair price charged by the manufacturer is no greater than both the used product price (so that consumers have an incentive to repair) and the cost of independent repair (so that consumers turn to the manufacturer for repair).

### 3.3. Discussion of Independent Repair

The independent repair option in our model has two interpretations: one is consumers performing self-repair; the other is consumers seeking repair from a third-party independent repair shop. In the first interpretation, consumers directly bear the exogenous independent repair cost $c_I$ (influenced by the right-to-repair legislation). In the second interpretation, it is the repair shops that directly bear $c_I$, and in principle, they could charge consumers a repair fee higher than $c_I$. However, the repair shops in our model are engaged in a Bertrand-type price competition with the manufacturer: they offer the same repair service as the manufacturer but face a cost disadvantage ($c_I > c_M$). Therefore, the equilibrium independent repair fee will be pushed down to the marginal cost $c_I$. 
(and this is true whether it be a single repair shop or multiple repair shops competing with the manufacturer). Thus, in either interpretation, consumers effectively face the same independent repair cost of $c_I$.\(^1\)

4. Equilibrium

In this section, we solve the manufacturer’s pricing problem and characterize the equilibrium outcomes.

**Proposition 1 (Equilibrium).** The manufacturer’s optimal new product price $p^*_n$, repair price $p^*_r$, the resulting used product price in the secondary market $p^*_u$, and consumers’ repair probability $\alpha^*$ are summarized in Table 2.

<table>
<thead>
<tr>
<th>Conditions</th>
<th>$p^*_n$</th>
<th>$p^*_r$</th>
<th>$p^*_u$</th>
<th>$\alpha^*$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$c_I \geq \frac{\delta + c_M}{2}$, $c &lt; \underline{c}$</td>
<td>$[c + 1 + \delta(1 - f)]/2$</td>
<td>$\infty$</td>
<td>$\frac{\delta[(c + 1 + \delta)(1 - f)]}{2[1 + \delta(1 - f)]}$</td>
<td>0</td>
</tr>
<tr>
<td>$c &lt; \underline{c}$</td>
<td>$[c + 1 + \delta(1 - f)]/2$</td>
<td>$\frac{\delta + c_M}{2}$</td>
<td>$\frac{\delta + c_M}{1 + \delta}$</td>
<td>$\alpha \in (0, 1)$</td>
</tr>
<tr>
<td>$c \geq \underline{c}$</td>
<td>$(c + 1 + \delta + c_M f)/2$</td>
<td>0</td>
<td>$\frac{\delta(2c + c_M f)}{1 + \delta}$</td>
<td>1</td>
</tr>
</tbody>
</table>

| $c_I < \frac{\delta + c_M}{2}$, $\hat{c}_1 \leq c \leq \hat{c}_2$ | $[c + 1 + \delta(1 - f)]/2$ | $\infty$ | $\frac{\delta[(c + 1 + \delta)(1 - f)]}{2[1 + \delta(1 - f)]}$ | 0 |
| $\hat{c}_2 < c < \hat{c}_3$ | $\frac{c + 1 + \delta + (2 - f)c_I - c_M f}{2}$ | $c_I$ | $\alpha \in (0, 1)$ |
| $c \geq \hat{c}_3$ | $(c + 1 + \delta + c_M f)/2$ | 0 | $\frac{\delta(2c + c_M f)}{1 + \delta}$ | 1 |

\(\hat{c}(p_n, p_r) \triangleq \frac{\delta[(2 - 4\delta + 3\delta + 1)pr - 8(2 - f)pr + 4(1 - \delta)(1 - f)]}{\delta f[pr - (2 - f)pr + 4(1 - \delta)]}. \underline{c}, \hat{c} \) are constant in $c_I$ and $\hat{c}_1, \hat{c}_2, \hat{c}_3$ are increasing in $c_I$.

Figure 1 illustrates Proposition 1, which, on the high level, shows that full repair arises when the production cost is high; partial repair arises when the production cost is intermediate; and no repair arises when the production cost is low. We explain the results in more detail below.

First consider the case of the independent repair cost being high enough ($c_I \geq (\delta + c_M)/2$) that it does not influence the manufacturer’s pricing decisions (i.e., since independent repair was such a costly option to consumers, the manufacturer can act as if it were unavailable).

When the production cost is low ($c \leq \underline{c}$), it is cheap to churn out new products and thus the new product price can be set relatively low to attract consumers. In this case, it is in the manufacturer’s best interest to sell new products but preclude repair. Repaired products increase the supply in the secondary market and put a downward pressure on the used product price, which, in turn, increases consumer demand for used products, cannibalizing new product sales. Hence, a manufacturer who

\(^1\) We will consider an extension in §6.2 of imperfect competition between the manufacturer and an independent repair shop who provide differentiated repair services. In that extension, the independent repair shop can potentially charge above the marginal cost and earn a positive profit in equilibrium.
can sell new products cost-effectively should eliminate repair altogether. One implementation of this strategy is to charge an exorbitantly high repair price that deters repair, which is indeed believed to be a common strategy used in practice (Gizmodo 2019).

When the production cost is intermediate \((c < \bar{c})\), relying on new product sales only is not advisable, because the manufacturer cannot set a product price as low as before to attract sufficient demand. While offering an after-sales repair service cannibalizes some new product sales, doing so generates additional profit that supplements the otherwise insufficient profit from selling new products. The new product and repair prices should be set to induce an equilibrium where the used product price is exactly equal to the repair price. Consumers are thus indifferent to repair and play a mixed repair strategy. Thus, a partial-repair equilibrium emerges.

When the production cost is high \((c \geq \bar{c})\), the new product price is inevitably high, causing low demand for new products. In this case, the manufacturer can offer the repair service as a means to strengthen consumers’ willingness to pay for the (high-priced) product and stimulate demand. It is optimal for the manufacturer to give out the repair service for free such that all consumers who experience a product failure seek repair from the manufacturer. A free repair service enhances the resale value of a product in the event of a product failure, enabling the manufacturer to charge a higher new product price up front that factors in the repair cost it may later incur in service provision. Thus, full repair emerges in equilibrium. Such a pricing strategy can be interpreted as implementing a warranty, another common practice adopted by many manufacturers.

In general, as new products become more expensive to produce, the manufacturer has a stronger incentive to make its product last longer and thus will increasingly facilitate repair.
Next, we consider the case where the independent repair cost is not too high \((c_I < (\delta + c_M)/2)\) and can pose a threat to the manufacturer. Notably, when the production cost is intermediate, the threat of independent repair indeed kicks in, compelling the manufacturer to adjust prices.

Specifically, when the production cost is intermediately low \((\hat{c}_1 \leq c \leq \hat{c}_2)\), a no-repair equilibrium arises, but the structure of the equilibrium is different than that of the previous no-repair equilibrium. Here, in order to prevent consumers from seeking independent repair (which is now less expensive), the manufacturer must lower the new product price (relative to the one charged in the previous no-repair equilibrium under \(c_I \geq (\delta + c_M)/2\)) to induce a low enough used product price \(p_u^*\) (such that consumers do not bother with repair) (i.e., \(p_u^* = c_I\)). Failure to do so (i.e., \(p_u^* > c_I\)) would trigger too much repair that cannibalizes new product sales. Likewise, when the production cost is intermediately high \((\hat{c}_2 < c < \hat{c}_3)\), there is a new partial-repair equilibrium, in which the manufacturer has to ensure not only that its repair price matches the independent repair cost (i.e., \(p_r^* = c_I\)) to capture the repair profit, but also that it cuts the new product price (relative to the one charged in the previous partial-repair equilibrium under \(c_I \geq (\delta + c_M)/2\)) low enough to induce a sufficiently low used product price that discourages some consumers from seeking repair.

We also show in Proposition 1 (see Figure 1 for an illustration) that the cutoff values on the production cost \((\hat{c}_1, \hat{c}_2, \hat{c}_3)\) are all weakly increasing in the independent repair cost \(c_I\). This implies that as independent repair gets less costly, the no-repair equilibrium becomes harder to maintain, whereas the full-repair equilibrium becomes more widespread, which is intuitive.\(^2\)

We acknowledge that in our model, the independent repair option acts as a credible threat that shapes the manufacturer’s pricing decisions but will never be exercised by consumers in equilibrium. This is because we assume away possible quality differentiation between manufacturer repair and independent repair or heterogeneity among consumers in their repair preference/ability. These simplifying assumptions enable analytical tractability. To address this limitation of the base model, we consider three extensions in §6.1, §6.2, and §6.3; in each of these extensions, independent repair can arise in equilibrium, yet our key insights remain robust.

5. Impact of the Right to Repair

This section builds on the equilibrium characterization in the previous section and studies how a decrease in the independent repair cost due to the right-to-repair legislation affects the manufacturer’s profit, product prices, consumer surplus, social welfare, and the environmental impact.

\(^2\) Our numerical exploration further suggests that as either \(\delta\) or \(f\) increases, the “no repair” region in the \((c_I, c)\) space shrinks and the “full repair” region expands. This is because, as \(\delta\) increases, the life-cycle value of a product increases; to take advantage of this, the manufacturer leans toward more repair. On the other hand, as \(f\) increases, the product is more likely to fail, and hence, it opens up more opportunities for repair, which the manufacturer capitalizes on to either generate additional profit or strengthen the life-cycle product valuation.
5.1. Profit and Prices

Proposition 2 below characterizes the impact of the right to repair on the manufacturer’s profit.

**Proposition 2 (Profit).** As independent repair cost $c_I$ decreases due to the right-to-repair legislation, the manufacturer’s profit $\pi$ weakly and continuously decreases.

Proposition 2 is intuitive: less costly independent repair undermines the manufacturer’s repair revenue and cannibalizes new product sales, which will only hurt the manufacturer’s profit. This result helps explain why many manufacturers fiercely lobby against the right-to-repair bills. While the profit implication is straightforward, how the manufacturer adjusts prices to mitigate the profit loss is more subtle, as shown in Proposition 3 below.

**Proposition 3 (Prices).** As independent repair cost $c_I$ decreases due to the right-to-repair legislation, the repair price $p^*_r$ (weakly) decreases; and there exists $\tilde{c}_I < \bar{c}_I$ such that

(i) if $c \geq \bar{c}_I$, then both new product price $p^*_n$ and used product price $p^*_u$ are constant;

(ii) if $\tilde{c}_I \leq c < \bar{c}_I$, then both $p^*_n$ and $p^*_u$ first (weakly) decrease and then (weakly) increase;

(technically, there exists $c_{I_0} \in (c_M, (\delta + c_M)/2)$ such that both $p^*_n$ and $p^*_u$ (weakly) decrease as $c_I$ decreases for $c_I > c_{I_0}$; $p^*_n(c_{I_0}) > p^*_n(c_I^+) > p^*_n(c_I^-)$, $p^*_u(c_{I_0}) > p^*_u(c_I^-)$; both $p^*_n$ and $p^*_u$ are constant in $c_I$ for $c_I \in [c_M, c_{I_0})$);

(iii) if $c < \tilde{c}_I$, then both $p^*_n$ and $p^*_u$ (weakly) decrease.

![Figure 2](image.png)

**Figure 2** Impact of RTR on the new and used product prices

Panel (a): New product price

Panel (b): Used product price

Note. $f = 0.8$, $\delta = 0.6$, $c_M = 0.2$, $\tilde{c}_I = 0.15$. 
Proposition 3 first shows that as independent repair cost falls, so does the manufacturer’s repair price. This result is consistent with the intuition that the right to repair legislation will regulate the repair market, not only making independent repair more accessible but also manufacturer repair more affordable. Next, we turn to the comparative statics of the new and used product prices, which are illustrated by Figure 2.

When the production cost is high ($c \geq \bar{c}$), recall from Proposition 1 that the manufacturer voluntarily offers full repair for free regardless of how costly it is to conduct independent repair. In this case, the right to repair legislation has no impact on the product prices.

When the production cost is low ($c < \tilde{c}_0$), a lower independent repair cost leads to lower new and used product prices. In this case, as shown by Proposition 1, a no-repair equilibrium arises if the independent repair cost is prohibitively high. However, as independent repair gets less costly, the threat of consumers turning to independent repair will eventually kick in, potentially ruining the no-repair equilibrium. To mitigate the cannibalization effect of repair on new product sales, the manufacturer adopts a volume strategy and cuts the new product price. Doing so has two effects: (1) prompting more consumers to buy new products, and this increased output of new products translates into more supply of used products in the secondary market; (2) dissuading more consumers from buying used products, and this substitution behavior reduces the demand in the secondary market. Both effects induce a lower used product price, thereby disincetivizing repair. As a result, the manufacturer may manage to maintain the no-repair outcome in equilibrium or at least limit the volume of repair (as also pointed out in the explanation of Proposition 1).

The most intriguing price response occurs when the production cost is intermediate ($\tilde{c}_0 \leq c < \bar{c}$), in which case, as independent repair gets cheaper, the new and used product prices initially decline and then jump up after the independent repair cost falls below a certain threshold. As in the case of a low production cost, the manufacturer initially responds to cheaper independent repair by lowering the new product price, but now producing new products becomes more expensive, and a lower price would further erode the already thin profit margin. Thus, price cutting can only do so much to guard against independent repair. As the cost of independent repair further declines, repair gets increasingly harder to prevent. At a certain point, instead of continuing the price cut to circumvent repair, the manufacturer would be better off giving away repair for free (through warranties). Doing so will enhance the life-cycle value of a new product by insuring consumers against the risk of product failure. This effect, in turn, enables the manufacturer to raise the new product price and adopt a margin strategy. While opening up repair potentially increases the supply of used products in the secondary market, a higher new product price implies fewer products produced in the first place, thereby counterbalancing the former effect and causing the
used product price to rise as well. Hence, knowing repair is too hard to eliminate, the manufacturer switches from a volume strategy that stymies repair to a margin strategy that embraces repair.\(^3\)

5.2. Consumer Surplus and Social Welfare

We define consumer surplus (per period) \(CS\) to be the sum of each individual’s expected utility:
\[
CS \triangleq \int_{\theta_1}^{\theta_2} V_u(\theta) d\theta + \int_{\theta_2}^{1} V_u(\theta) d\theta.
\]
We define social welfare (per period) \(SW\) to be the sum of the manufacturer’s profit and consumer surplus: \(SW \triangleq \pi + CS\). Note that in social welfare, all the money transfers (the new and used product prices as well as the repair price) cancel out with each other; thus, social welfare encapsulates the total net value generated. Proposition 4 below characterizes the impact of the right to repair on consumer surplus and social welfare.

**Proposition 4 (Consumer Surplus and Social Welfare).** As independent repair cost \(c_I\) decreases due to the right-to-repair legislation, there exists \(\tilde{c}_0 < \bar{c}\) such that
(i) if \(c \geq \bar{c}\), then both consumer surplus \(CS\) and social welfare \(SW\) are constant;
(ii) if \(\tilde{c}_0 \leq c < \bar{c}\), then both \(CS\) and \(SW\) first (weakly) increase and then (weakly) decrease;
(iii) if \(c < \tilde{c}_0\), then both \(CS\) and \(SW\) (weakly) increase.

Figure 3 illustrates Proposition 4. The welfare implications of the right to repair is closely tied to the manufacturer’s price response (characterized in Proposition 3). In particular, when the

\(^3\)On a technical note, when \(c_I = c_{I_0}\), the volume strategy (which has \(p_u^* = c_I\)) yields exactly the same profit as the full-repair margin strategy (which would take over to be optimal if \(c_I < c_{I_0}\)). Thus, although there is a price jump at \(c_I = c_{I_0}\), the profit is still a continuous function of \(c_I\), as pointed out in Proposition 2.
production cost is low \((c < \tilde{c}_0)\), the right to repair legislation prompts the manufacturer to lower the new product price, which also induces a lower used product price. These price cuts translate into higher consumer surplus. In fact, the improvement in consumer surplus is substantial enough to make up for the manufacturer’s profit loss, causing social welfare to also increase. In this case, the right to repair legislation works exactly as intended (insofar as the economic impact is concerned).

However, when the production cost is intermediate \((\tilde{c}_0 \leq c < \tilde{c})\), a decrease of the independent repair cost due to the right to repair legislation has a non-monotone welfare effect. Similar to the case above, consumer surplus and social welfare initially increase, but when the independent repair cost falls below a certain point, both consumer surplus and social welfare decline as the manufacturer raises the new product price in a switch to the margin strategy (see the explanation of Proposition 3). Note that the manufacturer complements the product price adjustment with free repair, i.e., the repair price drops to zero. Hence, if one looks at the repair market alone, it is tempting to conclude consumers are better off, but this reasoning fails to account for the higher product prices consumers have to pay up front.

5.3. Environmental Impact

We follow the convention of the literature and take a product life cycle approach (Agrawal et al. 2012, Atasu and Souza 2013, Agrawal and Bellos 2017) to the environmental impact. We consider three life-cycle phases: production, use (which includes repair), and disposal. The total environmental impact is the sum of the environmental impact in each phase, which is further given by the volume of products in each phase times the per-unit impact in each phase. We break down the per-unit impact by phase as follows:

*Production:* Let \(\gamma_p\) denote the per-unit impact due to production of a product. The per-period quantity of new products produced is \(Q \triangleq 1 - \theta_2\). Thus, the production impact per period is \(\gamma_p Q\).

*Use:* Let \(\gamma_{u1}\) and \(\gamma_{u2}\) denote the per-unit, per period impact due to using a new product and an old product, respectively. As noted by Agrawal et al. (2012), a particularly relevant case is \(\gamma_{u2} \geq \gamma_{u1}\), which indicates the use impact of an old product is higher than that of a new one, because energy efficiency tends to degrade with use, as observed in refrigerators and automobiles (Cooper and Gutowski 2017). The per-period quantity of new products in use is \(Q_u \triangleq \theta_2 - \theta_1\). Thus, the total use impact per period is \(\gamma_{u1} Q + \gamma_{u2} U_u\).\(^4\)

*Repair:* Let \(\gamma_r\) denote the per-unit impact of repairing a product, including the energy and materials used in the repair process as well as any possible impact generated in transporting the product for repair. The per-period quantity of failed products that get repaired is \(R \triangleq (1 - \theta_2) f \alpha\). Thus, the repair impact per period is \(\gamma_r R\).

\(^4\) For clarity, we separate the repair impact from the use impact.
Disposal: Let $\gamma_d$ denote the per-unit impact due to disposal of an end-of-life product (e.g., e-waste). Thus, the disposal impact per period is $\gamma_d Q$.

The resulting total environmental impact per period is given by

$$
\mathcal{E} = \gamma_p Q + (\gamma_{u1} Q + \gamma_{u2} U^u) + \gamma_u R + \gamma_d Q = (\gamma_p + \gamma_{u1} + \gamma_d) Q + \gamma_{u2} U^u + \gamma_u R.
$$

For convenience, denote $\gamma_q \triangleq \gamma_p + \gamma_{u1} + \gamma_d$. Proposition 5 below characterizes how the three components of the environmental impact, $Q, U^u$, and $R$, are affected by the right-to-repair legislation.

**Proposition 5 (Production, Use, and Repair Volume).** As independent repair cost $c_I$ decreases due to the right-to-repair legislation, the repair volume $R$ always (weakly) increases; and there exists $\tilde{c}_0 < \bar{c}$ such that

(i) if $c \geq \bar{c}$, then both new production volume $Q$, and volume of old products in use $U^u$ are constant;

(ii) if $\tilde{c}_0 < c < \bar{c}$, then $Q$ first (weakly) increases and then (weakly) decreases; $U^u$ (weakly) increases if $\delta < \frac{2f}{(1-f)^2 + f^2}$ and may first (weakly) increase and then (weakly) decrease otherwise;

(iii) if $c < \tilde{c}_0$, then both $Q$ and $U^u$ (weakly) increase.

Figure 4 illustrates Proposition 5. Naturally, the right-to-repair legislation will increase the repair volume and thus the repair impact. However, the changes of the new production volume and used product volume are more subtle, and again closely tied to the manufacturer’s price response.
(characterized in Proposition 3). In particular, when the production cost is low ($c < \tilde{c}_0$), the right to repair legislation lowers the product prices. Hence, consumers end up buying more, which increases both the new production volume and the volume of used products traded on the secondary market, and thus, the volume of old products in use also increases. In this case, the environmental impact in each phase increases, and the total environmental impact also unequivocally increases.

When the production cost is intermediate ($\tilde{c}_0 \leq c < \bar{c}$), a decrease of the independent repair cost due to the right-to-repair legislation has a non-monotone effect on the new production volume. It initially increases (as in the case above), but when the independent repair cost falls below a certain point, the manufacturer switches from the volume strategy to the margin strategy (see the explanation of Proposition 3), and therefore, the new production volume declines. As for the volume of old products in use, like the new production volume, it initially increases, but when the manufacturer switches from the volume strategy to the margin strategy, the volume of old products in use decreases in some cases but keeps increasing in others. The manufacturer’s switch to a high-price/free-repair strategy imposes competing forces on the volume of use products: a downward pressure from a higher used product price versus an upward pressure from more repair (i.e., as more failed products are recovered from repair, more old products are in use instead of being scrapped prematurely). We identify a sufficient condition for the upward pressure to outweigh the downward pressure: product durability $\delta$ being not too high. In such a case, a used product does not generate extremely high value, which limits the magnitude of its price hike, and therefore, the upward pressure from more repair becomes a dominant force, causing the used product volume to increase after the switch. Note that the analytical condition we identified on $\delta$ in the proposition is sufficient but not necessary.

Altogether, when the production cost is intermediate, if the production or disposal impact dominates a product’s life cycle, then the total environmental impact will first increase and then decrease as the independent repair cost falls; if the use impact of the old products dominates and a product is not highly durable, then the total environmental impact will monotonically rise in response to the right-to-repair legislation. Corollary 1 below builds on Proposition 5 and summarizes the change of the total environment impact in response to the right-to-repair legislation.

**Corollary 1 (Environmental Impact).** As independent repair cost $c_I$ decreases due to the right-to-repair legislation, there exists $\tilde{c}_0 < \bar{c}$ such that

(i) if $c \geq \bar{c}$, then the total environmental impact $E$ is constant;

(ii) if $\tilde{c}_0 \leq c < \bar{c}$, then $E$ first (weakly) increases and then (weakly) decreases if $\gamma_q$ is sufficiently high; $E$ (weakly) increases if $\gamma_{u_2}$ is sufficiently high and $\delta < \frac{2f}{(1-f)^2 + 3};$

(iii) if $c < \tilde{c}_0$, then $E$ (weakly) increases.
Corollary 1 follows straightforwardly from Proposition 5 and hence, we will not repeat our explanation. Instead, we would like to point out the following. Conventional wisdom may suggest that the right-to-repair legislation, by expanding the useful life of products, reduces the production and disposal impacts. Yet, the potential price response in the product market may instead drive higher production and disposal impacts. Therefore, the lifespan of a product is not necessarily an accurate indicator of the environmental impact in each phase of the product life cycle.

5.4. Summary
As a regulatory intervention, the right-to-repair legislation is understandably no good news to the manufacturer (Proposition 2); our analysis further suggests that it may not necessarily benefit consumers or the environment, either (Proposition 4 and Corollary 1). While our analysis so far has examined how consumer surplus and the environmental impact change with a continuous reduction of the independent repair cost $c_I$ (due to the right-to-repair legislation), policymakers may be particularly interested in comparing these metrics before and after the legislation is enforced (i.e., comparing them under two specific $c_I$’s). Proposition 6 conducts such a comparison.

**Proposition 6 (Joint Effects on Consumers and the Environment).**

(i) if $c \geq \bar{c}$, then RTR has no effect on consumers and the environment;

(ii) if $c < \tilde{c}_0$, then RTR (weakly) benefits consumers but harms the environment;

(iii) if $\tilde{c}_0 \leq c < \bar{c}$, there exists $\tilde{c}_I$ such that

1. if $c_I > \tilde{c}_I$ after RTR, then RTR (weakly) benefits consumers but harms the environment;

2. if $c_I < \tilde{c}_I$ after RTR, then RTR (weakly) harms consumers; RTR (weakly) benefits the environment if $\gamma_q$ is sufficiently high; RTR (weakly) harms the environment if $\gamma_u^2$ is sufficiently high and $\delta < \frac{2f}{(1-f)^2+3}$.

Proposition 6 shows that the right-to-repair legislation can lead to a win-lose, a lose-win, or a lose-lose outcome for consumers and the environment, depending on the market conditions. When the production cost is high, the right to repair does not affect consumer surplus or the environmental impact. When the production cost is low, the right to repair is a win-lose proposition, increasing consumer surplus but also increasing the environmental impact. This can be applicable to relatively low-cost products whose environmental impact is mostly generated in the production and disposal phases, such as cellphones (Kuehr et al. 2003) and LED monitors (Bhakar et al. 2015), or those whose environmental impact is dominated by the use phase, such as microwaves (Chen et al. 2017) or similar small kitchen appliances.

When the production cost is intermediate, the right to repair can still be a win-lose proposition provided that the independent repair cost after the right-to-repair legislation is not too low. Otherwise, consumers will be worse off, and the directional change of the environmental impact
Note. \( f = 0.8, \, c_M = 0.2, \, \delta = 0.6, \, c = 0.35 \). “Win-Lose” means RTR benefits consumers but hurts the environment; “Lose-Win” means RTR hurts consumers but benefits the environment. “Lose-Lose” means RTR hurts both consumers and the environment.

potentially depends on which phase of the product life cycle is the main contributor to the environmental impact. If the production and disposal phases dominate (e.g., high-end computers; Kuehr et al. 2003), then the right to repair reduces the environmental impact, leading to a lose-win outcome. Figure 5-(a) illustrates this case. If the environmental impact is primarily generated in the use phase (e.g., automobiles, refrigerators, tractors; Cooper and Gutowski 2017, Lee et al. 2000) and product durability is not too high, then the right to repair increases the environmental impact, leading to a lose-lose outcome. Figure 5-(b) illustrates this case.

To further illustrate how to put the above result into context, consider the following example. In 2012, the Motor Vehicle Owners’ Right to Repair Act was passed in Massachusetts, but it did not cover telematics systems. In 2020, an amendment to the law in 2020 addressed this issue and required an open access data platform for cars sold in the state starting with the model year 2022 (The Verge 2020b). In light of our analysis, one may argue that the initial bill in 2012 (which can be seen as reducing the independent repair cost to a moderate level) might have benefited consumers (albeit to the detriment of the manufacturer and the environment), but the amendment in 2020 (which can be seen as further reducing the independent repair cost) might have unintentionally created a lose-lose-lose outcome for all three parties.\(^5\)

\(^5\) The above illustration is merely to help the reader contextualize our theoretical results. It is not meant to calibrate actual outcomes in practice, which is beyond the scope of our paper.
Since reducing \( c_I \) can backfire, policymakers may be interested in finding the optimal \( c_I \) that balances economic and environmental interests, i.e., finding a \( c_I \) that maximizes the following aggregate metric: \( \kappa SW - (1 - \kappa)E \), where \( SW \) is social welfare (manufacturer profit plus consumer surplus) and \( E \) is the total environmental impact, \( \kappa \in [0, 1] \) is the weight on social welfare. Our existing results can provide guidance for the selection of the optimal \( c_I \). When the production cost is low, if economic efficiency is the primary concern (i.e., if \( \kappa \) is close to 1), then \( c_I \) should be as low as possible (since a lower \( c_I \) increases social welfare); as the emphasis shifts toward the environment (i.e., as \( \kappa \) decreases), the optimal \( c_I \) increases accordingly. However, this is not always the case. When the production cost is intermediate and the environmental impact is primarily generated in the production and disposal phases, a growing focus on the environment reduces the optimal \( c_I \) because the environmental impact is minimized when \( c_I \) is minimized. In sum, while reducing \( c_I \) is hardly welfare-improving and environmentally favorable at the same time, maximizing the aggregate metric nevertheless requires \( c_I \) to be as low as possible in many cases, which points to the value of the right to repair despite the challenge of pleasing all stakeholders. Appendix E presents further numerical illustration of how the optimal \( c_I \) varies with the model parameters.

6. Extensions

In this section, we consider four extensions. Since the base model is already very complex from an analytical standpoint, further extensions are analytically intractable. Therefore, the analysis in this section will be largely numerical.

6.1. Differentiated Repair Quality

In this extension, we allow repair quality to differ between manufacturer repair and independent repair. Specifically, manufacturer repair still yields a used product from which consumer \( \theta \) derives gross utility \( \delta \theta \), but independent repair yields a used product from which consumer \( \theta \) only derives gross utility \( \mu \theta; \mu < \delta \). Our model captures a setting where independent repair produces subpar products relative to manufacturer repair. In fact, it is a reason that manufacturers often cite in their pushback against the right to repair movement (Gizmodo 2019, Forbes 2020). The subtext is that allowing consumers to perform independent repair can be socially inefficient and make consumers worse off. We will explore the validity of this argument.

To begin with, we go over the equilibrium structure of this extended model (the formal formulation of the problem is relegated to Appendix A). In general, there can be four possible consumer segments in terms of purchasing decisions. Consumers with low valuations are inactive; those with intermediately low valuations buy a used product of quality \( \mu \) every period (supplied by independent repair); those with intermediately high valuations buy a used product of quality \( \delta \) every period (supplied by manufacturer repair and used products that do not encounter failure); those with high
valuations buy a new product every period and upon product failure, choose among manufacturer repair, independent repair, and no repair. Hence, there may be two secondary markets, one for used products of quality $\mu$ and one for quality $\delta$, each with its own equilibrium used product price.

One palatable feature of this model extension is that it can allow manufacturer repair and independent repair to coexist in equilibrium (which addresses the limitation of the base model that independent repair can never arise in equilibrium). Our numerical study suggests that all our main insights from the base model continue to hold in this extension. See Appendix A for an illustration. Our findings may cast a different light on the manufacturers’ insinuation that the right to repair can hurt consumers. As in the base model, we do find instances in which consumer surplus and social welfare are lower (and in general, change non-monotonically as the independent repair cost falls). However, this inefficient outcome is less attributed to independent repair being inferior than it is to the manufacturer’s strategic price response (because as shown in the base model, even if independent repair is not inferior, consumers are still not insulated from the potential welfare loss and as shown in this extension, even if independent repair is inferior, consumer surplus can still increase as a result of the manufacturer’s price cut). Hence, while manufacturers may have a point, they may also (intentionally or inadvertently) shift the blame away from the first-order driving force.

6.2. Holding on to Functional Used Products

In the base model, consumers who buy new always resell a functional used product after one period of use instead of holding on to it for a second period. This is because we assume away transaction frictions in the resale market, following the convention of the literature. In practice, such transaction frictions can be widespread, including the time and effort consumers must expend in selling their product, the commission charged by intermediaries, and information asymmetry between buyers and sellers (which potentially leads to adverse selection in the resale market). In the extreme case, a sufficiently high transaction cost would shut down the resale market altogether. This subsection considers such an extension. In addition to assuming a sufficiently high transaction cost that precludes a secondary market (which makes the analysis cleaner, albeit still highly nontrivial), we also keep the assumption from the previous extension that independent repair yields a product with gross utility $\mu \theta$ for consumer $\theta$; $\mu < \delta$.

In this revised model, for a consumer who buys a new product, after one period of use, if the product does not fail, then she must decide whether to keep using it for a second period or throw it away and buy a new one; if the product fails, then she must decide whether to seek manufacturer repair, independent repair, or forgo repair and buy new. Note that here, repair is for self-use, contrasting the base model, in which repair is for resale. There can be up to five segments of
consumers who differ in their actions: (i) inactive; (ii) seek independent repair upon product failure and hold on to a used product otherwise; (iii) seek manufacturer repair upon product failure and hold on to a used product otherwise; (iv) buy new upon product failure and hold on to a used product otherwise; (v) buy a new product every period. In this model, both independent repair and holding on to a functional used product can arise as equilibrium outcomes.

Our numerical study suggests that our main insights from the base model continue to hold. Interestingly, we do observe instances in which the right to repair benefits both consumers and the environment in this extension. It occurs when the manufacturer responds by cutting both the new product price and repair price; in such a case, it is possible that consumer surplus and social welfare increase, and the new production volume falls. However, numerically, the reduction of the new production volume looks quite modest; additionally, in this case, the used product volume and repair volume both increase, which implies that the total environmental impact is unlikely to decrease much, if at all. See Appendix B for more details.

The modeling framework in this extension also provide us with an opportunity to (nontrivially) endogenize the independent repair price. Specifically, we consider a simultaneous pricing game between a single independent repair shop who charges independent repair price $p^I_r$ and the manufacturer who charges manufacturer repair price $p_r$ and new product price $p_n$. Further, consumers who seek independent repair must turn to the repair shop. Since independent repair and manufacturer repair differ in quality ($\mu$ vs. $\delta$) and consumers considering repair vary in their sensitivity to repair quality, the independent repair shop can potentially charge above the independent repair cost $c_I$ and turn a profit in equilibrium. Our numerical study suggests that as $c_I$ falls, so does $p^I_r$ (when independent repair is chosen by some consumers). However, a decrease in $c_I$ does not necessarily increase the profit of the independent repair shop because when $c_I$ falls below a certain point, the manufacturer may switch to a high-product-price-free-repair strategy, driving the repair shop out of business. In this case, social welfare (which now includes the repair shop’s profit besides the manufacturer’s profit and consumer surplus) will still fall (due to the high product price), consistent with the findings from the base model. Note that while a single repair shop can charge above $c_I$, as long as multiple (at least two) identical independent repair shops are present, the price competition among them will again drive the equilibrium independent repair price down to the marginal cost $c_I$.

### 6.3. Heterogeneous Independent Repair Cost

In this extension, we allow consumers to differ in their independent repair cost (since in practice, we would expect some consumers to be more adept at fixing products than others). We consider the following model: an $\omega \in [0, 1]$ fraction of consumers have an independent repair cost $c_I$, whereas an
fraction of consumers are incapable of conducting independent repair; or equivalently, their independent repair cost is infinity. The independent repair cost distribution is independent of the valuation distribution. The right to repair legislation reduces $c_I$ for the first stream of consumers but does not change the fact that the second stream of consumers do not perform independent repair (one may argue that some consumers do not have the ability or time to tinker with their own products despite improved access to repair). Note that while the right to repair would not make independent repair any easier for the second stream of consumers, they could still be affected by the legislation as the manufacturer might revise its pricing decisions. Our numerical study suggests that our main insights continue to hold and importantly, independent repair can again arise as an equilibrium outcome in this extension. We refer the reader to Appendix C for more details.

6.4. Endogenizing Durability

In this extension, the manufacturer can determine the durability of the product $\delta$. A more durable product requires a higher unit cost of production, denoted by $c(\delta) \triangleq a_0 + c_0 \delta^2$, where $a_0 \geq 0$ and $c_0 \geq 0$. Parameter $a_0$ captures the unit production cost the manufacturer would bear for producing a nondurable product, whereas coefficient $c_0$ captures the rate of cost increase due to higher durability. Such a quadratic functional form reflects diminishing returns on durability and is commonly used in the literature (e.g., Agrawal et al. 2012, 2016, Huang et al. 2019).\(^6\) Generally, in designing product durability, the manufacturer faces the tradeoff between increasing consumer valuation of new products (which calls for a more durable product) and reducing cannibalization from old products (which calls for a less durable product).

We conduct a numerical study to investigate how a lower independent repair cost $c_I$ (as a result of the right to repair legislation) influences the manufacturer’s durability choice. We find that the right to repair will have an effect on product durability only if the base production cost $a_0$ is not too low nor too high. If $a_0$ is negligible (e.g., $a_0 = 0$), then the manufacturer will practice planned obsolescence by setting $\delta^* = 0$ regardless of the independent repair cost. In this case, since new products are so cheap to produce, the manufacturer will make products nondurable so that consumers will keep buying new ones (a similar result is established in Agrawal et al. 2016). If the base production cost is high (but not too high to drive the manufacturer out of the business), then the manufacturer will adopt a “bang-bang” strategy, setting $\delta^* = 0$ if $c_0$ is high but switching to $\delta^* = 1$ if $c_0$ is low (again regardless of the independent repair cost). If both $a_0$ and $c_0$ are high, then durable products are simply too costly to make, which again prompts the manufacturer to practice planned obsolescence. On the other hand, if $a_0$ is high (which implies new products are costly to

\(^6\)Our cost specification is exactly the same as that in Agrawal et al. (2012); slightly more general than that in Agrawal et al. (2016), who consider only the quadratic term but assume away the intercept. Huang et al. (2019) also incorporate an interaction term of durability and recyclability in addition to the intercept and quadratic term.
produce) but $c_0$ is low (which implies that once a product is made, increasing durability does not add much to the production cost), the manufacturer will make the product as durable as possible (and provide free repair service) to capitalize on consumers’ enhanced valuation.

When $a_0$ is intermediate, the independent repair cost plays a part in the manufacturer’s durability choice, as illustrated by Figure 6-(c). We observe that when $c_0$ is relatively high, the manufacturer chooses relatively low durability to keep the production cost in check, and since production is not cheap, the manufacturer provides free repair to prevent the product from being scrapped prematurely. Therefore, the independent repair cost does not influence the durability choice. However, when $c_0$ is intermediate, the manufacturer reduces product durability as the independent repair cost falls; when $c_0$ is low, the manufacturer initially reduces durability and then increases it.

We further observe from Figures 6-(a) and (b) that the product prices change with the independent repair cost in exactly the same pattern, resembling the price response in the base model where durability is fixed. The rationale is indeed similar. The manufacturer responds to the reduction of the independent repair cost by first cutting the new product price, and when durability is endogenous, complements the lower price with lower durability to further limit cannibalization from used products. When the independent repair cost further declines and falls below a certain threshold, the manufacturer raises the new product price, and when durability is endogenous, complements the higher price with higher durability to further strengthen consumer valuation. Note that the latter strategy will only materialize when $c_0$ is sufficiently low, which allows the manufacturer to generally maintain high durability, making it viable to charge a high price.
As both the product prices and durability can change non-monotonically with the independent repair cost, so can consumer surplus, social welfare and the environmental impact. These findings echo with the key insights from the base model. We report more details of our numerical observation in Appendix D. One observation to note is that even in cases where product prices monotonically decrease as the independent repair cost falls (e.g., the case of $c_0 = 0.03$ in Figure 6), consumer surplus and social welfare may still decline (or, in general, change non-monotonically) because products also become less durable.

The above analysis endogenizes durability $\delta$ for a fixed failure rate $f$. One may argue that these two parameters are linked: a higher $\delta$ may be associated with a lower $f$. To capture this association, we specify that $f$ depends on $\delta$ according to the following decreasing function: $f(\delta) = e^{-\lambda \delta}$, $\lambda > 0$. Leveraging this functional form, we study another extension that jointly endogenizes $\delta$ and $f$ in Appendix D and demonstrate the robustness of our insights.

7. Conclusion

The right to repair movement calls for government legislation that requires manufacturers to provide more support and make it easier for consumers to repair their own products. As a regulatory constraint, the right to repair legislation would understandably hurt manufacturer profit, but it is less clear how manufacturers would adjust product prices (and redesign product durability) in an attempt to mitigate the (inevitable) profit loss once the bill is enacted and what the associated welfare and environmental implications are. Conventional wisdom suggests that giving consumers the right to repair benefits consumers, improves overall social efficiency (although to the detriment of manufacturers), and reduces the environmental impact. Our research challenges these intuitive predictions. We find that when the unit production cost is on the low end (e.g., cellphones, microwaves), the right to repair benefits consumers but harms the environment. This continues to be the case for an intermediate production cost provided that the post-legislation independent repair cost is not too low. Otherwise, consumers are worse off, and the environmental impact decreases if it is mostly generated in the production and disposal phases (e.g., high-end computers), but can nevertheless increase if the use impact dominates (e.g., cars, tractors, refrigerators), leading to a lose-lose-lose outcome for manufacturers, consumers, and the environment. Our results tell a cautionary tale and urge legislative authorities to factor in the inextricable link between the repair market and the product market in their assessment of the right to repair.

Next, we discuss some of our modeling assumptions and future research directions. While our life-cycle analysis of the environmental impact captures the first-order effect, it is certainly not meant to incorporate all the ramifications. For instance, one may argue that if consumers opt to buy a new phone, they will fiddle with it more, potentially generating more use impact than if they
buy a used one. This effect could be easily captured in our model by making an upward adjustment of the per-unit, per-period use impact of new products. On a different note, real-world used goods markets may include more refined segments than what our model does. One example is refurbished products, which are likely to generate a higher consumption utility than non-refurbished ones but also sold at a higher equilibrium price (Oraiopoulos et al. 2012). Refurbishment will also generate additional environmental impact.

The current analysis assumes the manufacturer is a monopolist in the product market. It approximates settings where the manufacturer has sufficient market power, which most durable goods producers do (Waldman 2003). An interesting direction for future research would be to study competing manufacturers, and specifically whether the right to repair softens or stiffens competition in the product market. Relatedly, the focus of our paper is on the right to repair’s ability to make independent repair less costly, leaving open the possibility that the legislation may have other effects that are not captured through the independent repair cost. Our paper represents the first attempt to study this topic. We hope it can inspire more future research on this emerging yet fundamental issue and more broadly on sustainable product-service systems.

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Appendix A: Differentiated Repair Quality

This section presents the model formulation and illustrative numerical results for the extension in §6.1. In general, there may exist two secondary markets, one for used products that have not failed or have failed but been recovered from manufacturer repair, and the other for used products that have failed but been recovered from independent repair. We denote the used product price in the first secondary market by $p^\delta_u$ and the second by $p^\mu_u$. For consumers of type $\theta$, the expected utility per period for those who buy a used product of quality $\mu$ every period is $\mu\theta - p^\mu_u$; that for those who buy a used product of quality $\delta$ every period is $\delta\theta - p^\delta_u$; that for those who buy a new product every period is $\theta - p^n + (1 - f)p^\delta_u + f\max\{p^\delta_u - p_r, p^\mu_u - c_I, 0\}$.

There exist $\theta_0, \theta_1, \theta_2$ satisfying $\theta_0 \leq \theta_1 \leq \theta_2$ and

$$\mu\theta_0 - p^\mu_u = 0, \quad \mu\theta_1 - p^\mu_u = \delta\theta_1 - p^\delta_u, \quad \delta\theta_2 - p^\delta_u = \theta_2 - p_n + (1 - f)p^\delta_u + f\max\{p^\delta_u - p_r, p^\mu_u - c_I, 0\}$$

such that consumers with $\theta \in [0, \theta_0)$ are inactive; consumers with $\theta \in [\theta_0, \theta_1)$ buy a used product recovered from independent repair every period; consumers with $\theta \in [\theta_1, \theta_2)$ buy a used product every period that has not failed or has failed and been recovered from manufacturer repair; consumers with $\theta \in [\theta_2, 1]$ buy a new product every period.

Let $\alpha$ and $\beta$ be the probabilities of consumers seeking manufacturer repair and independent repair, respectively. Thus, given $(p_r, p^\delta_u, p^\mu_u)$,

$$(\alpha, \beta) \in \arg\max_{(x,y) \in [0,1]^2} x(p^\delta_u - p_r) + y(p^\mu_u - c_I), \quad \text{s.t.} \quad x + y \leq 1.$$ 

In equilibrium, both secondary markets clear. The market-clearing conditions give

$$\theta_1 - \theta_0 = \beta f(1 - \theta_2), \quad \theta_2 - \theta_1 = (1 - f + \alpha f)(1 - \theta_2).$$

The manufacturer’s pricing problem is

$$\pi \triangleq \max_{p_n, p_r} (p_n - c)(1 - \theta_2) + (p_r - c_M)f(1 - \theta_2)\alpha.$$ 

The new production volume $Q$, used product volume $U^u$, total use volume $U$, and repair volume $R$ are

$$Q = 1 - \theta_2, \quad U^u = \theta_2 - \theta_0, \quad U = 1 - \theta_0, \quad R = f(1 - \theta_2)(\alpha + \beta).$$
Figure A.1  \( \mu \) model: Illustration of the equilibrium

Note. \( f = 0.8, c_M = 0.2, \delta = 0.6, \mu = 0.55. \)

Figure A.2  \( \mu \) model: Impact of RTR on product prices

Note. \( f = 0.8, c_M = 0.2, \delta = 0.6, \mu = 0.55. \)

Figure A.1 illustrates the equilibrium structure in a representative case. Notably, in this example, when both \( c_t \) and \( c \) are intermediate (the shaded area), manufacturer repair and independent repair coexist in a partial-repair equilibrium.
Figures A.2, A.3, and A.4 show how the product prices, consumer surplus/social welfare, and various components of the environmental impact, respectively, change with the independent repair cost. We observe that these trends very much resemble those in the base model (cf. Figures 2, 3 and 4), thus confirming the robustness of our main insights.
Appendix B: Holding on to Functional Used Products (FUP)

This section presents the model formulation and illustrative numerical results for the extension in §6.2.

There exist $\theta_1, \theta_2, \theta_3, \theta_4$ with $0 \leq \theta_1 \leq \theta_2 \leq \theta_3 \leq \theta_4 \leq 1$ such that: consumers with $\theta \in [0, \theta_1)$ are inactive; consumers with $\theta \in [\theta_1, \theta_2)$ seek independent repair upon product failure and hold on to a used product otherwise; consumers with $\theta \in [\theta_2, \theta_3)$ seek manufacturer repair upon product failure and hold on to a used product otherwise; consumers with $\theta \in [\theta_3, \theta_4)$ buy new upon product failure and hold on to a used product otherwise; consumers with $\theta \in [\theta_4, 1]$ buy a new product every period, regardless of whether the current product is still functional. Let $V(\theta)$ be the long-run per-period utility of consumer $\theta$. Thus,

$$V(\theta) = \begin{cases} 
\theta - p_n, & \theta \in [\theta_4, 1] \\
\theta - p_n + (1-f)\delta \theta, & \theta \in [\theta_3, \theta_4] \\
\theta - p_n + \delta \theta - f_2, & \theta \in [\theta_2, \theta_3] \\
\theta - p_n + (1-f)\delta \theta + f(\mu - c_I), & \theta \in [\theta_1, \theta_2] \\
0, & \theta \in [0, \theta_1). 
\end{cases}$$

In particular, for $\theta \in [\theta_3, \theta_4)$, the expected utility from a product is $\theta - p_n + (1-f)\delta \theta$, and the expected lifespan of a product is $2 \times (1-f) + f = 2 - f$. Hence, the long-run per-period utility $[\theta - p_n + (1-f)\delta \theta]/(2-f)$ follows from the renewal-reward theorem. A formal proof can be found in Lemma 2 of Appendix F. The manufacturer’s pricing problem is

$$\max_{p_n \geq 0, p_r \geq 0} \pi(p_n, p_r) = (p_n - c)(1 - \theta_4) + \frac{(p_n - c)(\theta_4 - \theta_3)}{2-f} + \frac{[p_n - c + (p_r - c_M)f\theta_3 - \theta_2]}{2} + \frac{(p_n - c)(\theta_2 - \theta_1)}{2}.$$

Consumer surplus is $CS = \int_{\theta_1}^{\theta_4} V(\theta)d\theta$. The new production volume $Q$, used product volume $U^u$, total use volume $U$, and repair volume $R$ are

$$Q = 1 - \theta_4 + \frac{\theta_4 - \theta_3}{2 - f} + \frac{\theta_4 - \theta_3}{2}, \quad U = 1 - \theta_1, \quad U^u = U - Q, \quad R = (\theta_3 - \theta_1)f/2.$$

Figure B.1  Holding on to FUP: Illustration of the equilibrium

Note. $f = 0.8$, $c_M = 0.2$, $\delta = 0.6$, $\mu = 0.5$. 

![Figure B.1](image_url)
Figure B.2  Holding on to FUP: Impact of RTR on the new product price, consumer surplus, and social welfare

Figure B.3  Holding on to FUP: Impact of RTR on the environment

Note. $f = 0.8$, $c_M = 0.2$, $\delta = 0.6$, $\mu = 0.5$.

Figure B.1 illustrates the equilibrium structure. Notably, in the shaded area, independent repair arises as an equilibrium. Also, in this example, consumers always hold on to a functional used product.

Figures B.2 and B.3 show how the new product price, consumer surplus/social welfare, and various components of the environmental impact change with the independent repair cost. We observe that these trends are largely consistent with those in the base model (cf. Figures 2, 3 and 4), thus demonstrating robustness.
Appendix C: Heterogeneous Independent Repair Cost

This section presents the model formulation and illustrative numerical results for the extension of heterogeneous independent repair cost studied in §6.3. We refer to consumers whose independent repair cost is infinity as type-\(H\) consumers and those with independent repair cost \(c_i\) as type-\(L\) consumers. Let \(c_H = \infty\) and \(c_L = c_i\). For consumers of type \(i \in \{L, H\}\), there exist \(\theta_1, \theta_2\) such that consumers with \(\theta \in [0, \theta_1)\) are inactive; consumers with \(\theta \in [\theta_1, \theta_2)\) buy a used product every period; consumers with \(\theta \in [\theta_2, 1]\) buy a new product every period. For consumer \(\theta \in [\theta_2, 1]\) of type \(i \in \{L, H\}\), her expected utility per period is

\[
\theta - p_n + (1 - f)p_u + f[p_u - \min\{c_i, p_i\}]^+,
\]

\(i \in \{L, H\}\).

Let \(\alpha_i^M\) and \(\alpha_i^L\) be the probability of type-\(i\) consumers seeking manufacturer repair and independent repair, respectively. Thus, for \(i \in \{L, H\}\) and given \((p_r, p_u)\),

\[
(\alpha_i^M, \alpha_i^L) \in \arg \max_{(\alpha_i^M, \alpha_i^L) \in [0, 1]^2} \alpha_i(p_u - p_r) + \alpha_2(p_u - c_i), \quad \text{s.t.} \quad \alpha_1 + \alpha_2 \leq 1.
\]

The market-clearing condition gives

\[
(1 - \omega)(\theta_2^H - \theta_1) + \omega(\theta_2^L - \theta_1) = (1 - \omega)(1 - \theta_2^H)[1 - f + f(\alpha_i^M + \alpha_i^L)] + \omega(1 - \theta_2^L)[1 - f + f(\alpha_i^M + \alpha_i^L)].
\]

The indifference conditions give

\[
\delta \theta_1 - p_u = 0;
\]

\[
\delta \theta_2 - p_u = \theta_2^L - \theta_1 + (1 - f)p_u + f[p_u - \min\{c_i, p_i\}]^+,
\]

\(i \in \{L, H\}\).

The manufacturer’s pricing problem is

\[
\pi = \max_{p_n, p_r} (p_n - c)(1 - \omega)(1 - \theta_2^H) + \omega(1 - \theta_2^L) + (p_r - c_m)\omega(1 - \omega)\alpha_i^M(1 - \theta_2^H) + \omega\alpha_i^M(1 - \theta_2^L).
\]

The new production volume \(Q\), used product volume \(U^n\), total use volume \(U\), and repair volume \(R\) are

\[
Q = (1 - \omega)(1 - \theta_2^H) + \omega(1 - \theta_2^L), \quad U^n = (1 - \omega)(\theta_2^H - \theta_1) + \omega(\theta_2^L - \theta_1), \quad U = 1 - \theta_1,
\]

\[
R = (1 - \omega)(1 - \theta_2^H)\omega(\alpha_i^M + \alpha_i^L) + \omega(1 - \theta_2^L)\omega(\alpha_i^M + \alpha_i^L).
\]

Figure C.1 illustrates the equilibrium structure. Notably, in the shaded area, independent repair arises as an equilibrium outcome.

Figures C.2, C.3 and C.4 show how the product prices, consumer surplus/social welfare, and various components of the environmental impact, respectively, change with the independent repair cost. We observe that these trends very much resemble those in the base model (cf. Figures 2, 3 and 4), thus confirming the robustness of our main insights.
Figure C.1  \( \omega \) model: Illustration of the equilibrium

Note. \( f = 0.8, c_M = 0.2, \delta = 0.6, \omega = 0.8 \).

Figure C.2  \( \omega \) model: Impact of RTR on the new and used product prices

Note. \( f = 0.8, \delta = 0.6, c_M = 0.2, \omega = 0.5 \).
Figure C.3  \( \omega \) model: Impact of RTR on consumer surplus and social welfare

\[ \begin{align*}
\text{Panel (a): } CS(c_f) & \\
& \text{The effect of RTR becomes stronger}
\end{align*} \]

Note. \( f = 0.8, \delta = 0.6, c_M = 0.2, \omega = 0.5. \)

Figure C.4  \( \omega \) model: Impact of RTR on the environment

\[ \begin{align*}
\text{Panel (a): } Q(c_f) & \\
& \text{The effect of RTR becomes stronger}
\end{align*} \]

Note. \( f = 0.8, \delta = 0.6, c_M = 0.2, \omega = 0.5. \)
Appendix D: Endogenizing Durability

This section presents illustrative numerical results for the extension of endogenous durability studied in §6.4.

Figure D.1  Endogenous $\delta$: Impact of RTR on consumer surplus and social welfare

![Figure D.1](image1)

Panel (a): $CS(c_f)$  
Panel (b): $SW(c_f)$

Note. $f = 0.8$, $c_M = 0.2$, $a_0 = 0.2$.

Figure D.2  Endogenous $\delta$: Impact of RTR on the environment

![Figure D.2](image2)

Panel (a): $Q(c_f)$  
Panel (b): $C^*(c_f)$  
Panel (c): $U(c_f)$  
Panel (d): $R(c_f)$

Note. $f = 0.8$, $c_M = 0.2$, $a_0 = 0.2$.

Figures D.1 and D.2 continue the numerical example ($f = 0.8$, $c_M = 0.2$, $a_0 = 0.2$) of Figure 6 (which shows the product prices and durability). Figure D.1 shows that consumer surplus and social welfare are generally
non-monotone in the independent repair cost, echoing the insights from the base model. As pointed out in §6.4, since durability may also vary endogenously with the reduction of the independent repair cost, a drop in product prices (e.g., the case of \( c_0 = 0.03 \) in Figures 6 and D.1) does not necessarily translate into higher consumer surplus or higher social welfare. This is because durability can also decline along with the product prices.

### D.1. Endogenizing Durability and Failure Rate

In this subsection, we assume the following functional form that links durability \( \delta \) and failure rate \( f \): \( f(\delta) = e^{-\lambda \delta} \). We then jointly endogenize \( \delta \) and \( f \). Figures D.3 through D.5 show that our insights remain robust.

**Figure D.3** Endogenize \( \delta \) and \( f \): Impact of RTR on product prices, durability, and failure probability

![Graphs showing the impact of RTR on product prices, durability, and failure probability.](image)

**Note.** \( a_0 = 0.35, \ c_M = 0.2, \ \lambda = 5 \).
Figure D.4  Endogenize $\delta$ and $f$: Impact of RTR on consumer surplus and social welfare

Panel (a): $CS(\xi)$

Panel (b): $SW(\xi)$

Note. $a_0 = 0.35$, $c_M = 0.2$, $\lambda = 5$.

Figure D.5  Endogenize $\delta$ and $f$: Impact of RTR on environment

Panel (a): $Q(\xi)$

Panel (b): $U^e(\xi)$

Panel (c): $U(\xi)$

Panel (d): $R(\xi)$

Note. $a_0 = 0.35$, $c_M = 0.2$, $\lambda = 5$. 
Appendix E: Optimal Independent Repair Cost $c_I$

In this section, we provide numerical examples for how to determine the optimal $c_I$ to maximize $\kappa SW - (1 - \kappa)E$ in the base model. We set $\kappa \in \{0.25, 0.5, 0.75\}$. We set $\gamma_r = 0.1$ and $\gamma_q \in \{0.2, 0.7\}$ and $\gamma_{u2} = 1 - \gamma_r - \gamma_q$; hence $\gamma_{u2} \in \{0.7, 0.2\}$. The results are presented in Figure E.1. In the left panels, $\gamma_q = 0.2$ and $\gamma_{u2} = 0.7$, which can be interpreted as a scenario in which the use phase dominates the environmental impact. In the right panels, $\gamma_q = 0.7$ and $\gamma_{u2} = 0.2$, which can be interpreted as a scenario in which the production and disposal phases dominate the environmental impact. From top to bottom, $\kappa$ decreases and the focus of the optimal $c_I$ shifts from maximizing social welfare to minimizing the environmental impact. Vertical lines highlight the location of the optimal $c_I$ in each case. When multiple $c_I$'s are optimal, we highlight the smallest value.

![Figure E.1](image)

**Note.** $f = 0.8$, $\delta = 0.6$, $c_M = 0.2$, $\gamma_r = 0.1$.

We make the following observations from Figure E.1.

(i) When the production cost is low ($c = 0.05$, red line), decreasing $\kappa$ increases the optimal $c_I$.

(ii) When the production cost is intermediate ($c = 0.35$, blue line), decreasing $\kappa$ increases the optimal $c_I$ if $\gamma_q$ is low (the left panels), but decreases the optimal $c_I$ if $\gamma_q$ is high (the right panels).

(iii) The lowest possible value of $c_I$ ($c_M$, 0.2 in this example) is optimal if the production cost is low and $\kappa$ is high, or if the production cost is intermediate and $\kappa$ is low.
Appendix F: Proofs

Proof of Lemma 1

From (1a), we have \( V(0) \geq \delta \theta - p_u + \rho V(0) \), which gives

\[ p_u + V(0) \geq \delta \theta + \rho V(0). \]

Note that the left hand side is the total expected discounted utility from selling the used product in state \( \bar{1} \), whereas the right hand side is the total expected discounted utility from holding on to the used product in state \( \bar{1} \). The inequality implies that holding on to the used product is a dominated strategy. Hence, from (1b), we have

\[ V(\bar{1}) = p_u + V(0). \]

Plugging it into (1a) and (1c) gives

\[
\begin{align*}
V(0) &= \max \{ \rho V(0), \delta \theta - p_u + \rho V(0), \theta - p_u + \rho[(1 - f)(p_u + V(0)) + f V(\bar{1})] \}. \\
V(\bar{1}) &= \max \{ V(0), -\min\{c_1, p_r\} + p_u + V(0) \}.
\end{align*}
\]

Thus, \( V(\bar{1}) = V(0) + (p_u - \min\{c_1, p_r\})^+ \). Plugging it into (F.1) gives

\[ V(0) = \max \{ \rho V(0), \delta \theta - p_u + \rho V(0), \delta \theta - p_u + \rho[(1 - f)p_u + V(0) + f(p_u - \min\{c_1, p_r\})^+] \}. \]

- Letting \( V(0) = \rho V(0) \) gives zero utility \( V(0) = 0 \). This is the utility from staying inactive.
- Letting \( V(0) = (\delta \theta - p_u + \rho V(0) \) gives \( V(0) = (\delta \theta - p_u)/(1 - \rho) \). This is the total discounted utility from buying a used product every period.
- Letting \( V(0) = \theta - p_u + \rho[(1 - f)p_u + V(0) + f(p_u - \min\{c_1, p_r\})^+] \) gives

\[ V(0) = \frac{\theta - p_u + \rho[(1 - f)p_u + f(p_u - \min\{c_1, p_r\})^+]}{1 - \rho}. \]

This is the total expected discounted utility form buying a new product every period. Therefore, there exist \( \theta_1, \theta_2 \) satisfying \( \theta_1 \leq \theta_2 \)

\[ \delta \theta_1 - p_u = 0, \quad \theta_2 - p_u + \rho[(1 - f)p_u + f(p_u - \min\{c_1, p_r\})^+] = \delta \theta_2 - p_u \]

such that consumers with \( \theta \in [0, \theta_1) \) are inactive; consumers with \( \theta \in [\theta_1, \theta_2) \) purchase a used product every period; consumers with \( \theta \in [\theta_2, 1] \) purchase a new product every period. (\( \theta_1 \leq \theta_2 \) will be guaranteed by the market-clearing condition.)

Note that for \( \theta > \theta_2 \), \( V(0) > (\delta \theta - p_u)/(1 - \rho) \), which is equivalent to \( p_u + V(0) > \delta \theta + \rho V(0) \). It means that consumers with \( \theta > \theta_2 \) strictly prefer selling their (functional) used product after one period of use to holding on to it for a second period. Only consumers with \( \theta_2 \) are indifferent between holding on to a used product and selling it, but this is an infinitesimal singleton that does not affect the equilibrium outcome. \( \square \)

Proof of Proposition 1

We solve the manufacturer’s profit maximization problem for general \( c_I \geq 0, \ c_M \geq 0, \ c \geq 0, \ f \in [0,1] \) and \( \delta \in [0,1] \), which incorporates Assumption 1 as a special case. The manufacturer needs to
consider six sub optimization problems in general (which can be reduced to five problems eventually) and choose the price \((p^*_n, p^*_r)\) that generates the maximum profit among all of the sub optimization problems.

The structure of the proof is the following:

(a) formulate all of the sub optimization problems;

(b) solve each sub optimization problems and compare the optimal value of the objective functions of these sub optimization problems and select the one with the maximum value.

The six sub optimization problems are organized as the following: (i) \(p_r > c_t\), in which case, no consumer seeks manufacturer repair; (ii) \(p_r \leq c_t\), in which case, no consumer performs independent repair. Note that (i) and (ii) can be further divided into three sub categories: (a) \(p_u < \min\{c_t, p_r\}\), in which case, no consumer repairs their failed products; (b) \(p_u > \min\{c_t, p_r\}\), in which case, all consumers who encounter a product failure repair their products; (c) \(p_u = \min\{c_t, p_r\}\), in which case, only a fraction of consumers who encounter a product failure repair their products.

**Step 1. Formulating the sub optimization problems.**

We now formulate the six optimization problems. The global optimal solution is the \((p^*_n, p^*_r)\) that yields the maximum profit among the six maximization problems.

**Problem 1: Assume \(p_r > c_t\) and \(p_u < c_t\).** This implies that no consumer repairs their failed product and hence the manufacturer abandons the repair market. The per-period utility for consumer \(\theta\) in segment \((\theta_2, 1)\) is \(\theta - p_u + p_u(1 - f)\). The per-period utility for consumers in segment \((\theta_1, \theta_2)\) is \(\delta \theta - p_u\). The cutoff values \(\theta_1\) and \(\theta_2\) must satisfy \(\theta_2 - p_u + p_u(1 - f) = \delta \theta_2 - p_u\) and \(\delta \theta_1 - p_u = 0\); the market-clearing condition gives \(\theta_2 - \theta_1 = (1 - \theta_2)(1 - f)\). Hence, \(p_u = \frac{\delta (\theta_2) - (1 - \theta_2)(1 - f)}{(1 - f)(3 - f) \delta + 1}\), \(\theta_1 = \frac{p_u(2 - f) - (1 - \theta_2)(1 - f)}{(1 - f)(3 - f) \delta + 1}\), \(\theta_2 = \frac{(1 - f)(2 - f) \delta + p_u}{(1 - f)(3 - f) \delta + 1}\).

It is clear that \(\frac{(1 - f)(1 - \delta)}{2 - f} < 1 + \delta (1 - f)\). It is easy to verify that \(\frac{(1 - f)(1 - \delta)}{2 - f} < \frac{c_t(1 - f) \delta (3 - f) + 1 + \delta (1 - \delta)(1 - f)}{\delta (2 - f)}\) if \(\frac{(1 - f)(1 - \delta)}{2 - f} \leq p_u < 1 + \delta (1 - f)\); when \(c_t \leq \delta\), we have \(\frac{(1 - f)(1 - \delta)}{2 - f} \leq p_u < 1 + \delta (1 - f)\); when \(c_t > \delta\), we have \(\frac{(1 - f)(1 - \delta)}{2 - f} \leq p_u < \frac{c_t(1 - f) \delta (3 - f) + 1 + \delta (1 - \delta)(1 - f)}{\delta (2 - f)}\). When \(p_u < \frac{(1 - f)(1 - \delta)}{2 - f}\), the supply of used products exceeds the demand of used products, which results in \(p_u = 0\).

In this case, we have \(\theta_2 = \frac{p_u}{1 - \delta}\). Therefore, Problem 1 is:

\[
\begin{align*}
\max_{p_u:p_r} & \quad \pi_1(p_u) = (p_u - c)^+ [1 - \theta_2(p_u)] \\
\text{s.t.} & \quad \theta_2(p_u) = \begin{cases} 
\frac{p_u}{1 - \delta}, & \text{if } p_u < \frac{(1 - f)(1 - \delta)}{2 - f}, \\
\frac{(1 - f)(2 - f) \delta + p_u}{1 + \delta (1 - f)(1 - f)}, & \text{if } p_u \geq \frac{(1 - f)(1 - \delta)}{2 - f}
\end{cases}, \\
& \quad c \leq p_u < \min \left\{ 1 + \delta (1 - f), \frac{c_t(1 - f) \delta (3 - f) + 1 + \delta (1 - \delta)(1 - f)}{\delta (2 - f)} \right\} \\
& \quad p_r > c_t.
\end{align*}
\]
Problem 2: Assume \( p_r > c_I \) and \( p_u = c_I \). This implies no consumer seeks repair from the manufacturer. When the product fails, \( \alpha \) fraction of these consumers perform independent repair and \( 1 - \alpha \) fraction of these consumers scrap the product. We have

\[
\begin{align*}
\begin{cases}
p_u = c_I, \\
p_\theta - p_\theta = (1 - p_\theta)(1 - f) + \alpha f (1 - p_\theta), \\
\delta \theta_I - p_\theta = 0, \\
\theta_2 - p_n + p_n (1 - f) = \delta \theta_2 - p_n
\end{cases}
\end{align*}
\]

\[
\begin{align*}
\begin{cases}
\theta_1 < 1 & \iff c_I < \delta, \\
\theta_2 < \theta_1 & \iff p_n > c_I (1/\delta + 1 - f), \\
\theta_2 < 1 & \iff p_n < 1 - \delta + c_I (2 - f), \\
\alpha \in [0, 1] & \iff c_I (1 - f) \delta (3 - f) + 1 + f (1 - \delta) (1 - f) \leq p_n \leq (3\delta - 2\delta f + 1)c_I - \delta^2 + \delta.
\end{cases}
\end{align*}
\]

Note that a necessary condition to ensure the feasibility of Problem 2 is \( c_I < \delta \). When it is feasible, \( p_n > c_I (1/\delta + 1 - f) \) is implied by \( p_n \geq c_I (1 - f) (1 + (2 - f) \delta (3 - f) + 1 + f (1 - \delta) (1 - f)) \), since \( c_I (1 - f) (1 + (2 - f) \delta (3 - f) + 1 + f (1 - \delta) (1 - f)) > c_I (1/\delta + 1 - f) \iff c_I < \delta \). Also, note that \( p_n < 1 - \delta + c_I (2 - f) \) is implied by \( p_n \leq (3\delta - 2\delta f + 1)c_I - \delta^2 + \delta \), since \( (3\delta - 2\delta f + 1)c_I - \delta^2 + \delta < 1 - \delta + c_I (2 - f) \iff c_I < \delta \). When the Problem 2 is feasible, we have

\[
\begin{align*}
\max_{p_n: p_r, p_u = c_I} \pi_2(p_n) &= (p_n - c)^+ [1 - \theta_2(p_n)] \\
\text{s.t. } \theta_2(p_n) &= \frac{p_n - c_I (2 - f)}{1 - \delta} \\
&\text{max} \left\{ \frac{c_I [(1 - f) \delta (3 - f) + 1 + f (1 - \delta) (1 - f)]}{\delta (2 - f)} \right\} \leq p_n \leq \frac{(3\delta - 2\delta f + 1)c_I - \delta^2 + \delta}{2\delta},
\end{align*}
\]

\[ (F.4) \]

Problem 3: Assume \( p_r > c_I \) and \( p_u > c_I \). This implies consumers perform independent repair when they encounter a product failure and hence, the manufacturer abandons the repair market. The per-period utility for consumer \( \theta \) in segment \([\theta_2, 1]\) is \( \theta - p_n + p_n (1 - f) + (p_u - c_I) f \). The per-period utility for consumer \( \theta \) in segment \([\theta_1, \theta_2]\) is \( \delta \theta - p_n \). The cutoff values \( \theta_1 \) and \( \theta_2 \) must satisfy \( \theta_2 - \theta_1 = 1 - \theta_2 \). Therefore, \( p_n = \frac{(3\delta - 2\delta f + 1)c_I - \delta^2 + \delta}{2\delta} \), \( \theta_1 = \frac{\delta + 2p_n + 2f c_I - 1}{1 + 3\delta} \), \( \theta_2 = \frac{\delta + 2p_n + f c_I}{1 + 3\delta} \).

\[
\begin{align*}
\begin{cases}
p_u > c_I & \iff p_n > \frac{(3\delta - 2\delta f + 1)c_I - \delta^2 + \delta}{2\delta}, \\
\theta_2 > \theta_1 & \iff p_n < 1 + \delta - f c_I \iff \theta_2 < 1, \\
\end{cases}
\end{align*}
\]

\[
\Rightarrow \frac{(3\delta - 2\delta f + 1)c_I - \delta^2 + \delta}{2\delta} < p_n < 1 + \delta - f c_I.
\]

When it is feasible, we have

\[
\begin{align*}
\max_{p_n: p_r, p_u > c_I} \pi_3(p_n) &= (p_n - c)^+ [1 - \theta_2(p_n, p_r)] \\
\text{s.t. } \theta_2(p_n, p_r) &= \frac{2\delta + p_n + f c_I}{1 + 3\delta} \\
&\text{max} \left\{ \frac{c_I [(3\delta - 2\delta f + 1)c_I - \delta^2 + \delta]}{2\delta} \right\} \leq p_n < 1 + \delta - f c_I
\end{align*}
\]

\[ (F.5) \]

Problem 4: Assume \( p_r \leq c_I \) and \( p_u > p_r \). This implies consumers seek repair from the manufacturer when they encounter a product failure. The per-period utility for consumer \( \theta \) in segment \([\theta_2, 1]\) is \( \theta - p_n + p_n (1 - f) + (p_u - p_r) f \). The per-period utility for consumer \( \theta \) in segment \([\theta_1, \theta_2]\) is \( \delta \theta - p_n \). The cutoff values
\( \theta_1 \) and \( \theta_2 \) must satisfy \( \theta_2 - p_n + p_u(1 - f) + (p_u - p_r)f = \delta \theta_2 - p_u \) and \( \delta \theta_1 - p_u = 0 \); The market-clearing condition gives \( \theta_2 - \theta_1 = 1 - \theta_2 \). Therefore, \( p_u = \frac{2 \theta_2 + 2 p_r - 1}{1 + 3 \delta}, \theta_1 = \frac{\delta + 2 p_u - 1}{1 + \delta}, \theta_2 = \frac{2 \theta_2 + p_u + p_r - 1}{1 + 3 \delta} \).

\[
\begin{cases}
\theta_1 \geq 0 & \iff p_r \geq 0 \iff p_r + f p_r \geq (1 - \delta)/2, \\
\theta_2 \geq \theta_1 & \iff \theta_2 \leq 1 \iff p_r + f p_r \leq 1 + \delta, \\
 p_n > p_r & \iff p_r < \frac{\delta (\delta + 2 p_u - 1)}{1 + \delta + 3 \delta (1 - f)}.
\end{cases}
\]

\[
\Rightarrow \max \left\{ 0, \frac{(1 - \delta)/2 - p_n}{f} \right\} \leq p_r < \min \left\{ \frac{1 + \delta - p_n}{f}, \frac{\delta (\delta + 2 p_u - 1)}{1 + \delta + 2 \delta (1 - f)} \right\}
\]

which implies \( p_r \leq \delta \). Note that if \( c_l \geq \delta \), then the condition \( p_r \leq c_l \) holds trivially. The manufacturer’s optimization problem is thus

\[
\max_{p_n, p_r} \pi_4(p_n, p_r) = (p_n - c)[1 - \theta_2(p_n, p_r)] + (p_r - c_M)[1 - \theta_2(p_n, p_r)]f
\]

s.t. \( \theta_2(p_n, p_r) = \frac{2 \delta + p_n + f p_r}{1 + 3 \delta} \), \( 0 \leq p_r \leq \min \left\{ c_l, \frac{1 + \delta - p_n}{f}, \frac{\delta (\delta + 2 p_u - 1)}{1 + \delta + 2 \delta (1 - f)} \right\} \), \( (1 - \delta)/2 \leq p_n \leq 1 + \delta \).

**Problem 5: Assume \( p_r \leq c_l \) and \( p_n = p_r \). This implies no consumer performs independent repair. When the product fails, \( \alpha \) fraction of these consumers seek repair from the manufacturer and \( 1 - \alpha \) fraction of these consumers scrap the product. The per-period utility for consumers in segment \((\theta_2, 1)\) is \( \theta - p_n + p_u(1 - f) \).**

The per-period utility for consumers in segment \((\theta_1, \theta_2)\) is \( \delta \theta - p_u \). We have

\[
\begin{cases}
\theta_1 = p_r/\delta, \\
\theta_2 - p_n + p_u(1 - f) = \delta \theta_2 - p_u, \\
\delta \theta_1 - p_u = 0, \\
\theta_2 - \theta_1 = (1 - \theta_2)(1 - f) + \alpha f (1 - \theta_2).
\end{cases}
\]

\[
\Rightarrow \begin{cases}
\theta_1 < 1 & \iff p_r < \delta, \\
\theta_2 < 1 & \iff p_r < \frac{\delta p_u}{1 + \delta (1 - f)}, \\
\alpha \in [0, 1] & \iff \frac{\delta (2 p_n + 2 p_r - 1)}{1 + 3 \delta - 2 f} \leq p_n \leq \frac{\delta (2 p_n - 1) (1 - f)}{1 + \delta (1 - f) (3 - f)}.
\end{cases}
\]

Note that \( \frac{\delta p_n}{1 + \delta (1 - f)} < p_r < \frac{\delta p_n}{1 + \delta (1 - f)} \) is implied by \( \frac{\delta (2 p_n + 2 p_r - 1)}{1 + 3 \delta - 2 f} \leq p_n \leq \frac{\delta (2 p_n - 1) (1 - f)}{1 + \delta (1 - f) (3 - f)} \), since

\[
\left\{ \begin{array}{l}
\frac{\delta p_n}{1 + \delta (1 - f)} > \frac{\delta (2 p_n - 1) (1 - f)}{1 + \delta (1 - f) (3 - f)} \\
\frac{\delta (2 p_n + 2 p_r - 1)}{1 + 3 \delta - 2 f} > \frac{\delta p_n}{1 + \delta (1 - f)}
\end{array} \right. \Rightarrow \begin{array}{l}
(1 - \delta)(1 - f)[1 + \delta (1 - f) - p_n] > 0, \quad \iff p_n < 1 + \delta (1 - f), \\
(1 - \delta)[1 + \delta (1 - f) - p_n] > 0, \quad \iff p_n < 1 + \delta (1 - f),
\end{array}
\]

Note that if \( c_l \geq \delta \), then the condition \( p_r \leq c_l \) holds trivially. When \( c_l < \delta \), \( 0 \leq p_r \leq c_l \) together with \( \frac{\delta (2 p_n + 2 p_r - 1)}{1 + 3 \delta - 2 f} \leq p_r \leq \frac{\delta (2 p_n - 1) (1 - f)}{1 + \delta (1 - f) (3 - f)} \) implies that

\[
\frac{(1 - f)(1 - \delta)}{2 - f} \leq p_n < \frac{(3 \delta - 2 \delta f + 1) c_l - \delta^2 + \delta}{2 \delta}
\]

Note that

\[
1 + \delta (1 - f) - \frac{(3 \delta - 2 \delta f + 1) c_l - \delta^2 + \delta}{2 \delta} = \frac{(\delta - c_l)(1 + 3 \delta - 2 \delta f)}{2 \delta}.
\]
The manufacturer’s optimization problem is thus

\[
\max_{p_n, p_r} \pi_5(p_n, p_r) = (p_n - c)[1 - \theta_2(p_n, p_r)] + (p_r - c_M)[1 - \theta_2(p_n, p_r)]f \alpha
\]

s.t. \[
\theta_2(p_n, p_r) = \frac{p_n - p_r - p_r(1 - f)}{1 - \delta}
\]

\[
\alpha = \frac{(\delta f^2 - 4\delta f + 3\delta + 1)p_r - \delta(2 - f)p_n + \delta(1 - \delta)(1 - f)}{(1 - f)(1 - \delta)}
\]

\[
\frac{1 - f}{2 - f} \leq p_n < \min \left\{ 1 + \delta(1 - f), \frac{3\delta - 2\delta f + 1}{2\delta} \right\}
\]

\[
\frac{\delta(\delta + 2p_r - 1)}{1 + 2\delta(1 - f)} \leq p_r \leq \min \left\{ c_l, \frac{\delta[p_n(2 - f) - (1 - f)(1 - \delta)]}{(1 - f)(3 - \delta)f + 1} \right\}.
\]

**Problem 6:** Assume \( p_r \leq c_l \) and \( p_n < p_r \). This implies that no consumer repairs their failed product and hence the manufacturer abandons the repair market. The per-period utility for consumer \( \theta \) in segment \((\theta_2, 1] \) is \( \theta - p_n + p_n(1 - f) \). The per-period utility for consumer \( \theta \) in segment \((\theta_1, \theta_2] \) is \( \delta \theta - p_n \). The cutoff values \( \theta_1 \) and \( \theta_2 \) must satisfy \( \theta_2 - p_n + p_n(1 - f) = \delta \theta_2 - p_n \) and \( \delta \theta_1 - p_n = 0 \); the market-clearing condition gives \( \theta_2 - \theta_1 = (1 - \theta_2)(1 - f) \). Therefore, \( p_n = \frac{\delta p_n(2 - f) - (1 - \delta)(1 - \delta)}{(1 - f)(1 + 2\delta)(1 - f) + 1}, \theta_1 = \frac{p_n(n - (1 - \delta)(1 - \delta))}{(1 - f)(1 + 2\delta)(1 - f) + 1}, \delta \theta_2 = \frac{(1 - f)[1 + (1 - f)]p_n}{(1 - f)(1 + 2\delta)(1 - f) + 1}. \)

\[
\begin{align*}
\begin{cases}
\theta_1 \geq 0 & \iff p_n \geq 0 \iff p_n \geq \frac{(1 - f)(1 - \delta)}{2 - f}, \\
\theta_1 < \theta_2 & \iff p_n < \frac{1 + \delta(1 - f)}{2 - f} \iff p_n < 1 + \delta(1 - f).
\end{cases}
\end{align*}
\]

It is clear that \( \frac{(1 - f)(1 - \delta)}{2 - f} < 1 < 1 + \delta(1 - f) \). Note that when \( p_n \geq \frac{(1 - f)(1 - \delta)}{2 - f} \), \( c_l \geq p_r > p_n \) becomes \( c_l \geq p_r > \frac{\delta p_n(2 - f) - (1 - f)(1 - \delta)}{(1 - f)(1 + 2\delta)(1 - f) + 1} \), which implies \( p_n < \frac{c_l[1 - \theta_2](3 - f) + 1 + \delta(1 - \delta)(1 - f)}{\delta(2 - f)} \). Note that

\[
\frac{c_l[1 - \theta_2](3 - f) + 1 + \delta(1 - \delta)(1 - f)}{\delta(2 - f)} \geq 1 + \delta(1 - f) \iff c_l \geq \delta.
\]

The manufacturer’s optimization problem is thus

\[
\max_{p_n, p_r} \pi_6(p_n) = (p_n - c)[1 - \theta_2(p_n)]
\]

s.t. \[
\theta_2(p_n) = \frac{(1 - f)(2 - f)\delta + p_n}{1 + \delta(1 - f)(3 - f)}
\]

\[
c \leq p_n < \min \left\{ 1 + \delta(1 - f), \frac{c_l[1 - \theta_2](3 - f) + 1 + \delta(1 - \delta)(1 - f)}{\delta(2 - f)} \right\}
\]

\[
\max \left\{ 0, \frac{\delta[p_n(2 - f) - (1 - f)(1 - \delta)]}{(1 - f)(1 + 2\delta)(1 - f) + 1} \right\} < p_r \leq c_l.
\]

Note that Problem 1 and 6 can be combined, because their objective functions are the same and their feasible regions are continuous. With a slight abuse of notation, we combine Problem 1 and 6 together and denote the new problem as Problem 1.

\[
\max_{p_n, p_r} \pi_1(p_n) = (p_n - c)[1 - \theta_2(p_n)]
\]

s.t. \[
\theta_2(p_n) = \begin{cases}
\frac{p_n}{1 + \delta(1 - f)(3 - f)}, & \text{if } p_n < \frac{(1 - f)(1 - \delta)}{2 - f}, \\
\frac{1 + \delta(1 - f)(3 - f)}{1 + \delta(1 - f)(3 - f)}, & \text{if } p_n \geq \frac{(1 - f)(1 - \delta)}{2 - f},
\end{cases}
\]

\[
c \leq p_n < \min \left\{ 1 + \delta(1 - f), \frac{c_l[1 - \theta_2](3 - f) + 1 + \delta(1 - \delta)(1 - f)}{\delta(2 - f)} \right\}
\]

\[
p_r > \left[ \frac{\delta[p_n(2 - f) - (1 - f)(1 - \delta)]}{1 + \delta(1 - f)(3 - f)} \right]^{+}.
\]
In addition, note that when \( p_n \geq 1 + \delta - fc_I \) and \( p_r \geq (1 + \delta - p_n)^+ / f \), the manufacturer has profit zero. This is because the expected maximum utility that a consumer with \( \theta = 1 \) can obtain is

\[
(1 + \max\{\delta, p_u\})(1 - f) + f(1 + [p_u - \min\{p_r, c_I\}]^+) = (1 + \delta)(1 - f) + f(1 + [p_u - \min\{p_r, c_I\}]^+),
\]

since otherwise, no one will buy used product. And we have

\[
\begin{aligned}
\begin{cases}
p_n \geq 1 + \delta - fc_I, \\
p_r \geq (1 + \delta - p_n)^+ / f,
\end{cases}
\Rightarrow 1 + \delta \geq p_n \geq (1 + \delta)(1 - f) + f(1 + \delta - \min\{p_r, c_I\}).
\end{aligned}
\]

It is clearly that \( \min\{p_r, c_I\} \leq \delta \) and \( \delta - \min\{p_r, c_I\} \geq p_n - \min\{p_r, c_I\} \). Thus, \( p_n \) is no less than than the maximum willingness to pay of the consumer with \( \theta = 1 \). Therefore, for any given \( (p_n, p_r) \in \mathbb{R}^2_+ \), it is either the case that the manufacturer obtains zero profit or there is unique equilibrium in the secondary market.

**Step 2. Solving the sub optimization problems and comparing the optimal values of their objective functions.**

**Case 1:** \( c_I \geq \delta \). In this case, we only need to consider Problem 1, 4 and 5, as Problem 2 and 3 are infeasible.

- **For Problem 1, (F.9),** one can verify that the optimal solution is

\[
(p_{n_1}, p_{r_1}, \pi_1) = \begin{cases}
\text{unprofitable}, & \text{if } c \geq 1 + \delta(1 - f), \\
\left( \frac{c + \delta + (1 - f)}{2}, + \infty, \frac{(c - \delta + (1 - f))^2}{4(\delta^2 - 4\delta f + 3\delta + 1)} \right), & \text{if } 0 \leq c < 1 + \delta(1 - f).
\end{cases}
\]

- **For Problem (4), (F.6),** to ensure \( \pi_4(p_n, p_r) \geq 0 \), we also need \( p_n - c + f(p_r - c_M) \geq 0 \), or equivalently, \( p_n + fp_r \geq c + fc_M \). Combining \( p_n + fp_r \geq c + fc_M \) with the constraints of (F.6), one can verify that

\[
\pi_4(p_n, p_r) > 0 \text{ if and only if } c + fc_M < 1 + \delta.
\]

It is easy to verify that the optimal solution of the unconstrained (F.6) is \((p_{n_4}, p_{r_4})\) such that \( p_{n_4} + fp_{r_4} = (c + \delta + c_M f + 1)/2 \). Note that

\[
\begin{cases}
(c + \delta + c_M f + 1)/2 < 1 + \delta \Leftrightarrow c + c_M f < 1 + \delta, \\
(c + \delta + c_M f + 1)/2 > (1 - \delta)/2 \text{ is obviously true.}
\end{cases}
\]

Therefore, the constraints of (F.6) are inactive when \( c + fc_M < 1 + \delta \). Hence, the optimal solution of Problem 4, (F.6), is

\[
(p_{n_4}, p_{r_4}, \pi_4) = \begin{cases}
\text{unprofitable,} & \text{if } c \geq 1 + \delta - fc_M, \\
p_{n_4} + fp_{r_4} = (c + \delta + c_M f + 1)/2, & \pi_4(p_{n_4}, p_{r_4}) = \frac{(c - \delta + c_M f + 1)^2}{4(1 - \delta)} , & \text{if } 0 \leq c < 1 + \delta - fc_M.
\end{cases}
\]

- **For Problem 5, (F.7),** the optimal solution of the unconstrained (F.7) is given by

\[
\begin{aligned}
\frac{\partial \pi_5(p_n, p_r)}{\partial p_n} = 0, \quad & \Rightarrow \quad p_n = [c + \delta(1 - f) + 1]/2, \\
\frac{\partial \pi_5(p_n, p_r)}{\partial p_r} = 0. \quad & \Rightarrow \quad p_r = (\delta + c_M)/2.
\end{aligned}
\]

The second partial derivative test implies that \( \pi_5(p_n, p_r) \) is jointly concave, i.e.,

\[
\frac{\partial^2 \pi_5(p_n, p_r)}{\partial p_n^2} = 0, \quad \frac{\partial^2 \pi_5(p_n, p_r)}{\partial p_r^2} = 0, \quad \left( \frac{\partial^2 \pi_5(p_n, p_r)}{\partial p_n \partial p_r} \right)^2 = \frac{4}{\delta(1 - \delta)} > 0 \quad \text{and} \quad \frac{\partial^2 \pi_5(p_n, p_r)}{\partial p_n^2} = -\frac{2}{1 - \delta} < 0.
\]

Hence, \((p_{n_5}, p_{r_5})\) is the unique global maximum solution of the unconstrained (F.7).

Note that if the optimal solution of (F.7) is not an interior solution, then it is clearly dominated by \( \max\{\pi_1(p_{n_1}, p_{r_1}), \pi_4(p_{n_4}, p_{r_4})\} \), since one can verify that the objective functions of Problem 1, 4 and 5 are continuous at the boundaries.
It can be verified that if \((p_{n_5}, p_{r_5})\) is an interior solution of (F.7), then
\[
\begin{align*}
\pi_5(p_{n_5}, p_{r_5}) - \pi_4(p_{n_4}, p_{r_4}) &= \frac{c_m + \delta - 2\delta + 3c_m\delta - \delta^2 - 2c_m\delta f}{4(1 - \delta)(1 + 3\delta)} \geq 0, \\
\pi_5(p_{n_5}, p_{r_5}) - \pi_1(p_{n_1}, p_{r_1}) &= \frac{c_m + \delta - 2\delta + 3c_m\delta - \delta^2 - 2c_m\delta f}{4(1 - \delta)[1 + (1 - f)(3 - f)]} \geq 0.
\end{align*}
\]

(H.12)

Hence, if \((p_{n_5}, p_{r_5})\) is an interior solution of (F.7), then the manufacturer will choose \((p_{n_5}, p_{r_5})\) to maximize its profit. If \((p_{n_5}, p_{r_5})\) is not an interior solution of (F.7), then the manufacturer will choose between \((p_{n_1}, p_{r_1})\) and \((p_{n_4}, p_{r_4})\) to maximize its profit.

The comparison between \(\pi_1(p_{n_1}, p_{r_1})\) and \(\pi_4(p_{n_4}, p_{r_4})\) yields:

When \(0 \leq c_M < \delta\), max \{\(\pi_1(p_{n_1}, p_{r_1})\), \(\pi_4(p_{n_4}, p_{r_4})\)\} yields:
\[
\begin{align*}
\pi_4(p_{n_4}, p_{r_4}) &= \frac{(c - \delta + c_M f - 1)^2}{4(1 + 3\delta)}, \\
\pi_1(p_{n_1}, p_{r_1}) &= \frac{(c - \delta + c_M f - 1)^2}{4(\delta^2 - 4f + 3\delta + 1)}.
\end{align*}
\]

When \(c_M \geq \delta\),
\[
\begin{align*}
\max \{\pi_1(p_{n_1}, p_{r_1}), \pi_4(p_{n_4}, p_{r_4})\} &= \left\{
\begin{array}{ll}
\text{unprofitable,} & \text{if } c \geq 1 + \delta - c_M f, \\
\pi_1(p_{n_1}, p_{r_1}) &= \frac{(c - \delta + c_M f - 1)^2}{4(\delta^2 - 4f + 3\delta + 1)}, & \text{if } 0 \leq c < 1 + \delta - c_M f,
\end{array}
\right.
\end{align*}
\]

Note that when \(0 \leq c_M < \delta\), we have \(1 + \delta(1 - f) < 1 + \delta - c_M f\) and \(\frac{(1 - c_c + c_M f)^2}{4(1 + 3\delta)} > \frac{(1 - c_c + \delta f)^2}{4(\delta^2 - 4f + 3\delta + 1)} \iff c > 1 + \delta - c_M f - \frac{(4 - c_M f)}{1 - \sqrt{1 - 4f + 3\delta}}\). Further, \(1 + \delta - c_M f - \frac{(4 - c_M f)}{1 - \sqrt{1 - 4f + 3\delta}} < 1 + \delta(1 - f) \iff f(\delta - c_M) \sqrt{1 - 2f(4 - f)} > 0\), since \(c_M < \delta\) and \(\frac{1 - \delta}{1 + 3\delta} < 1\), which is equivalent to \(\delta(1 - f)(3 - f) + 1 > 0\). When \(c_M \geq \delta\), we have \(1 + \delta(1 - f) \geq 1 + \delta - c_M f\) and \(\frac{(1 - c_c + c_M f)^2}{4(1 + 3\delta)} < \frac{(1 - c_c + \delta f)^2}{4(\delta^2 - 4f + 3\delta + 1)} \iff \frac{(1 - \delta + c_M f)^2}{4(\delta^2 - 4f + 3\delta + 1)} < \frac{(1 - \delta + \delta f)^2}{4(\delta^2 - 4f + 3\delta + 1)}\), which is obviously true. This leads to (F.13).

To combine \(\pi(p_{n_5}, p_{r_5})\) with (F.13), we derive the sufficient and necessary conditions for \((p_{n_5}, p_{r_5})\) being an interior solution of (F.7):

\[
\begin{align*}
p_{n_5} &\geq \frac{(1 - f)(1 - \delta)}{2f} \quad \iff c \geq -\frac{\delta(1 - f)(4 - f) + f}{2f}, \quad \text{which is negative and hence can be ignored}, \\
p_{n_5} &< 1 + \delta(1 - f) \quad \iff c < 1 + \delta(1 - f), \\
p_{r_5} &\geq \frac{\delta(\delta + 2c - 1)}{1 + 2\delta(1 - f)} \quad \iff c \leq \frac{c_m - \delta^2 + \delta[1 + c_M(3 - 2f)]}{\delta(1 - f)}, \\
p_{r_5} &\leq \frac{\delta(\delta + 2c - 1)}{1 + 2\delta(1 - f)}(1 - f)(1 - \delta) \quad \iff c \geq \frac{\delta(1 - f)(3 - f) + c_M + \delta(1 - \delta)(1 - f)}{\delta(1 - f)} \leq c < \min \left\{1 + \delta(1 - f), \frac{c_M - \delta^2 + \delta[1 + c_M(3 - 2f)]}{2\delta} \right\}.
\end{align*}
\]

Note that \(\frac{c_m - \delta^2 + \delta[1 + c_M(3 - 2f)]}{2\delta} \geq 1 + \delta(1 - f) \iff (c_M - \delta)[\delta(3 - 2f)] + 1 > 0 \iff c_M \geq \delta\). Hence, when \(c_M \geq \delta\), for \((p_{n_5}, p_{r_5})\) being the interior solution of (F.7), it requires \(\frac{\delta(1 - f)(3 - f) + 1 + c_M + \delta(1 - \delta)(1 - f)}{\delta(1 - f)} \leq c < 1 + \delta(1 - f)\). Further, \(1 + \delta(1 - f) > \frac{\delta(1 - f)(3 - f) + 1 + c_M + \delta(1 - \delta)(1 - f)}{\delta(1 - f)} \iff (c_M - \delta)[\delta(1 - f)(3 - f) + 1] < 0 \iff c_M < \delta\).

Hence, when \(c_M \geq \delta\), \((p_{n_5}, p_{r_5})\) is not an interior solution of (F.7). When \(c_M < \delta\), \((p_{n_5}, p_{r_5})\) being the interior solution of (F.7), it requires \(c \leq \frac{\delta(1 - f)(3 - f) + 1 + c_M + \delta(1 - \delta)(1 - f)}{\delta(1 - f)} \leq c < \frac{c_m - \delta^2 + \delta[1 + c_M(3 - 2f)]}{2\delta} \equiv \bar{c}\). Note that

\[
\begin{align*}
\bar{c} &> \bar{c} \iff c_M < \delta, \\
\bar{c} &< 1 + \delta - c_M f \iff (c_M - \delta)(1 + 3\delta) < 0 \iff c_M < \delta, \\
c &< 1 + \delta - c_M f - \frac{(\delta - c_M f)}{1 - \sqrt{1 - 4f + 3\delta}} \iff \frac{(1 - \delta)(1 - f)(3 - f) + 1}{2\delta} > 0, \\
\bar{c} &< 1 + \delta - c_M f - \frac{(\delta - c_M f)}{1 - \sqrt{1 - 4f + 3\delta}} \iff f(1 - f)(4 - f) > 0.
\end{align*}
\]
Therefore, the global optimal solution of the manufacturer is summarized in Table F.1.

**Case 2: 0 ≤ c_I < δ.** In this case, we need to solve all of the five optimization problems and compare the optimal values of their objective functions.

- **For Problem 1, (F.9),** one can verify that the optimal solution is

  When 0 < c_I < \bar{c}_I,
  \[(p_{n_1}, \pi_1) = \begin{cases} \text{unprofitable,} & \text{if } c \geq c_1, \\ \left( \frac{c_I - \delta}{\delta(2 - f)} (1 - f), \left( \frac{(1 + \delta(1 - f))(1 - f)}{\delta(2 - f)} \right) \right), & \text{if } 0 < c < c_1. \end{cases} \]

  When \bar{c}_I \leq c_I < \delta,
  \[(p_{n_1}, \pi_1) = \begin{cases} \text{unprofitable,} & \text{if } c \geq c_3, \\ \left( \frac{c_I - \delta}{\delta(2 - f)} (1 - f) + \frac{\delta(1 - \delta)}{\delta(2 - f)}, \left( \frac{(1 + \delta(1 - f))(1 - f)}{\delta(2 - f)} \right) \right), & \text{if } 0 < c < c_2. \end{cases} \]

  where

  \[
  \bar{c}_I = \frac{2}{2f} \left[ 1 - \frac{1 + \delta(1 - f)}{1 + \delta(1 - f)(2 - f)} \right], \quad c_1 = \frac{c_I (1 - \delta)}{\delta(2 - f)} (1 - \delta), \quad c_2 = \frac{2}{2f} \left( \frac{c_I (1 - \delta)}{\delta(2 - f)} (1 - \delta) + \frac{\delta(1 - \delta)(1 - f)}{\delta(2 - f)} \right) - \delta(1 - f). \]

- **For Problem 2, (F.4),** one can verify that the optimal solution is

  When 0 ≤ c_I < \bar{c}_I,
  \[(p_{n_2}, \pi_2) = \begin{cases} \text{unprofitable,} & \text{if } c \geq c_3, \\ \left( \frac{3(3 - 2\delta f + 1) c_I - \delta^2 + \delta}{2}, \left( \frac{c_I (1 - \delta)}{\delta(2 - f)} (1 - \delta) + \frac{\delta(1 - \delta)(1 - f)}{\delta(2 - f)} \right) \right), & \text{if } 0 < c < \frac{1 + \delta(1 - f)}{2} c_4. \end{cases} \]

  When \bar{c}_I \leq c_I < \delta,
  \[(p_{n_2}, \pi_2) = \begin{cases} \text{unprofitable,} & \text{if } c \geq c_3, \\ \left( \frac{3(3 - 2\delta f + 1) c_I - \delta^2 + \delta}{2}, \left( \frac{c_I (1 - \delta)}{\delta(2 - f)} (1 - \delta) + \frac{\delta(1 - \delta)(1 - f)}{\delta(2 - f)} \right) \right), & \text{if } 0 < c < \frac{1 + \delta(1 - f)}{2} c_4. \end{cases} \]

  where

  \[
  \bar{c}_I = \frac{2}{2f} \left[ 1 - \frac{1 + \delta(1 - f)(2 - f)}{1 + \delta(1 - f)(2 - f)} \right], \quad c_3 = \frac{3(3 - 2\delta f + 1) c_I - \delta^2 + \delta}{2f}, \quad c_4 = 1 - \delta + c_I (2 - f) - \frac{2(\delta - c_I)(1 - \delta)}{\delta(2 - f)}. \]
• For Problem 3, (F.5), one can verify that the optimal solution is

\[
\begin{align*}
(p_{n_3}, \pi_3) &= \begin{cases} 
\text{unprofitable}, & \text{if } c \geq 1 + \delta - f c_1, \\
\left(1 + c + \delta - c f_1, \frac{(c - \delta + c f_1 - 1)^2}{2}, \frac{4(1 + 3\delta)}{4(1 + 3\delta)} \right), & \text{if } 0 \leq c < 1 + \delta - f c_1.
\end{cases}
\end{align*}
\]

When \( \frac{2\delta^2}{1 + 3\delta - f} \leq c_f < \delta, \)

\[
\begin{align*}
(p_{n_3}, \pi_3) &= \begin{cases} 
\text{unprofitable}, & \text{if } c \geq 1 + \delta - f c_1, \\
\left(1 + c + \delta - c f_1, \frac{(c - \delta + c f_1 - 1)^2}{2}, \frac{4(1 + 3\delta)}{4(1 + 3\delta)} \right), & \text{if } c \geq 1 + \delta - f c_1,
\end{cases}
\end{align*}
\]

\[
\frac{c (1 + 3\delta - f) - 2\delta^2}{\delta} \leq c < 1 + \delta - f c_1,
\]

\[
\frac{(3\delta - 2f + 1)c_f - \delta^2 + \delta}{2\delta}, \frac{4(1 + 3\delta)}{4(1 + 3\delta)}, \text{ if } 0 \leq c < \frac{c (1 + 3\delta - f) - 2\delta^2}{\delta}.
\]

• For Problem 4, (F.6), based on the same analysis in Case 1, we still have

\[
\begin{align*}
(p_{n_4}, \pi_4) &= \begin{cases} 
\text{unprofitable}, & \text{if } c \geq 1 + \delta - f c_M, \\
\left(p_n + f p_{e_4} = (c + \delta + c_{M}f + 1)/2, \pi_4(p_{n_4}, p_{e_4}) = \frac{(c + 3\delta + c_{M}f - 1)^2}{4(1 + 3\delta)}, \right. & \text{if } 0 \leq c < 1 + \delta - f c_M.
\end{cases}
\end{align*}
\]

• For Problem 5, (F.7), based on the same analysis in Case 1, the optimal solution of the unconstrained problem is

\[
\begin{align*}
p_{n_5} &= [c + \delta(1 - f) + 1]/2, \\
p_{r_5} &= (\delta + c_M)/2, \\
\pi_5(p_{n_5}, p_{r_5}) &= \frac{c_{M}(1-f)-c}{2} + \frac{2c_{M}}{\delta} + \frac{(c_{M} + c_{M}f)^2}{4}.
\end{align*}
\]

By (F.12), we know that if Problem 5 has interior solution, then Problem 1 and 4 are dominated. The sufficient and necessary conditions for \((p_{n_5}, p_{r_5})\) being an interior solution of Problem 5 are

\[
\begin{align*}
p_{n_5} &\geq \frac{(1-f)(1-\delta)}{2}, \quad \Leftrightarrow \quad c \geq -\frac{(1-f)(4-f)+f}{2-f}, \quad \text{which is negative and hence can be ignored}, \\
p_{n_5} &< \frac{(3\delta - 2f + 1)c_f - \delta^2 + \delta}{2\delta}, \quad \Leftrightarrow \quad c < \frac{c_{M}(1 + 3\delta - 2f) - \delta^2 (2-f)}{\delta}, \\
p_{r_5} &> \frac{\delta + 2\delta}{2} - \frac{\delta(p_{n_5}(2-f) - (1-\delta)(1-\delta)}{2\delta}, \quad \Leftrightarrow \quad c > \frac{(1-f)(3-f) - 1+c_{M} + \delta(1-f)(1-f)}{\delta(2-f)} > 0, \\
p_{r_5} &\leq c_f, \quad \Leftrightarrow \quad (\delta + c_M)/2 \leq c_f, \\
\Rightarrow \quad &\left\{ \frac{\delta (1-f)(3-f) + \delta(1-f)(1-f)}{2-f} \right\} < c < \frac{c_{M}(1 + 3\delta - 2f) - \delta^2 (2-f)}{\delta}, \quad \frac{c_{M} - \delta^2 + \delta + c_{M}(3-2f)}{2\delta}.
\end{align*}
\]

Note that

\[
\frac{c_{M} - \delta^2 + \delta + c_{M}(3-2f)}{2\delta} - \frac{c_{M}(1 + 3\delta - 2f) - \delta^2 (2-f)}{\delta} \propto 3\delta - 2f + 1)(c_{M} + \delta - 2c_{f}) \leq 0.
\]

Hence, Problem 5 has interior solution if and only if

\[
\left\{ \frac{\delta (1-f)(3-f) + \delta(1-f)(1-f)}{2-f} \right\} < c < \frac{c_{M}(1 + 3\delta - 2f) - \delta^2 (2-f)}{\delta}
\]

\[
\frac{c_{M} - \delta^2 + \delta + c_{M}(3-2f)}{2\delta}, \quad (\text{which is non-empty if and only if } c_{M} < \delta)
\]

One can verify that the objective functions of Problem 1, 4, and 5 are continuous at their boundaries. If \((p_{n_5}, p_{r_5})\) is on \(p_{n_5} = \frac{\delta + 2\delta_{n_5} - 1}{1 + \delta + 2\delta(1-f)}, \) then \((p_{n_5}, p_{r_5})\) is dominated by \((p_{n_1}, p_{r_1}).\) Similarly, if \((p_{n_5}, p_{r_5})\) is on \(p_{r_5} = \frac{\delta + 2\delta_{r_5} - 1}{1 + \delta + 2\delta(1-f)}, \) then \((p_{n_5}, p_{r_5})\) is dominated by \((p_{n_4}, p_{r_4}).\)

\textbf{Case 2.1: }\(\delta + c_M)/2 \leq c_f < \delta.\) In this case, we show that if \((p_{n_5}, p_{r_5})\) is an interior solution of Problem 5, then it is the manufacturer’s global optimal solution; when \((p_{n_5}, p_{r_5})\) is not an interior solution of Problem 5, the manufacturer’s global optimal solution is either \(p_{n_1}\) or \((p_{n_4}, p_{r_4}).\)
Case 2.1.1: \( \xi = \frac{\delta(1-f)(3-f)+1}{\delta(2-f)}c_M + \delta(1-\delta)(1-f) < c < \frac{c_M - \delta^2 + \delta(1+c_M)(3-2f)}{2\delta} = \bar{c} \). In this case, \((p_{n_5}, p_{r_5})\) is interior solution of Problem 5. We show \((p_{n_5}, p_{r_5})\) is the manufacturer’s global optimal solution. Based on (F.12), it suffices to show \(\pi_5(p_{n_5}, p_{r_5}) \geq \max\{\pi_2(p_{n_2}), \pi_3(p_{n_3})\} \). Note that \(\delta > c_l = (\delta + c_M)/2 \Rightarrow c_l \geq \min\{c_M, \delta/2\} \).

We first show that \(\pi_5(p_{n_5}, p_{r_5}) \geq \pi_2(p_{n_2})\). Note that

\[
\begin{align*}
1 - \delta + c_l(2 - f) - \frac{2(\delta - c_l)(1-\delta)}{\delta(2-f)} &< \frac{c_M - \delta^2 + \delta[1+c_M(3-2f)]}{2\delta} < \frac{1+\delta(1-f)}{\delta(2-f)}, \\
\frac{\delta(1-f)(3-f)+1}{\delta(2-f)}c_M + \delta(1-\delta)(1-f) &< 1 - \delta + c_l(2 - f) - \frac{2(\delta - c_l)(1-\delta)}{\delta(2-f)}.
\end{align*}
\]

This is because

\[
\begin{align*}
\frac{c_M - \delta^2 + \delta[1+c_M(3-2f)]}{2\delta} &< \frac{1+\delta(1-f)}{\delta(2-f)} c_l \iff [1 + \delta(3-2f)]c_M + \delta - \delta^2 - 2c_l[1 + \delta(1-f)] < 0 \\
[1 + \delta(3-2f)]c_M + \delta - \delta^2 - 2c_l[1 + \delta(1-f)] &\leq [1 + \delta(3-2f)](2c_l - \delta) + \delta - \delta^2 - 2c_l[1 + \delta(1-f)] \\
&= -2\delta(\delta - c_l)(2 - f) < 0.
\end{align*}
\]

And

\[
\frac{c_M - \delta^2 + \delta[1+c_M(3-2f)]}{2\delta} > 1 - \delta + c_l(2 - f) - \frac{2(\delta - c_l)(1-\delta)}{\delta(2-f)} \iff c_l < \delta.
\]

Also,

\[
\begin{align*}
\frac{\delta(1-f)(3-f)+1}{\delta(2-f)}c_M + \delta(1-\delta)(1-f) &< 1 - \delta + c_l(2 - f) - \frac{2(\delta - c_l)(1-\delta)}{\delta(2-f)} \\
\iff [1 + \delta(1-f)(3-f)]c_M + [2\delta - 2\delta(2-f)^2]c_l + \delta(1-\delta) &< 0 \\
[1 + \delta(1-f)(3-f)]c_M + [2\delta - 2\delta(2-f)^2]c_l + \delta(1-\delta) &\leq [1 + \delta(1-f)(3-f)](2c_l - \delta) - 2\delta - 2\delta(2-f)^2]c_l + \delta(1-\delta) = -4\delta(\delta - c_l)(2 - f)^2 < 0.
\end{align*}
\]

Finally, as \(\delta/2 > \frac{\delta(1-f)(3-f)+1}{\delta(2-f)} \iff \delta f^2 + 2(1+\delta)(1-f) > 0\), (F.14) becomes

When \(\delta/2 \leq c_l < \delta\),

\[
(p_{n_2}, \pi_2(p_{n_2})) = \left\{ \begin{array}{ll}
\frac{c + 2c_l - \delta - c_l f + 1}{\delta(2-f)}, & c_4 \leq c < \bar{c}, \\
\frac{c_4 + 2c_l - \delta + c_l f - 1}{\delta(2-f)}, & c_4 \leq c < c_4,
\end{array} \right.
\]

where

\[
\begin{align*}
c_4 &= \frac{c_M - \delta^2 + \delta[1+c_M(3-2f)]}{2\delta}, \\
\bar{c} &= 1 - \delta + c_l(2 - f) - \frac{2(\delta - c_l)(1-\delta)}{\delta(2-f)}.
\end{align*}
\]

It is easy to verify that \(\pi_5(p_{n_5}, p_{r_5})\) is a quadratic convex function in \(c_M\) and it achieves its minimum at \(c_M = \frac{\delta(2-f)(1-\delta)(1-f)}{1+\delta(1-f)(3-f)}\). Meanwhile, \(\frac{\delta[1-f](3-f)+1}{\delta(2-f)}c_M + \delta(1-\delta)(1-f) < c \iff c \leq \bar{c} \leq \frac{\delta[2-f](1-\delta)(1-f)}{1+\delta(1-f)(3-f)}\). Hence, \(\pi_5(p_{n_5}, p_{r_5}) > \frac{c_4 + \delta(1-f)(3-f) + \delta(1-\delta)(1-f)}{\delta(2-f)}\), which is obtained by setting \(c_M = \frac{\delta[2-f](1-\delta)(1-f)}{1+\delta(1-f)(3-f)}\). Therefore

* When \(0 \leq c < c_4\), it is obvious that

\[
\pi_5(p_{n_5}, p_{r_5}) > \frac{c - c_l}{\delta(2-f)} \left( c_4 + \frac{\delta[1+f(1-f)(3-f)] + \delta(1-\delta)(1-f)}{\delta(2-f)} - c \right),
\]

since \(\frac{(c - c_l)(\delta f - 1)^2}{4[1+\delta(1-f)(3-f)]}\) is the upper bound of \(\pi_1(p_{n_1})\) and \(\frac{\delta - c_l}{\delta(2-f)} \left( c_4 + \frac{\delta[1+f(1-f)(3-f)] + \delta(1-\delta)(1-f)}{\delta(2-f)} - c \right)\) is attainable in Problem 1 and 2.
When \( c_4 \leq c < \bar{c} \), to show \( \pi_5(p_{n_5}, p_{r_5}) > \frac{(2-f)\epsilon_2(1-\delta-c)^2}{4(1-\delta)} \), which is the upper bound of \( \pi_2 \), it suffices to show
\[
\frac{(c_4 + \epsilon_2 - \delta - f + \epsilon_2)^2}{4(1-\delta)} \geq \frac{(2-f)\epsilon_2(1-\delta-c)^2}{4(1-\delta)}.
\]
Note that
\[
(2-f)c_1 + 1 - \delta - c \geq (2-f)(\delta + c_M)/2 + 1 - \delta - c_M\delta^2 + \delta[1 + c_M(3 - 2f)] = \frac{(\delta - c_M)[1 + \delta(1 - f)]}{2\delta} \geq 0.
\]
Hence, \( \frac{(2-f)c_1 + 1 - \delta - c}{4(1-\delta)} \) is increasing in \( c_1 \). As \( c_4 \leq c \Leftrightarrow c_1 < \frac{2[2-f(1-c-\delta)]}{2+2\delta - 4\delta f + \delta^2 f^2} \), we have
\[
\frac{(2-f)(1-\delta - c)^2}{4(1-\delta)} \leq \frac{(2-f)c_1 + 1 - \delta - c}{4(1-\delta)}.
\]
Since
\[
\frac{(1+\delta-c-\delta)^2}{4[1+\delta(1-f)(3-f)]} - \frac{(2-f)c_1 + 1 - \delta - c}{4(1-\delta)} \leq \frac{\delta^2(1-\delta)(2-f)^4(c-\delta+\delta f - 1)^2}{(\delta^2 f^2 - 4\delta f + 2\delta + 2)^2} \geq 0,
\]
we have \( \frac{(c_4 + \epsilon_2 + f + \epsilon_2 - 1)^2}{4[1+\delta(1-f)(3-f)]} \geq \frac{(c_4 + \epsilon_2 + f + \epsilon_2 - 1)^2}{4[1+\delta(1-f)(3-f)]} \). Therefore, when Problem 5 has interior solution, \( \pi_5(p_{n_5}, p_{r_5}) > \pi_2(p_{n_2}) \).

Second, we show that \( \pi_5(p_{n_5}, p_{r_5}) > \pi_2(p_{n_3}) \). As \( \pi_5(p_{n_3}, p_{r_3}) > \frac{(c_4 + \epsilon_2 + f + \epsilon_2 - 1)^2}{4[1+\delta(1-f)(3-f)]} \), it suffices to show that \( \frac{(c_4 + \epsilon_2 + f + \epsilon_2 - 1)^2}{4[1+\delta(1-f)(3-f)]} \) is the upper bound of \( \pi_3(p_{n_3}) \). Because
\[
(c_4 + \epsilon_2 + f + \epsilon_2 - 1)^2 < (1 - \delta - c_1 f) < (1 - \delta - f - \delta) = \frac{(\delta - c_M)(1 + 3\delta - 2\delta f)}{2\delta} < 0,
\]
hence, \( 1 - c + \delta - c_1 f > 0 \) for \( c < \frac{\delta - c_M + \delta f}{3\delta - 2\delta f} \). This together with \( c_7 \geq (\delta + c_M)/2 \) imply
\[
\frac{(1 - c_7 - \delta f)^2}{4[1+\delta(1-f)(3-f)]} - \frac{(1 - c_7 - f - \delta)^2}{4[1+\delta(1-f)(3-f)]} \geq \frac{(1 - c_7 - (\delta + c_M)f/2)^2}{4(1+\delta(1-f)(3-f))} \geq 0.
\]

It is easy to verify that \((- f^3 + 10 f^2 - 24 f + 18)\delta^2 + (2 f^2 - 11 f + 12)\delta - f + 2 > 0, \forall \delta \in [0, 1] \). Hence, \( \pi_5(p_{n_3}, p_{r_3}) > \pi_3(p_{n_3}) \). Therefore, when \( (\delta + c_M)/2 \leq c_1 < \delta \) and Problem 5 has interior solution, the manufacturer’s global optimal solution is
\[
(p_{n_5}, p_{r_5}) = \left( \frac{c + \delta(1-f) + 1}{2}, \frac{\delta + c_1 f}{2} \right).
\]

Case 2.1.2: \( c \leq \frac{(1-f)(3-f)\epsilon_2 + \delta + c_1 f}{\delta(2-f)} = \xi \). In this case, we show that \( p_{n_3} = [c + 1 + (\delta(1-f))/2] \) is the manufacturer’s global optimal solution.

Because \( c_7 \geq (\delta + c_M)/2 = p_{r_3} \) and \( c \leq \xi \) (which is equivalent to \( p_{r_3} \geq \frac{\delta[p_{r_3}(2-f)-(1-f)(1-\delta)]}{(1-f)(3-f)^{\delta + 1}} \)), the optimal solution of Problem 5 must satisfy \( p_r = \frac{\delta[p_r(2-f)-(1-f)(1-\delta)]}{(1-f)(3-f)^{\delta + 1}} \) and \( \frac{(1-f)(1-\delta)}{2-f} \leq p_n \leq \frac{\delta(1-f)(3-f)\epsilon_2 + \delta + (1-f)(1-\delta)}{\delta(2-f)} \), since the objective function of the Problem 5 has been proven to be global concave (F.11) and the feasible region is clearly convex. Hence, Problem 5 is dominated by Problem 1, since their objective functions are the same when \( p_r = \frac{\delta[p_r(2-f)-(1-f)(1-\delta)]}{(1-f)(3-f)^{\delta + 1}} \).

Because \( c_7 \geq \delta/2 \) and
\[
\xi \leq \frac{2[c_7(1+\delta(1-f)(3-f)] + \delta(1-\delta)(1-f)]}{\delta(2-f)} - 1 - \delta(1-f)
\]
\[
\xi \leq \frac{2[c_7(1+\delta(1-f)(3-f)] + \delta(1-\delta)(1-f)]}{\delta(2-f)} - 1 - \delta(1-f)
\]
\[
\xi \leq \frac{1 - \delta + c_1 f}{\delta(2-f)} \leq \xi - \frac{2(\delta - c_1 f)(1-\delta)}{\delta(2-f)} \leq -\delta - c_1 f(2 - f) < 0,
\]

\[
(2-f)c_1 + 1 - \delta - c \geq (2-f)(\delta + c_M)/2 + 1 - \delta - c_M\delta^2 + \delta[1 + c_M(3 - 2f)] = \frac{(\delta - c_M)[1 + \delta(1 - f)]}{2\delta} \geq 0.
\]
hence, we have
\[
\begin{align*}
(p_{n1}, \pi_1(p_{n1})) &= \left( \frac{c+\delta(1-f)}{2}, \frac{(c+\delta+\delta f-1)^2}{3(\delta f^2+3\delta+3+1)} \right), & \text{for } 0 \leq c \leq \xi, \\
(p_{n2}, \pi_2(p_{n2})) &= \left( \frac{c}{\delta(2-f)} \left( \frac{c(1+\delta(1-f)^{-3}(3-f))}{\delta(2-f)} \right), \frac{(\delta-c_1)\left(\frac{c(1+\delta(1-f)(3-f))}{\delta(2-f)}-c\right)}{\delta(2-f)} \right), & \text{if } 0 \leq c < \xi.
\end{align*}
\]
Therefore, it is clear that \( \pi_1(p_{n1}) \geq \pi_2(p_{n2}) \).

To show \( \pi_1(p_{n1}) \geq \pi_3(p_{n2}) \), it suffices to show that \( \frac{(1-c+\delta-\delta f)^2}{4(\delta f^2+3\delta+3+1)} \geq \frac{(1-c+\delta-\delta f)^2}{4(1+3\delta)} \). Note that \( \xi < 1 + \delta - f c_1 \), since
\[
\begin{align*}
\frac{cM[1+\delta(1-f)(3-f)]+\delta(1-\delta)(1-f)}{\delta(2-f)} &\leq \frac{(2c_1-\delta)[1+\delta(1-f)(3-f)]+\delta(1-\delta)(1-f)}{\delta(2-f)} - [1+\delta-f c_1] = -(\delta-c_1) \left( \frac{2(1-\delta)}{\delta(2-f)} + 4 - f \right) < 0.
\end{align*}
\]
Thus, \( 1 + \delta - f c_1 > 0 \) for \( \xi < \xi \). As a result,
\[
\begin{align*}
\frac{(1-c+\delta-\delta f)^2}{4(\delta f^2+3\delta+3+1)} &\geq \frac{(1-c+\delta-\delta f)^2}{4(1+3\delta)} - \frac{(1-c+\delta-\delta c_1 f f/2)^2}{4(1+3\delta)} \\
\geq \frac{(1-c+\delta-\delta f)^2}{4(\delta f^2+3\delta+3+1)} - \frac{(1-c+\delta-\delta[2(1-f)-\delta(1-f)] f f)}{4(1+3\delta)} \\
\geq \frac{(1-c+\delta-\delta f)^2}{4(\delta f^2+3\delta+3+1)} - \frac{(1-c+\delta-\delta f)^2}{4(1+3\delta)}
\end{align*}
\]
Note that one can verify that \(-f^3+12f^2-36f+24 \geq -1 \). Hence, \( \frac{(1-c+\delta-\delta f)^2}{4(\delta f^2+3\delta+3+1)} \geq \frac{(1-c+\delta-\delta c_1 f f^2)}{4(1+3\delta)} \).

To show \( \pi_1(p_{n1}) \geq \pi_4(p_{n2}) \), first notice that \( \xi < 1 + \delta - f c_M \), as \( \xi - (1 + \delta - f c_M) = -\frac{(\delta-c_1)M[1+\delta(1-f)][3-f)]}{\delta(2-f)} < 0 \).

This implies that \( 1 + \delta - f M c \geq 0 \) for \( \xi < \xi \). Also notice that \( c \leq \xi \) implies \( 1 + \delta - (1 - f) \geq 0 \) for \( c \leq \xi \), as \( 1 + \delta - (1 - f) \geq 0 \). Also notice that \( c \leq \xi \) implies \( 1 + \delta - (1 - f) \geq 0 \). Also notice that \( c \leq \xi \) implies \( 1 + \delta - (1 - f) \geq 0 \). Therefore,
\[
\begin{align*}
\frac{(1-c+\delta-\delta f)^2}{4(\delta f^2+3\delta+3+1)} &\leq \frac{(1-c+\delta-\delta f)^2}{4(1+3\delta)} \geq \frac{(1-c+\delta-\delta f)^2}{4(\delta f^2+3\delta+3+1)} - \frac{(1-c+\delta-\delta[2(1-f)-\delta(1-f)] f f)}{4(1+3\delta)} \\
\geq \frac{(1-c+\delta-\delta f)^2}{4(\delta f^2+3\delta+3+1)} - \frac{(1-c+\delta-\delta f)^2}{4(1+3\delta)}
\end{align*}
\]
Therefore, when \( (\delta + c_M)/2 \leq c_1 < \delta \) and \( c \leq \xi \), the manufacturer’s global optimal solution is
\[
p_{n1} = \frac{c+1+\delta(1-f)}{2}.
\]

**Case 2.1.3:** \( c \geq \frac{cM-c^2+\delta(1+c+\delta M^{-2}f)}{23} = \bar{c} \). In this case, we show that \( p_{n1} + f p_{n1} = (1 + c + \delta + c_M f)/2 \) is the manufacturer’s global optimal solution, i.e., \( \pi_4(p_{n1}, p_{n1}) = \frac{(c+\delta+\delta c_M f^{-1})^2}{4(1+3\delta)} \) (for \( 0 \leq c < 1 + \delta - f c_M \)) is the maximum profit.

Because \( c_1 \geq (\delta + c_M)/2 = p_{n1} \) and \( c \geq \bar{c} \) (which is equivalenle to \( p_{n1} \leq \frac{\delta(\delta+2\bar{c}^{-1})}{1+\delta+2\bar{c}(1-f)} \)), the optimal solution of Problem 5 must satisfy \( p_{n1} = \frac{\delta(\delta+2\bar{c}^{-1})}{1+\delta+2\bar{c}(1-f)} \) and \( (1 - \delta)/2 \leq p_{n1} \leq \frac{(3\delta-2\delta^2 f + 1)\bar{c} - \delta^2 + \delta}{23} \), since the objective function of the Problem 5 has been proven to be global concave (F.11) and the feasible region is clearly convex. Hence, Problem 5 is dominated by Problem 4, since their objective functions are the same when \( p_{n1} = \frac{\delta(\delta+2\bar{c}^{-1})}{1+\delta+2\bar{c}(1-f)} \).

To show \( \pi_1(p_{n1}, p_{n1}) \geq \pi_3(p_{n2}) \), it suffices to show that \( \frac{(1-c+\delta-c_M f)^2}{4(1+3\delta)} \geq \frac{(1-c+\delta-c_M f)^2}{4(1+3\delta)} \), as \( \frac{(1-c+\delta-c_M f)^2}{4(1+3\delta)} \) is the upper bound of \( \pi_3(p_{n2}) \) and \( \bar{c} < 1 + \delta - f c_M \Leftrightarrow -\frac{(\delta-c_M f^{-1})}{23} < 0 \). Because \( c_M < c_1 \), it is clear that
$1 + c - f c_l < 1 + c - f c_M$ and hence, $1 - c + \delta - c_M f > 1 - c + \delta - c_l f > 0$ for $c < 1 + \delta - f c_l$, which implies $(1 - c + \delta - c_M f)^2 \geq (1 - c + \delta - c_l f)^2$. When $c > 1 + \delta - f c_l$, $\pi_3(p_{n_3}) = 0$. Consequently, we have $\pi_4(p_{n_4}, p_{n_4}) \geq \pi_3(p_{n_3})$.

To show $\pi_4(p_{n_4}, p_{n_4}) \geq \max\{\pi_1(p_{n_1}), \pi_2(p_{n_2})\}$, we derive $\max\{\pi_1(p_{n_1}), \pi_2(p_{n_2})\}$. As $(\delta + c_M)/2 \leq c_l < \delta \Rightarrow c_l > \max\{c_M, \delta/2\}$, $\pi_1(p_{n_1})$ and $\pi_2(p_{n_2})$ becomes

When $\max\{c_M, \delta/2\} \leq c_l < \delta$,

$$\begin{align*}
(p_{n_1}, \pi_1) &= \begin{cases}
\text{unprofitable}, & \text{if } c \geq c_1, \\
\left(\frac{c_l}{\delta(2-f)}, \frac{(\delta - c_l)(3f - 3f + c_l f + c_l)}{\delta(2-f)} - c\right), & \text{if } c_2 \leq c < c_1, \\
\left(\frac{c_l + \delta(1-f)}{\delta(2-f)}, \frac{(\delta - c_l)(3f - 3f + c_l f + c_l)}{\delta(2-f)} - c\right), & \text{if } 0 \leq c < c_2,
\end{cases}
\end{align*}$$

$$\begin{align*}
(p_{n_2}, \pi_2) &= \begin{cases}
\text{unprofitable}, & \text{if } c \geq c_3, \\
\left(\frac{c_l}{\delta(2-f)}, \frac{(\delta - c_l)(3f - 3f + c_l f + c_l)}{\delta(2-f)} - c\right), & \text{if } c_4 \leq c < c_3,
\end{cases}
\end{align*}$$

where

$$\begin{align*}
c_1 &= \frac{c_l}{\delta(2-f)}(3f - 3f + c_l f + c_l), \\
c_2 &= \frac{c_l}{\delta(2-f)}(3f - 3f + c_l f + c_l) - 1 - \delta(1 - f), \\
c_3 &= \frac{(c_l - 2f + 1) f + (\delta - c_l)(3f - 3f + c_l f + c_l)}{\delta(2-f)}, \\
c_4 &= 1 - \delta + c_l (2 - f) - \frac{2(\delta - c_l)(1-f)}{\delta(2-f)}.
\end{align*}$$

Note that $c_4 - c_2 = (\delta - c_l)(2 - f) > 0$ and $c_4 - c_1 = \frac{-(\delta - c_l)(1-f)}{\delta(2-f)} < 0$. Therefore, by continuity of the objective functions of Problem 1 and 2 at their boundaries, we have

$$\pi_2(p_{n_2}) = \begin{cases}
\text{unprofitable}, & \text{if } c \geq c_3, \\
\left(\frac{c_l}{\delta(2-f)}, \frac{(\delta - c_l)(3f - 3f + c_l f + c_l)}{\delta(2-f)} - c\right), & \text{if } c_4 \leq c < c_3,
\end{cases}$$

$$\pi_1(p_{n_1}) = \begin{cases}
\text{unprofitable}, & \text{if } c \geq c_1, \\
\left(\frac{c_l}{\delta(2-f)}, \frac{(\delta - c_l)(3f - 3f + c_l f + c_l)}{\delta(2-f)} - c\right), & \text{if } c_2 \leq c < c_1, \\
\left(\frac{c_l + \delta(1-f)}{\delta(2-f)}, \frac{(\delta - c_l)(3f - 3f + c_l f + c_l)}{\delta(2-f)} - c\right), & \text{if } 0 \leq c < c_2.
\end{cases}$$

Since

$$\begin{align*}
c_M - \delta^2 + \delta(1 + c_M(3 - 2f)) - \frac{[1 + \delta(1-f)]c_l}{\delta} & \leq \frac{(2c_l - \delta) - \delta^2 + \delta[1 + (2c_l - \delta)(3 - 2f)] - \frac{[1 + \delta(1-f)]c_l}{\delta}}{2\delta} = -(\delta - c_l)(2 - f) < 0 \text{ and } 1 - \delta - c_M f - c_3 \geq 1 + \delta - (2c_l - \delta) f - c_3 = \frac{(\delta - c_l)(1 + 3\delta + 2\delta f)}{2\delta} \geq 0,
\end{align*}$$

it is clear that $\pi_4(p_{n_4}, p_{n_4}) \geq \max\{\pi_1(p_{n_1}), \pi_2(p_{n_2})\}$ when $c \geq c_3$.

One can verify that $\pi_4(p_{n_4}) - \frac{(\delta - c_l)(3f - 2f f + c_l f + c_l)}{\delta(2-f)} \geq \frac{I(\delta - c_l)(c_l - c_M)}{2\delta^2} > 0$, as it is obvious that $\pi_4(p_{n_4}) - \frac{(\delta - c_l)(3f - 2f f + c_l f + c_l)}{\delta(2-f)}$ is a convex quadratic function of $c$ with minimum value $\frac{I(\delta - c_l)(c_l - c_M)}{2\delta^2}$. Hence, $\pi_4(p_{n_4}, p_{n_4}) \geq \max\{\pi_1(p_{n_1}), \pi_2(p_{n_2})\}$, when $[1 + \delta(1-f)]c_l/\delta \leq c < c_3$.

By (F.13), we know that $\pi_4(p_{n_4}, p_{n_4}) > \frac{(c_l - c_1 f)}{\delta(2-f)}$ if $c > 1 + \delta - c_M f - \frac{(\delta - c_M)}{2\delta}$. In this case, as $c \geq \bar{c}$ and it is easy to verify that $\bar{c} > 1 + \delta - c_M f - \frac{(\delta - c_M)}{1 - \sqrt{1 - \frac{1}{\delta(2-f)^2}}}$, we have $\pi_4(p_{n_4}, p_{n_4}) > \max\{\pi_1(p_{n_1}), \pi_2(p_{n_2})\}$ when $c < \bar{c} < c_2$. 
As \(\frac{(c-\delta+\delta f-1)^2}{4(\delta f^2-4\delta f+2\delta+2)}\) is the upper bound of \(\pi_1(p_{n_1})\) and \(\frac{(\delta-c_f)}{\delta(2-f)}\left(\frac{c_f(1+\delta(1-f)(3-f)+\delta(1-\delta)(1-f))}{\delta(2-f)}-c\right)\) is attainable in Problem 1, hence, we have \(\pi_1(p_{n_1},p_{r_3}) > \frac{(\delta-c_f)}{\delta(2-f)}\left(\frac{c_f(1+\delta(1-f)(3-f)+\delta(1-\delta)(1-f))}{\delta(2-f)}-c\right)\). Therefore, we have \(\pi_4(p_{n_4},p_{r_4}) > \max\{\pi_1(p_{n_1}), \pi_2(p_{n_2})\}\) when \(\bar{c} \leq c < c_4\).

One can verify that \(\pi_4(p_{n_4},p_{r_4}) > \frac{(c-\delta+\delta c_f-1)^2}{4(\delta f^2-3\delta f+3\delta+3)}\Leftrightarrow c > 1 - \frac{\delta-(\delta-c_f)(2-f)+\delta(\delta-c_M f)}{1-\sqrt{\frac{1-\delta}{\delta(2-f)}}}\) is attainable, as \(1-c-\delta+c_f(2-f) > 0\) \(\Leftrightarrow c < 1-\delta+\delta c_f(2-f)\) and \(1-\delta+c_f(2-f) - [1+\delta(1-f)]c_f/\delta = (1-\delta)(\delta-c_f)/\delta > 0\). Now we show that \(1 - \frac{\delta-(\delta-c_f)(2-f)+\delta(\delta-c_M f)}{1-\sqrt{\frac{1-\delta}{\delta(2-f)}}} < \max\{c_4, \bar{c}\}\). Note that \(c_4 - \bar{c} \geq 0 \Leftrightarrow c_4 \geq c_M + \frac{(f+2)(1-\delta)(\delta-c_M f)}{2\delta(\delta+2)(\delta f^2-4\delta f+2\delta+2)}\).

- **When** \(c_4 \geq \bar{c}\), we define
  \[
  F_1(c_f) \triangleq c_4 - \frac{1 - \delta - c_f(2-f) + (\delta - c_M f) \sqrt{\frac{1-\delta}{\delta(2-f)}}}{1 - \sqrt{\frac{1-\delta}{\delta(2-f)}}}.
  \]

One can verify that \(F_1(c_f)\) is linear in \(c_f\). Hence, to show \(F_1(c_f) \geq 0\), it suffices to examine the points \(c_{f} \in \left\{\delta, c_{M} + \frac{(f+2)(1-\delta)(\delta-c_M f)}{2\delta(\delta+2)(\delta f^2-4\delta f+2\delta+2)}\right\}\). Note that

\[
F_1(c_f) = \frac{\delta - c_f(2-f) + (\delta - c_M f) \sqrt{\frac{1-\delta}{\delta(2-f)}}}{1 - \sqrt{\frac{1-\delta}{\delta(2-f)}}}
\]

\[
\Rightarrow F_1(c_f) \triangleq \left(3(4-f)^2 + 2(4+f)^2(3f^2+4f+6)(2(3f^2+4f+6)f^2-6f+12)\delta^2 + 2(4+f)^2(3f^2+4f+6)(2(3f^2+4f+6)f^2-6f+12)\delta + (4+f)^2 - (4-f^2)^2 \geq 0.\right.
\]

Note that \(F_1(\delta)\) is a convex quadratic function of \(\delta\), which achieves its minimum at \(\delta_0 \triangleq \frac{3(3f^2-6f+4)(f^2-6f+12)}{14f^2} - 1/6\). If \(\delta_0 < 0\), then \(F_1(\delta) \geq F_4(0) = (4+f)^2 - (4-f^2)^2 \geq 0\). If \(\delta_0 \geq 0\), then \(F_4(\delta) \geq F_4(\delta_0) = \frac{(2-f)^2(4+f)^2(12f^2-4-f^2)}{12f^2-4-f^2}\). One can verify that for \(f \in [0,1]\), \(\delta_0 \geq 0 \Leftrightarrow f \geq 2 + \sqrt{2} - \sqrt{2(1+2\sqrt{2})} \approx 0.6471\). This implies \(12-f-4-f^2 \geq 0\) and \((3f^2-6f+4)(f^2-6f+12) \geq 0\). Hence, \(F_4(\delta_0) \geq 0\) when \(\delta_0 \geq 0\).

- **When** \(c_4 < \bar{c}\), we define
  \[
  F_2(c_f) \triangleq \bar{c} - \frac{1 - \delta - c_f(2-f) + (\delta - c_M f) \sqrt{\frac{1-\delta}{\delta(2-f)}}}{1 - \sqrt{\frac{1-\delta}{\delta(2-f)}}}.
  \]

One can verify that that \(F_2(c_f)\) is linearly decreasing in \(c_f\). Hence, to show \(F_2(c_f) > 0\) it suffices to examine the point \(c_f = c_M + \frac{f+2(1-\delta)(\delta-c_M f)}{2\delta(\delta+2)(\delta f^2-4\delta f+2\delta+2)}\). We have

\[
F_2(c_f) = \frac{1 - \delta + (1+3\delta) \sqrt{\frac{1-\delta}{\delta(2-f)}}}{2} - \frac{1 - \delta(1+3\delta - 2\delta f) \sqrt{\frac{1-\delta}{\delta(2-f)}}}{2 \delta f^2 - 4\delta f + 2\delta + 2} \geq 0,
\]

as proved in the case of \(c_4 \geq \bar{c}\).

Therefore, when \((\delta + c_M)/2 \leq c_f < \delta c_f < \delta f < \delta\), the manufacturer’s global optimal solution is

\[
p_{n_4} + f p_{r_4} = (c + \delta + c_M f + 1)/2.
\]

**Case 2.2: 0 \leq c_f < \min\{\delta + c_M/2, \delta\}**. In this case, we first combine the optimal solutions of Problem 1, 2, and 3 together. Then we compare it with the optimal solutions of Problem 4 and 5.

For Problem 1 and 2, we have

\[
\begin{align*}
\left\{ \begin{array}{l}
\frac{\delta}{2} \left(1 - \frac{3}{1+3\delta(1-f)(3-f)}\right) \geq \frac{\delta f(1-\delta)}{2-3\delta(1-f)(3-f)} \Leftrightarrow \delta f(1-\delta) > 0,
\end{array} \right.
\end{align*}
\]

\[
\begin{align*}
c_4 - c_f & = (\delta - c_f)(2-f) > 0,
\end{align*}
\]

\[
\begin{align*}
c_4 - c_f & = \frac{(\delta-c_f)(1-\delta)}{\delta(2-f)} < 0.
\end{align*}
\]
Thus, by continuity of the objective functions of Problem 1 and 2 at their boundaries, we know that

\[
\begin{align*}
\text{When } 0 & \leq c_l < \bar{c}_l, \\
\max\{\pi_1, \pi_2\} &= \left\{ \begin{array}{ll}
\text{unprofitable,} & \text{if } c \geq c_3, \\
\pi_2(p_{n_2}) = \frac{(\delta - c_l)(3c_l - \delta - 2c_l f + c_l f + \frac{1}{\delta} + 1 - 2c_l)}{4c_l^2}, & \text{if } \frac{1 + \delta(1 - f)c_l}{\delta} \leq c < c_3, \\
\pi_2(p_{n_2}) = \frac{\delta - c_l}{4(1 - \delta)} & \text{if } 0 \leq c < \frac{1 + \delta(1 - f)c_l}{\delta}.
\end{array} \right.
\end{align*}
\]

\[
\begin{align*}
\text{When } \bar{c}_l & \leq c_l < \bar{c}_l, \\
\max\{\pi_1, \pi_2\} &= \left\{ \begin{array}{ll}
\text{unprofitable,} & \text{if } c \geq c_3, \\
\pi_2(p_{n_2}) = \frac{(\delta - c_l)(3c_l - \delta - 2c_l f + c_l f + \frac{1}{\delta} + 1 - 2c_l)}{4c_l^2}, & \text{if } \frac{1 + \delta(1 - f)c_l}{\delta} \leq c < c_3, \\
\pi_2(p_{n_2}) = \frac{\delta - c_l}{4(1 - \delta)} & \text{if } c_4 \leq c < \frac{1 + \delta(1 - f)c_l}{\delta}.
\end{array} \right.
\end{align*}
\]

\[
\begin{align*}
\text{When } \bar{c}_l & \leq c_l < \min\{\delta, (\delta + c_M)/2\}, \\
\max\{\pi_1, \pi_2\} &= \left\{ \begin{array}{ll}
\text{unprofitable,} & \text{if } c \geq c_3, \\
\pi_2(p_{n_2}) = \frac{(\delta - c_l)(3c_l - \delta - 2c_l f + c_l f + \frac{1}{\delta} + 1 - 2c_l)}{4c_l^2}, & \text{if } \frac{1 + \delta(1 - f)c_l}{\delta} \leq c < c_3, \\
\pi_2(p_{n_2}) = \frac{\delta - c_l}{4(1 - \delta)} & \text{if } c_4 \leq c < \frac{1 + \delta(1 - f)c_l}{\delta}.
\end{array} \right.
\end{align*}
\]

To combine the optimal solution of Problem 3 with those of Problem 1 and 2, first we notice that

\[
\begin{align*}
\left\{ \begin{array}{l}
1 + \delta - f c_l - c_3 = \frac{1 + \delta(1 - f)c_l}{\delta} > 0, \\
c_4(1 + 3\delta - 4f f + c_l f) - 2\delta^2 - \frac{(1 + \delta(1 - f)c_l)}{\delta} = -2(\delta - c_l) < 0.
\end{array} \right.
\end{align*}
\]

As \(\pi_3(p_{n_3}) > 0\) when \(c < 1 + \delta - f c_l\) and \(\pi_2(p_{n_2}) = \frac{(1 - c_l f)^2}{4(1 - \delta)}\) when \(c < 1 + \delta(1 - f)c_l/\delta\), we have
\[
\frac{(1 - c_l f)^2}{4(1 + 3\delta)} > \frac{1 - c_l f}{4(1 + 3\delta)} \Leftrightarrow c < 1 + \delta - c_l f - \frac{2(\delta - c_l)}{\delta} \Leftrightarrow \frac{c - c_l}{\delta} < \frac{2(\delta - c_l)}{\delta} \Leftrightarrow c_5 \leq c < c_5 < 1 + \delta(1 - f)c_l/\delta, \text{ since}
\]

\[
\begin{align*}
\left\{ \begin{array}{l}
c_5 - \frac{1 + \delta(1 - f)c_l}{\delta} = (\delta - c_l) \left( \frac{2}{\sqrt{\frac{1 + 3\delta}{2\delta}}} + 1 + \frac{1 + \delta}{\delta} \right) < 0, \\
\left( \begin{array}{l}
\frac{1 + \delta}{\delta} < 2 \Leftrightarrow \frac{2(1 + \delta)}{1 + 3\delta} < 1 \Leftrightarrow \frac{1 + 3\delta}{1 + 3\delta} < -\frac{4(1 - f)c_l}{(1 + 3\delta)} < 0,
\end{array} \right.
\end{array} \right.
\end{align*}
\]

In addition, \(c_5(c_l)\) is linear in \(c_l\) and one can verify that

\[
\begin{align*}
\left\{ \begin{array}{l}
c_5(0) = \frac{\sqrt{1 + \delta}}{\sqrt{3 - 2\sqrt{3}}} (\sqrt{1 - \delta - \sqrt{1 + 3\delta}}) < 0, \\
c_5 \left( \frac{\delta(1 - \delta)(f f - (f f - 1))}{2 - \delta + \delta f} \right) \propto 1 + \delta(1 - f) - \frac{\delta(f f - 1)}{1 - \sqrt{\frac{1 + 3\delta}{2\delta}}} < 0 \Leftrightarrow \delta > \frac{2f}{\sqrt{2f - 2f + 4}}.
\end{array} \right.
\end{align*}
\]

Note that

\[
\begin{align*}
c_5 - c_4 & \propto 1 + \delta(1 - f) + \frac{2\delta - f}{\sqrt{\frac{1 + 3\delta}{2\delta} - 1}} > 0 \Leftrightarrow \left( \frac{4[1 + \delta(1 - f)]}{(2 - f)(1 + 3\delta)} - 1 \right)^2 > \frac{1 - \delta}{1 + 3\delta} \Leftrightarrow \delta < \frac{2f}{\sqrt{2f - 2f + 4}}
\end{align*}
\]

\[
\begin{align*}
F_3(c) & \triangleq (\delta - c_l) \left( \frac{2[1 + \delta(1 - f)(3 - f) + \delta(1 - (f f)) - c]}{(2 - f) (1 + 3\delta)} - \frac{(c_l - c_l f - 1)^2}{4(1 + 3\delta)} \right), \\
F_3(c_4) & = -\frac{(c_l - \delta)^2 \left[ \frac{\delta(4 - 2f + f^2 - 2f)}{\delta(1 + 3\delta)(2 - f)^2} \right]}{2f} > 0 \Leftrightarrow \delta < \frac{2f}{f^2 - 2f + 4}, \\
F_3(c_2) & = \frac{1 + \delta}{4\delta(1 + 3\delta)(2 - f)^2} > 0.
\end{align*}
\]
It is easy to verify that \( F_3(c) \) is concave and quadratic in \( c \).

In addition, \[
\frac{(1-c+c_\delta-cf)^2}{4(1+3\delta)} > \frac{(1-c+c_\delta-cf)^2}{4(1+3\delta)} \iff c < 1 + \delta - cf - \frac{f(\delta-cf)}{1 - \sqrt{1 + \delta(1-f)(3-f)}}.
\]

We have

\[
1 + \delta - cf - \frac{f(\delta-cf)}{1 - \sqrt{1 + \delta(1-f)(3-f)}} - c_2 > 0 \iff \frac{(1 + 3\delta)(1 + \sqrt{1 + \delta(1-f)(3-f)})}{4-f} < \frac{\delta^2 - 6\delta f + 6\delta + 2}{2-f}.
\]

\[
\frac{(4-f)\delta^2 - 6\delta f + 6\delta + 2}{2-f} - 1 \iff \frac{(f-4)(1+\delta(1-f)(3-f))|\delta((f-6)^2f-24)\{2\} - 8}{(1 + 3\delta)^2(2-f)^2} > 0,
\]

since \((f-6)^2f\) is increasing in \(f \in [0, 1]\) and hence \((f-6)^2f \leq 25\), which implies \(\delta[(f-6)^2f-24] - 8 \leq \delta - 8 < 0\).

Also we have

\[
\frac{(1-c+c_\delta-cf)^2}{4(1+3\delta)} > \frac{(\delta-cf)(3c_\delta - \delta - 2cf + cf/\delta + 1 - 2c)}{4\delta} \iff c > c_6 \triangleq \frac{1}{2-f} \left[ 2(\delta-cf)(1+3\delta)\sqrt{2f(1+3\delta)}/\delta + cf(f^2 - 2f + 6 - 2/\delta) - f - \delta(4+f) \right].
\]

Therefore, \(\max\{\pi_1(p_{n_2}), \pi_2(p_{n_2}), \pi_3(p_{n_2})\}\) are given by:

- **If** \(0 < \delta < \frac{2f}{f^2-2f+4}\):

  When \(0 \leq c_\delta < c\)

  \[
  \max\{\pi_1, \pi_2, \pi_3\} = \begin{cases} 
  \text{unprofitable,} & \text{if } c \geq 1 + \delta - cf, \\
  \pi_3(p_{n_2}) = \frac{(c-\delta+cf-1)^2}{4(1+3\delta)}, & \text{if } \max\{0, c_5\} \leq c < 1 + \delta - cf, \\
  \pi_2(p_{n_2}) = \frac{(c-\delta+cf-1)^2}{4(1+3\delta)}, & \text{if } 0 \leq c < c_5.
  \end{cases}
  \]

  When \(c_6 \leq c < c_\delta\)

  \[
  \max\{\pi_1, \pi_2, \pi_3\} = \begin{cases} 
  \text{unprofitable,} & \text{if } c \geq 1 + \delta - cf, \\
  \pi_3(p_{n_2}) = \frac{(c-\delta+cf-1)^2}{4(1+3\delta)}, & \text{if } c_6 \leq c < 1 + \delta - cf, \\
  \pi_2(p_{n_2}) = \frac{(c-\delta+cf-1)^2}{4(1+3\delta)}, & \text{if } c_4 \leq c < c_6, \\
  \pi_2(p_{n_2}) \text{ and } \pi_1(p_{n_1}) = \frac{(\delta-cf)}{\delta(2-f)} \left( \frac{c_1(1+\delta(1-f)(3-f) + \delta(1-\delta)(1-f))}{\delta(2-f)} - c \right), & \text{if } 0 \leq c < c_4.
  \end{cases}
  \]

  When \(c_\delta \leq c < c_\delta \min\{\delta, (\delta + c_M)/2\}\)

  \[
  \max\{\pi_1, \pi_2, \pi_3\} = \begin{cases} 
  \text{unprofitable,} & \text{if } c \geq 1 + \delta - cf, \\
  \pi_3(p_{n_2}) = \frac{(c-\delta+cf-1)^2}{4(1+3\delta)}, & \text{if } c_6 \leq c < 1 + \delta - cf, \\
  \pi_2(p_{n_2}) = \frac{(c-\delta+cf-1)^2}{4(1+3\delta)}, & \text{if } c_4 \leq c < c_6, \\
  \pi_2(p_{n_2}) \text{ and } \pi_1(p_{n_1}) = \frac{(\delta-cf)}{\delta(2-f)} \left( \frac{c_1(1+\delta(1-f)(3-f) + \delta(1-\delta)(1-f))}{\delta(2-f)} - c \right), & \text{if } c_2 \leq c < c_4, \\
  \pi_1(p_{n_1}) = \frac{(c-\delta+cf-1)^2}{4(\delta^f-4f^3+4f+1)}, & \text{if } 0 \leq c < c_2.
  \end{cases}
  \]

(F.15)
• If \( \frac{2f}{\delta^2 - 2f + 1} \leq \delta < 1 \),

When \( 0 \leq c_l < c_l \)

\[
\max\{\pi_1, \pi_2, \pi_3\} = \begin{cases} \text{unprofitable}, & \text{if } c \geq 1 + \delta - f c_l, \\ \pi_3(p_n) = \frac{(c - \delta + c_l f - 1)^2}{4(1 + \delta)}, & \text{if } 0 \leq c < 1 + \delta - f c_l. \end{cases}
\]

When \( c_l \leq c_l < \tilde{c}_l \)

\[
\max\{\pi_1, \pi_2, \pi_3\} = \begin{cases} \text{unprofitable}, & \text{if } c \geq 1 + \delta - f c_l, \\ \pi_3(p_n) = \frac{(c - \delta + c_l f - 1)^2}{4(1 + \delta)}, & \text{if } c_l \leq c < 1 + \delta - f c_l, \\ \pi_2(p_n) \text{ and } \pi_1(p_n) = \frac{(\delta - c_l)}{(2 - f)} \left( \frac{c_l (1 + \delta (1 - f)(3 - f)) + 4(1 - \delta)(1 - f) - c}{\delta (2 - f)} \right), & \text{if } 0 \leq c \leq c_6. \end{cases}
\]

When \( \tilde{c}_l \leq c_l < \min\{\delta, (\delta + c_M)/2\} \)

\[
\max\{\pi_1, \pi_2, \pi_3\} = \begin{cases} \text{unprofitable}, & \text{if } c \geq 1 + \delta - f c_l, \\ \pi_3(p_n) = \frac{(c - \delta + c_l f - 1)^2}{4(1 + \delta)}, & \text{if } c_6 \leq c < 1 + \delta - f c_l, \\ \pi_2(p_n) \text{ and } \pi_1(p_n) = \frac{(\delta - c_l)}{(2 - f)} \left( \frac{c_l (1 + \delta (1 - f)(3 - f)) + 4(1 - \delta)(1 - f) - c}{\delta (2 - f)} \right), & \text{if } c_2 \leq c \leq c_6, \\ \pi_1(p_n) = \frac{(c - \delta + c_l f - 1)^2}{4(1 + \delta)}, & \text{if } 0 \leq c \leq c_2. \end{cases}
\]

\[\text{(F.16)}\]

**Case 2.2.1:** \( 0 \leq c_l < \min\{\delta, (\delta + c_M)/2, \delta \} \) and \( c_M \geq c_l \). In this case, we show that the optimal solution of Problem 4 and 5 are dominated by \( \max\{\pi_1(p_n), \pi_2(p_n), \pi_3(p_n)\} \).

Since \( c_M \geq c_l \) implies \( 1 + \delta - f c_M < 1 + \delta - f c_l \) and \( \frac{(1 - c_M - c_l f)^2}{4(1 + \delta)} < \frac{(1 - c_M - c_l f)^2}{4(1 + \delta)} \), hence, by the analysis in Case 2.2, we know that \( \pi_4(p_n) < \max\{\pi_1(p_n), \pi_2(p_n), \pi_3(p_n)\} \).

For Problem 5, in this case, it does not have interior solution, as \( c_l < (c_M + \delta)/2 \). Therefore, the optimal solution of Problem 5 is on \( p_r = \frac{\delta |(2 - f)(1 - f)(1 - \delta)|}{4(1 + \delta)} \) or on \( p_r = \frac{\delta (1 + 2p_n - 1)}{4(1 + \delta + 2(1 - f))} \), then it is clearly dominated by the optimal solution of Problem 1 or Problem 4, due to the continuity of the objective functions of Problem 1, 4 and 5 on these two boundaries. Therefore, we only need to consider the situation when the optimal solution of Problem 5 is on \( p_r = c_l \).

It is easy to verify that given \( p'_{n_5} = c_l \), the \( p_n \) that maximizes Problem 5 is \( p'_{n_5} = [1 + c - \delta + (2 - f)(2c_l - c_M)]/2 \). The corresponding value of the objective function is

\[
\pi_5(p'_{n_5}, p'_{r_5}) = c_l - \frac{c - \delta}{2} - \frac{c_M f}{2} + \frac{(c - 2c_M + c_M f)^2}{4(1 - \delta)} - \frac{c_l (c_l - c_M)}{\delta} + \frac{4}{2} \triangleq F_5(c_M).
\]

To ensure the feasibility of \( (p'_{n_5}, p'_{r_5}) \), we need

\[
\frac{c_l (1 - f) \delta (3 - f) + 1 + \delta (1 - \delta)(1 - f)}{\delta (2 - f)} < p'_{n_5} < \frac{2(\delta - 2f + 1)c_l - \delta^2 + \delta}{2\delta}
\]

\( \iff \quad c_l \triangleq 2c_M - c_M f + 1 - \frac{2(\delta - c_l)(1 - \delta)}{\delta (2 - f)} < c < 2c_M - c_l - c_M f + c_l / \delta \triangleq c' \)

\( \iff \quad \frac{c + c_l (1 - 1/\delta)}{2 - f} < c_M < \frac{2(1 - \delta)(\delta - c_l)}{\delta (2 - f)^2} + \frac{c + \delta - 1}{2 - f} = c_M'. \)

It is easy to verify that \( \pi_5(p'_{n_5}, p'_{r_5}) \) is convex and quadratic in \( c_M \) and it achieves its minimum at \( c_M' \). Note that \( c_l > \frac{c + c_l (1 - 1/\delta)}{2 - f} \iff c < c_l [1 + \delta (1 - f)]/\delta \). Hence, when \( c < [1 + \delta (1 - f)]c_l / \delta \), we have

\[\pi_5(p'_{n_5}, p'_{r_5}) < \pi_5(p'_{n_5}, p'_{r_5}) < F_5(c_l) = \frac{(1 - c - \delta + c_l f)^2}{4(1 - \delta)}.\]

Thus, Problem 5 is dominated by Problem 2. When \( c \geq \frac{[1 + \delta (1 - f)]c_l / \delta \), we have

\[\pi_5(p'_{n_5}, p'_{r_5}) < \pi_5(p'_{n_5}, p'_{r_5}) < F_5 \left( \frac{c + c_l (1 - 1/\delta)}{2 - f} \right) = \frac{\delta - c_l}{\delta^2} \left( \frac{\delta (5c_l - \delta + 1) - c_l - c_l / \delta}{2 - f} \right) - \frac{c_l}{2 - f}.\]
Note that
\[
F_b \left( \frac{c + c_I (1 - 1/\delta)}{2 - f} \right) - \frac{(\delta - c_I)(3c_I - \delta - 2c_I f + c_I/\delta + 1 - 2c)}{4\delta} = \frac{f(\delta - c_I)[c_I (1 + \delta (1 - f)) - c]}{2\delta^2 (2 - f)} < 0
\]
\[
\Leftrightarrow c > [1 + \delta (1 - f)] c_I / \delta.
\]

Thus, Problem 5 is again, dominated by Problem 2. Therefore, in Case 2.2.1, the manufacturer’s global optimal solution is characterized by \( \max \{ \pi_1(p_{n_1}), \pi_2(p_{n_2}), \pi_3(p_{n_3}) \} \), i.e., (F.15) and (F.16).

**Case 2.2.2: \( 0 < c_I < c^M < \min \{ (\delta + c_M)/2, \delta \} \) and \( c_M < c_I \).** In this case, we need to compare Problem 1, 2, 3, 4, and 5. We further divide this case into 3 subcases: (a) \( c \leq \gamma \); (b) \( c \geq \gamma \); (c) \( \gamma < c < \gamma \). In case (a) and (b), Problem 5 is dominated by either Problem 1 or Problem 4; in case (c), we need to compare all five problems. In this case, we have
\[
\begin{cases}
  c_2 \leq \gamma < 1 + \delta - f c_I, \\
  c_2 \leq \gamma < c_4.
\end{cases}
\]

\( c_2 \leq \gamma < 1 + \delta - f c_I \) is because \( F_b(c_M) \triangleq \gamma - (1 + \delta - f c_I) \) is linearly increasing in \( c_M \). It is easy to verify that \( F_b(c_I) = - (\delta - c_I)(1 + \delta) / \delta < 0 \). And
\[
c' > c_2 \Leftrightarrow c_M (2 - f) - 5c_I + 3\delta + 2c_I f - \delta f - 1 + c_I / \delta + \frac{2(\delta - c_I)(1 - \delta)}{\delta (2 - f)} > 0,
\]
which is due to \( c_I < (\delta + c_M)/2 \Leftrightarrow c_M > 2c_I - \delta \) that leads to
\[
c_M (2 - f) - 5c_I + 3\delta + 2c_I f - \delta f - 1 + \frac{c_I}{\delta} + \frac{2(\delta - c_I)(1 - \delta)}{\delta (2 - f)} > f(1 - \delta)(\delta - c_I) > 0.
\]

\( c_2 \leq \gamma < c_4 \) is because \( \gamma < c_4 \Leftrightarrow -(2 - f)(c_I - c_M) < 0 \) and \( F_M(c_M) \triangleq \gamma - c_2 \) is linearly increasing in \( c_M \) and \( F_M(2c_I - \delta) = 0 \).

We define
\[
F_b(c) \triangleq \frac{(\delta - c_I) \left[ \frac{c_I (1 + \delta (1 - f)) (3 - f) + \delta (1 - \delta) (1 - f)}{4(2 - f)} \right] - c}{\delta (2 - f)} = \frac{(1 - c + \delta - c_M f)^2}{4(1 + \delta f)}.
\]

One can verify that \( F_b(c) \) is concave quadratic in \( c \). We denote \( F_b(c_2) \triangleq F_b(c_M) \), which is concave and quadratic in \( c_M \). We have
\[
\begin{cases}
  F_b(2c_I - \delta) = \frac{f^2 (\delta - c_I) (1 - \delta)}{4(1 + \delta f)(2 - f)^2} > 0, \\
  F_b(c_I) = \frac{f^2 (\delta - c_I) (\delta + \delta (24 - 36f f^2 - f^3)}{4(1 + \delta f)(2 - f)^2} > 0.
\end{cases}
\]

It is easy to verify that \(-f^3 + 12f^2 - 36f + 24 \geq -1 \) and hence, \( 8 + \delta (24 - 36f f^2 - f^3) > 0 \). One can show that
\[
F_b(c) > 0 \Leftrightarrow c < 1 + \delta - c_M f + \frac{2(1 + 3\delta)}{\delta (2 - f)} (c_I - \delta + \delta \sqrt{\frac{f^2 (\delta - c_I) (2c_M - 4c_I + 2\delta + c_I + c_M f)}{4(1 + \delta f)}}) \Leftrightarrow c_9.
\]

Recall
\[
\frac{(1 - c + \delta - \delta f)^2}{4(\delta f^2 - 4\delta f + 3\delta + 1)} > \frac{(1 - c + \delta - c_M f)^2}{4(1 + 3\delta)} \Leftrightarrow c < 1 + \frac{\delta (1 - f) + (\delta - c_M f) \sqrt{\frac{1 + \delta (1 - f)(3 - f)}{1 + 3\delta}}}{1 - \sqrt{\frac{1 + \delta (1 - f)(3 - f)}{1 + 3\delta}}} \Leftrightarrow c_8.
\]
Hence,

\[
c_8 - c_2 > 0 \iff \frac{1 + \delta(1-f)(3-f)}{1 + 3\delta} < \frac{2[1 + \delta(1-f)(3-f)](\delta - c_I)}{\delta(f^2 - 6\delta f + 6\delta + 2) - 2[1 + \delta(1-f)(3-f)]c_I - \delta f(2-f)c_M}
\]

(F.19)

Because \(2c_I - \delta < c_M < c_I\), we have

\[
\delta(f^2 - 6\delta f + 6\delta + 2) - 2[1 + \delta(1-f)(3-f)]c_I - \delta f(2-f)c_M
\]

\[
\delta(f^2 - 6\delta f + 6\delta + 2) - 2[1 + \delta(1-f)(3-f)]c_I - \delta f(2-f)c_I = (\delta - c_I)(\delta(f - 3) - 3) > 0, \forall f \in [0, 1].
\]

Hence, to show (F.19), it suffices to show

\[
\frac{1 + \delta(1-f)(3-f)}{1 + 3\delta} < \frac{2[1 + \delta(1-f)(3-f)](\delta - c_I)}{\delta(f^2 - 6\delta f + 6\delta + 2) - 2[1 + \delta(1-f)(3-f)]c_I - \delta f(2-f)(2c_I - \delta)}
\]

\[
\iff \frac{1}{\sqrt{1 + 3\delta}} < \frac{1 + \delta(1-f)(3-f)}{1 + 3\delta} \iff \delta - (1 - \delta)^2 < 0,
\]

which implies that \(\pi_4(p_{n_4}, p_{r_4})\) is dominated for \(0 \leq c < c_2\).

Note that as \(c_M < c_I\), it is obvious that \(1 + \delta - f c_M > 1 + \delta - f c_I\) and \(\frac{(1-c+\delta-c_{M})^2}{4(1+3\delta)} < \frac{(1-c+\delta-c_{M})^2}{4(1+3\delta)}\).

Recall

\[
\frac{(1-c+\delta-c_{M})^2}{4(1+3\delta)} > \frac{(1-c+2c_I-c_{-c_{I}})^2}{4(1-\delta)} \iff c > 1 - \frac{\delta - c_I(2-f) + (\delta - c_{M}f)\sqrt{\frac{1-\delta}{1+3\delta}}}{1 - \sqrt{\frac{1-\delta}{1+3\delta}}}
\]

Thus, we have

\[
c_7 - c_5 = -\frac{(1+\delta)[1 - \delta + f(c_I - c_{M})]}{4} \left(1 + \frac{1 - \delta}{\sqrt{1 + 3\delta}}\right) \sqrt{\frac{1 - \delta}{1 + 3\delta}} < 0.
\]

Therefore, \(\max\{\pi_1(p_{n_1}), \pi_2(p_{n_2}), \pi_3(p_{n_3}), \pi_4(p_{n_4}, p_{r_4})\}\) is given by

- If \(\delta < \frac{2f}{f^2 - 2f + 4}\) and \(c_4 \leq c_7\),
• If $\delta < \frac{2f}{f^2 - 2f + 4}$ and $c_4 > c_7$

When $0 \leq c_I < \bar{c}_I$,

$$\max_{1 \leq i \leq 4} \{\pi_i\} = \begin{cases} \text{unprofitable}, & \text{if } c \geq 1 + \delta - f c_M, \\ \pi_4 = \frac{(1 - c - \delta - c_M f)^2}{4(1 + \delta)}, & \text{if } c_7 \leq c < 1 + \delta - f c_M, \\ \pi_2 = \frac{(1 - 2c + \delta + c_I f - 1)^2}{4(1 + \delta)}, & \text{if } 0 \leq c < c_7. \end{cases}$$

When $\bar{c}_I \leq c_I < \bar{c}_I$,

$$\max_{1 \leq i \leq 4} \{\pi_i\} = \begin{cases} \text{unprofitable}, & \text{if } c \geq 1 + \delta - f c_M, \\ \pi_4 = \frac{(1 - c - \delta - c_M f)^2}{4(1 + \delta)}, & \text{if } c_9 \leq c < 1 + \delta - f c_M, \\ \pi_2 \text{ and } \pi_1 = \frac{\delta - c_I}{\delta(2 - f)} \left(\delta \frac{(1 - \delta)(1 - f)}{\delta(2 - f)} - c\right), & \text{if } 0 \leq c < c_9. \end{cases}$$

When $\bar{c}_I \leq c_I < \min\{\delta, (\delta + c_M)/2\}$,

$$\max_{1 \leq i \leq 4} \{\pi_i\} = \begin{cases} \text{unprofitable}, & \text{if } c \geq 1 + \delta - f c_M, \\ \pi_4 = \frac{(1 - c - \delta - c_M f)^2}{4(1 + \delta)}, & \text{if } c_9 \leq c < 1 + \delta - f c_M, \\ \pi_2 \text{ and } \pi_1 = \frac{\delta - c_I}{\delta(2 - f)} \left(\delta \frac{(1 - \delta)(1 - f)}{\delta(2 - f)} - c\right), & \text{if } 0 \leq c < c_9, \end{cases}$$

• If $\delta \geq \frac{2f}{f^2 - 2f + 4}$,

When $0 \leq c_I < \bar{c}_I$,

$$\max_{1 \leq i \leq 4} \{\pi_i\} = \begin{cases} \text{unprofitable}, & \text{if } c \geq 1 + \delta - f c_M, \\ \pi_4 = \frac{(1 - c - \delta - c_M f)^2}{4(1 + \delta)}, & \text{if } 0 \leq c < 1 + \delta - f c_M. \end{cases}$$

When $\bar{c}_I \leq c_I < \bar{c}_I$,

$$\max_{1 \leq i \leq 4} \{\pi_i\} = \begin{cases} \text{unprofitable}, & \text{if } c \geq 1 + \delta - f c_M, \\ \pi_4 = \frac{(1 - c - \delta - c_M f)^2}{4(1 + \delta)}, & \text{if } c_9 \leq c < 1 + \delta - f c_M, \\ \pi_2 \text{ and } \pi_1 = \frac{\delta - c_I}{\delta(2 - f)} \left(\delta \frac{(1 - \delta)(1 - f)}{\delta(2 - f)} - c\right), & \text{if } 0 \leq c < c_9. \end{cases}$$

When $\bar{c}_I \leq c_I < \min\{\delta, (\delta + c_M)/2\}$,

$$\max_{1 \leq i \leq 4} \{\pi_i\} = \begin{cases} \text{unprofitable}, & \text{if } c \geq 1 + \delta - f c_M, \\ \pi_4 = \frac{(1 - c - \delta - c_M f)^2}{4(1 + \delta)}, & \text{if } c_9 \leq c < 1 + \delta - f c_M, \\ \pi_2 \text{ and } \pi_1 = \frac{\delta - c_I}{\delta(2 - f)} \left(\delta \frac{(1 - \delta)(1 - f)}{\delta(2 - f)} - c\right), & \text{if } 0 \leq c < c_9, \end{cases}$$

Case 2.2.2.1: $c \leq 2c_M - \delta - c_M f + 1 - \frac{2(\delta - c_I)(1 - \delta)}{\delta(2 - f)} = c'$. In this case, we have $0 \leq c_I < \frac{\delta f(1 - \delta)}{2 - \delta + \delta(1 - f)(3 - f)}$. Hence, we have

• If $\delta < \frac{2f}{f^2 - 2f + 4}$,

When $\bar{c}_I \leq c_I < \bar{c}_I$,

$$\max_{1 \leq i \leq 4} \{\pi_i\} = \pi_2 = \pi_1 = \frac{(\delta - c_I)}{\delta(2 - f)} \left(\delta \frac{(1 - \delta)(1 - f)}{\delta(2 - f)} - c\right), \text{ if } 0 \leq c < \bar{c}_I.$$
\[
\max_{1 \leq i \leq 4} \{ \pi_i \} = \begin{cases} 
\pi_4 = \frac{(1-c+\bar{c}-c_M f)^2}{4(1+3\delta)}, & \text{if } \min\{c_9, \bar{c}'\} \leq c < \bar{c}', \\
\pi_2 = \pi_1 = \frac{\delta_c}{2(2-f)} \bigg( \frac{c(\delta-c)+\delta(1-\delta)(1-f)}{\delta(2-f)} - c \bigg), & \text{if } 0 \leq c < \min\{c_9, \bar{c}'\}. 
\end{cases}
\]

When \( \bar{c}_I \leq c_I < \min\{\delta, \delta + c_M/2\}, \)
\[
\max_{1 \leq i \leq 4} \{ \pi_i \} = \begin{cases} 
\pi_4 = \frac{(1-c+\bar{c}-c_M f)^2}{4(1+3\delta)}, & \text{if } \min\{c_9, \bar{c}'\} \leq c < \bar{c}', \\
\pi_2 = \pi_1 = \frac{\delta_c}{2(2-f)} \bigg( \frac{c(\delta-c)+\delta(1-\delta)(1-f)}{\delta(2-f)} - c \bigg), & \text{if } c_2 \leq c < \min\{c_9, \bar{c}'\}, \\
\pi_1 = \frac{(c-c'\delta-f)^2}{4(2-2f+4)}, & \text{if } 0 \leq c \leq c_2. 
\end{cases}
\]

**Case 2.2.2.2:** \( c \geq 2c_M - c_I - c_M f + c_I/\delta = \bar{c}' \). In this case, we will combine Problem 4 with (F.15) and (F.16).

When \( \delta < \frac{2f}{f^2-2f+4} \), we show that \( \bar{c}' \geq c_7 \). Note that
\[
\bar{c}' \geq c_7 \iff \sqrt{\frac{1-\delta}{1+3\delta}} \geq \frac{(2-f)(c_I-c_M) + (1-\delta)(\delta - c_I)/\delta}{1+2c_M - c_I(1-\delta)/\delta} \triangleq F_{11}(c_M) \Rightarrow \frac{dF_{11}(c_M)}{dc_M} \propto f - \delta(4-f).
\]

Therefore, when \( \delta < \frac{2f}{f^2-2f+4} \), \( F_{11}(c_M) \) is increasing in \( c_M \). Thus, to show \( \bar{c}' \geq c_7 \), it suffices to show that
\[
\sqrt{\frac{1-\delta}{1+3\delta}} \geq F_{11}(c_I) \iff \frac{1-\delta}{1+3\delta} \geq \frac{1-f}{1+3\delta} \iff (1-\delta)^2 > (1-\delta)(1+3\delta) \iff \delta^2 > -3\delta^2.
\]

When \( \delta \geq \frac{2f}{f^2-2f+4} \), \( F_{11}(c_M) \) is decreasing in \( c_M \). Thus, to show \( \bar{c}' \geq c_7 \), it suffices to show that
\[
\sqrt{\frac{1-\delta}{1+3\delta}} \geq F_{11}(c_I) \iff \frac{1-\delta}{1+3\delta} \geq \frac{1+\delta(1-f)}{1+3\delta} \iff \delta < \frac{2f}{f^2-2f+4}.
\]

By the previous analysis, we know that when \( c_4 > c_7 \), we have \( c_4 > c_9 \). Hence, we have
\[
\max_{1 \leq i \leq 4} \{ \pi_i \} = \begin{cases} 
\text{unprofitable}, & \text{if } c \geq 1 + \delta - f c_M, \\
\pi_4 = \frac{(1-c+\bar{c}-c_M f)^2}{4(1+3\delta)}, & \text{if } c \leq c < 1 + \delta - f c_M. 
\end{cases}
\]

**Case 2.2.2.3:** \( 2c_M - \delta - c_M f + 1 - \frac{2(\delta-c_f)(1-\delta)}{\delta(2-f)} < c < 2c_M - c_I - c_M f + c_I/\delta \). In this case, we show that Problem 1, 2 and 3 are dominated either by Problem 4 or by Problem 5. Because of (F.17), we only need to compare \( \pi_4(p_5', p_5') \) with \( \frac{\delta_c}{2(2-f)} \left( \frac{c(\delta-c)+\delta(1-\delta)(1-f)}{\delta(2-f)} - c \right) \) and \( \frac{(c-c_9+c_M f-1)^2}{4(1-\delta)} \).

One can verify that \( F_{10}(c) \triangleq \pi_5(p_5', p_5') - \frac{\delta_c}{2(2-f)} \left( \frac{c(\delta-c)+\delta(1-\delta)(1-f)}{\delta(2-f)} - c \right) \) is concave and quadratic in \( c \) with
\[
\begin{cases}
F_{10} \left( 2c_M - \delta - c_M f + 1 - \frac{2(\delta-c_f)(1-\delta)}{\delta(2-f)} \right) = 0, \\
F_{10}(c_4) = \frac{(c_4-c_M f)^2(2-f)^2}{4(1-\delta)} > 0.
\end{cases}
\]

Hence, \( F_{10}(c) \geq 0 \).

One can verify that
\[
\pi_5(p_5', p_5') > \frac{(1-c+2c_I-\delta-c_f)^2}{4(1-\delta)} \iff c > 1 - \delta + c_I + c_M - \frac{c_I f}{2} - \frac{c_M f}{2} - \frac{2(1-\delta)(\delta-c_I)}{\delta(2-f)}.
\]

It is easy to show that
\[
1 - \delta + c_I + c_M - \frac{c_I f}{2} - \frac{c_M f}{2} - \frac{2(1-\delta)(\delta-c_I)}{\delta(2-f)} < c_4 \iff -(2-f)(c_I - c_M)/2 < 0.
\]

When \( 0 \leq c_I < \frac{\delta_c}{2(\delta-c_f)\delta(2-f)} \), we also have \( 1 - \delta + c_I + c_M - \frac{c_I f}{2} - \frac{c_M f}{2} - \frac{2(1-\delta)(\delta-c_I)}{\delta(2-f)} < 0 \). This is because
\[
1 - \delta + c_I + c_M - \frac{c_I f}{2} - \frac{c_M f}{2} - \frac{2(1-\delta)(2c_M + \delta f)}{\delta f^2 - 4\delta f + 4} < c_4.
\]
And
\[
\frac{2(1-\delta)(2c_M + \delta f) - c_M - \delta f(1-\delta)}{\delta f^2 - 4\delta f + 4} = \frac{\delta f(1-\delta)}{\delta f^2 - 4\delta f + 4}
\]
\[
= \frac{(2-\delta) c_M}{\delta f^2 - 4\delta f + 4} = \frac{2\delta f(1-\delta)}{\delta f^2 - 4\delta f + 4} \frac{2}{\delta f^2 - 4\delta f + 4}
\]
\[
\geq \frac{2}{\delta f^2 - 4\delta f + 4} = \frac{2}{\delta f^2 - 4\delta f + 4}
\]
Thus, Problem 1,2 and 3 are dominated either by Problem 4 or by Problem 5.

We now compare Problem 4 with Problem 5. Note that
\[1 + \delta - f c_M - (2c_M - c_I - c_M f) \delta = 1 + \delta + c_I (1-\delta) - 2c_M \]
which is decreasing in \(c_M\)
\[
1 + \delta + c_I (1-\delta) - 2c_M > 1 + \delta + c_I (1-\delta) - 2c_M = (1 + \delta)(\delta - c_I) \delta > 0.
\]
One can verify that \(\pi_4(p_{n_k}) - \pi_5(p'_{n_k}, p'_{n_k})\) is a concave and quadratic function of \(c\) and at \(c = 2c_M - c_I - c_M f + c_I \delta\), we have \(\pi_4(p_{n_k}) - \pi_5(p'_{n_k}, p'_{n_k}) = \frac{(c_M c_I + \delta f)^2}{(1 + 3\delta)} > 0\) Thus we have
\[
\pi_4(p_{n_k}) > \pi_5(p'_{n_k}, p'_{n_k}) \iff c > \frac{\delta f(1 - \delta) + c_M (1 + 3\delta - 2\delta f) + (2c_I - c_M) \sqrt{(1 - \delta)(1 + 3\delta)}}{2\delta} \equiv c_{10} < c'.
\]
In this case we have
\[
\text{max}_{1 \leq i \leq 5} \pi_i = \begin{cases} 
\pi_4 = \frac{(1-c+\epsilon-c_M f)^2}{4(1+3\delta)}, & \text{if } \max\{c_{10}, c'\} \leq c < c', \\
\pi_5 = c_I - \frac{\epsilon}{2} - \frac{\delta f}{4} - \frac{c_M f}{2} + \frac{(c_M c_I + \delta f)^2}{4(1+3\delta)} - \frac{c_I (c_I - c_M)}{4} + \frac{1}{4}, & \text{if } c' < c < \max\{c_{10}, c'\}.
\end{cases}
\]
Note that when \(\delta \geq \frac{2f}{2 - 2f + 4}\) and \(c_I < \frac{\delta f(1 - \delta)}{2 - \delta + (1-f)(3-f)}\), we have \(c_{10} < 0\). This is because
\[
c_{10} < 0 \iff \frac{\sqrt{(1 - \delta)(1 + 3\delta)}}{c_M - 2c_I + \delta} > \frac{\delta f(1 - \delta) + c_M (1 + 3\delta - 2\delta f)}{c_M - 2c_I + \delta} \equiv F_{12}(c_M)
\]
\[
\Rightarrow \frac{dF_{12}(c_M)}{dc_M} = \frac{2\delta f(1 - \delta) - c_M (1 + 3\delta - 2\delta f)}{c_M - 2c_I + \delta} > 0 \iff c_I < \frac{(2-f) c}{1 + 3\delta - 2\delta f}, \quad \text{as implied by } c_I < \frac{\delta f(1 - \delta)}{2 - \delta + (1-f)(3-f)}.
\]
\[
\Rightarrow \frac{\delta f(1 - \delta)}{2 - \delta + (1-f)(3-f)} < \frac{(2-f) c}{1 + 3\delta - 2\delta f} \iff \delta > \frac{f}{4 - 3f + f^2}, \quad \text{as implied by } \delta \geq \frac{2f}{f^2 - 2f + 4}.
\]
\[
\Rightarrow \delta \geq \frac{2f}{f^2 - 2f + 4} \iff (f - 2)^2 \geq 0.
\]
Hence, \(F_{12}(c_M)\) is increasing in \(c_M\). Therefore, to show \(\sqrt{(1 - \delta)(1 + 3\delta)} > F_{12}(c_M)\), it suffices to show
\[
\sqrt{(1 - \delta)(1 + 3\delta)} > F_{12}(c_I) = \frac{\delta f(1 - \delta) + c_I (1 + 3\delta - 2\delta f)}{\delta f(1 - \delta) + c_I (1 + 3\delta - 2\delta f)} > 0.\]
Note that \(\frac{dF_{12}(c_I)}{dc_I} = \frac{2\delta f(1 - \delta)}{(1-f)^2} > 0\). Hence, \(F_{12}(c_I)\) is increasing in \(c_I\). Therefore, to show \(\sqrt{(1 - \delta)(1 + 3\delta)} > F_{12}(c_I)\), it suffices to show that
\[
\sqrt{(1 - \delta)(1 + 3\delta)} > F_{12} \left( \frac{\delta f(1 - \delta)}{2 - \delta + (1-f)(3-f)} \right) \equiv F_{12} \left( \frac{\delta f(1 - \delta)}{2 - \delta + (1-f)(3-f)} \right).
\]
Note that
\[
\sqrt{(1 - \delta)(1 + 3\delta)} > F_{12} \left( \frac{\delta f(1 - \delta)}{2 - \delta + (1-f)(3-f)} \right) \iff \frac{\delta f(1 - \delta)}{2 - \delta + (1-f)(3-f)} = \frac{(f + 2)^2}{3(2-f)^2 + (f + 2)^2} = \frac{2f}{f^2 - 2f + 4}.
\]
As a result, we have \(c_{10} < 0\) when \(\delta \geq \frac{2f}{2 - 2f + 4}\) and \(c_I < \frac{\delta f(1 - \delta)}{2 - \delta + (1-f)(3-f)}\).
Finally, we show that when \(c_M < c_I\) and \(\delta \geq \frac{2f}{2 - 2f + 4}\), we have
\[
c_I \geq c' \quad \text{and} \quad c_{10} \geq c'.
\]
It is easy to verify that $c_{10}$ and $\zeta'$ are linearly increasing in $c_I$ and $c_{10} = \zeta' \Leftrightarrow c_I = \hat{c}_I$. One can verify that $c_{10}(\delta) - \zeta'(\delta) = \frac{\sqrt{1 + 3\delta - \sqrt{1 - \delta}}}{\delta + \sqrt{1 - \delta}} \geq 0$. Hence, $c_{10} \geq \zeta'$ if and only if $c_I \geq \hat{c}_I$.

For $c_9$ and $\zeta$, one can verify that
\[
c_9 = \zeta' \Leftrightarrow \begin{cases} s_1 = \frac{c_{14} + (\delta - c_M)}{2} \sqrt{\frac{1 - \delta}{1 + 3\delta}} - \frac{8\delta}{1 + 3\delta} f[(3 - f)\sqrt{1 + 3\delta - \sqrt{1 - \delta}}] (\delta - c_M) \sqrt{1 - \delta} - \frac{16\delta - f(1 + 3\delta)(4 - f)}{16\delta - f(1 + 3\delta)(4 - f)} > 0, \\
s_2 = \delta - c_I, \end{cases}
\]
with $s_1 > (c + c_M)/2 > \hat{c}_I$.

This is because
\[
s_1 - \frac{\delta + c_M}{2} = \frac{(\delta - c_M)\sqrt{1 - \delta}}{1 + 3\delta} \left[ \frac{8\delta}{1 + 3\delta} - f[(3 - f)\sqrt{1 + 3\delta - \sqrt{1 - \delta}}] (\delta - c_M) \sqrt{1 - \delta} - \frac{16\delta - f(1 + 3\delta)(4 - f)}{16\delta - f(1 + 3\delta)(4 - f)} \right] > 0
\]
\[
\Leftrightarrow 1 - \frac{16\delta - 2f[(3 - f)(1 + 3\delta) - \sqrt{(1 - \delta)(1 + 3\delta)}]}{16\delta - f(1 + 3\delta)(4 - f)} > 0 \Leftrightarrow \delta[1 + 3(2 - f)^2] > 1 - (2 - f)^2,
\]
which is obviously true, as $1 - (2 - f)^2 \leq 0$. And
\[
\frac{\delta + c_M}{2} - \hat{c}_I = \frac{-(\delta - c_M)\sqrt{1 - \delta}}{1 + 3\delta} \left[ \frac{8\delta}{1 + 3\delta} - f[(1 + 3\delta - \sqrt{1 - \delta}) (\delta - c_M) \sqrt{1 - \delta} - \frac{16\delta - f(1 + 3\delta)(4 - f)}{16\delta - f(1 + 3\delta)(4 - f)} \right] > 0
\]
\[
\Leftrightarrow \frac{8\delta - f[(1 + 3\delta - \sqrt{1 - \delta}) \sqrt{1 + 3\delta}]}{16\delta - f(1 + 3\delta)(4 - f)} > \frac{1}{2} \Leftrightarrow f(1 + 3\delta)(2 - f) + 2f \sqrt{1 - \delta} > 0.
\]

In addition, we have $c_9$ is increasing in $c_I$ for $c_M < c_I < \min\{\delta, (\delta + c_M)/2\}$. Note that to show $c_9$ is increasing in $c_I$, it suffices to show that $F_{14}(c_M) \equiv c_I - \delta + \delta \sqrt{f[(3 - c_I)(2\delta - c_I(4 - f) + c_M - 2\delta - f)]} / (1 + 3\delta)$ is increasing in $c_I$.

Note that
\[
\frac{dF_{14}(c_I)}{dc_I} = 1 + \frac{8c_I f - 2c_M f - 6f f - 2c_I f^2 + c_M f^2 + \delta f^2}{2(1 + 3\delta) \sqrt{f[(3 - c_I)(2\delta - c_I(4 - f) + c_M - 2\delta - f)]}}
\]
and $\frac{dF_{14}(c_I)}{dc_I} > 0 \Leftrightarrow F_{14}(c_M) > 0$.

where $F_{14}(c_M) \equiv 2f[c_I(4 - f) - f(2 - f)c_M + \delta f(f - 6) + (2 + 6\delta) \sqrt{f[(3 - c_I)(2\delta - c_I(4 - f) + c_M - 2\delta - f)]} / (1 + 3\delta)$.

\[
\frac{dF_{14}(c_M)}{dc_M} \propto \frac{1}{\sqrt{f\delta[2\delta - c_I(4 - f) + c_M - 2\delta - f]}} - 1 > 0 \Leftrightarrow (\delta - c_I)(1 + 3\delta) - f\delta[2\delta - c_I(4 - f) + c_M - 2\delta - f] > 0
\]
\[
\Leftrightarrow -\delta f(2 - f)c_M - (3\delta + 1)(c_I - \delta) - \delta f(2\delta + c_I f - 4)) > 0,
\]
which is implied by
\[
-\delta f(2 - f)c_M - (3\delta + 1)(c_I - \delta) - \delta f(2\delta + c_I f - 4)) = (\delta - c_I)(1 + 3\delta - 2\delta f) > 0.
\]

Hence, $F_{14}(c_M)$ is increasing in $c_M$. Thus, to show $F_{14}(c_M) > 0$, it suffices to show $F_{14}(2c_I - \delta) > 0$. Note that $F_{14}(2c_I - \delta) = f(\delta - c_I) \sqrt{\frac{1 + 3\delta}{\delta - 2}} > 0$, as $1/\delta + 3 > 4$.

Furthermore, when $c_I = c_M$, we have $\zeta'(c_M) - c_9(c_M) > 0$. This is because
\[
\zeta'(c_M) - c_9(c_M) = \frac{2\delta - c_M}{2 - f} \left( 2 + f - (1 + \delta) \sqrt{\frac{2f}{\delta(1 + 3\delta)}} \right) > 0 \Leftrightarrow 2 + f > \sqrt{\frac{2f(1 + \delta)^2}{\delta(1 + 3\delta)}}.
\]

Note that $\frac{(1 + \delta)^2}{\delta(1 + 3\delta)}$ is decreasing in $\delta$, as its derivative with respect to $\delta$ is $\left( \frac{2}{1 + 3\delta} - \frac{1}{2} \right) \left( \frac{2}{1 + 3\delta} + \frac{1}{2} \right) < 0$. Hence, to show $2 + f > \sqrt{\frac{2(1 + \delta)^2}{\delta(1 + 3\delta)}}$, it suffices to show $2 + f > \sqrt{\frac{2f(1 + \delta)^2}{\delta(1 + 3\delta)}}$, when $\delta = \frac{2f}{f^2 - 2f + 4}$. In this case, we have
\[
2 + f > \sqrt{\frac{2f(1 + \delta)^2}{\delta(1 + 3\delta)}} \Leftrightarrow \frac{16f^2(f^2 + 2f + 4)}{(f^2 - 2f + 4)^2} > 0.
\]
Consequently, we have, when $\delta \geq \bar{\delta}$,

$$c_9 < c' \quad \text{and} \quad c_{10} < c' \quad \text{if} \quad c_M < c_I < \hat{c}_I; \quad c_9 \geq c' \quad \text{and} \quad c_{10} \geq c' \quad \text{if} \quad \hat{c}_I \leq c_I < \min\{\delta, (\delta + c_M)/2\}.$$

Then the global optimal solution of the manufacturer is summarized in Table F.2, below are the notations used in Table F.2.

$$
\begin{align*}
\delta &= \frac{2\sqrt{f}}{f^2 - f + 4}, \\
\hat{c}_I &= \frac{1}{2} \left( c_M + \tilde{\delta} + \frac{f(\delta - c_M)(1 - \delta)}{2(1 - \delta) - (2 - f)\sqrt{(1 - \delta)(1 + 3\tilde{\delta})}} \right), \\
\tilde{c}_I &= \frac{\tilde{\delta}}{2} \left( 1 - \frac{(1 - \delta)(1 - f)}{1 + \delta(1 - f)(1 - f)} \right), \\
\bar{c} &= \frac{2c_I[1 + \delta(1 - f)(3 - f)] + (3 - f)(3 - f)}{\delta(2 - f)}, \\
\tilde{c} &= c_M - \delta^2 + \delta(1 + c_M)(3 - 2f)], \\
c' &= 2c_M - \delta - c_M f + 1 - \frac{2(\delta - c_I)(1 - \delta)}{\delta(2 - f)}, \\
c'^\prime &= 2c_M - c_I - c_M f + c_I/\delta, \\
c_2 &= \frac{2(c_I[1 + \delta(1 - f)(3 - f)] + (3 - f)(3 - f))}{\delta(2 - f)} - 1 - \delta(1 - f), \\
c_4 &= 1 - \delta + c_I(2 - f) - \frac{2(\delta - c_I)(1 - \delta)}{\delta(2 - f)}, \\
c_5 &= 1 + \tilde{\delta} - c_M f - \frac{2(\delta - c_I)}{\delta(2 - f)}, \\
c_6 &= \frac{1}{2 - f} \left[ 64\tilde{\delta} + \frac{2\tilde{\delta} - c_M}{\delta} + c_I \left( 6 - 2f - 2\sqrt{\frac{2\tilde{\delta}}{1 + 3\delta}} + f^2 \right) - \delta \left( f + 2(3c_I - 1)\sqrt{\frac{2\tilde{\delta}}{\delta(1 + 3\tilde{\delta})}} + 4 \right) \right], \\
c_{10} &= \frac{2\tilde{\delta} - c_M}{\delta} + \frac{c_I - \delta - c_M}{\delta(1 - \delta)(1 + 3\tilde{\delta}) + (2c_I - c_M)\sqrt{(1 - \delta)(1 + 3\tilde{\delta})}}.
\end{align*}
$$

Some of the cases in Table F.2 can be combined after we define several auxiliary functions.

First, notice that

$$c' - c_2 = c_M(2 - f) + (1 - \delta) \left( 1 - \frac{2(\delta - c_I)}{\delta(2 - f)} \right) - \left( \frac{2(c_I[1 + \delta(1 - f)(3 - f)] + (1 - \delta)(1 - f))}{\delta(2 - f)} - 1 - \delta(1 - f) \right)$$

$$= (2 - f)[c_M - (2c_I - \delta)] > 0.$$  \hfill (F.22)

(F.22) is true when $c_M < c_I < \min\{\delta, (\delta + c_M)/2\}$.

Second, we prove that $c_{10} > c'$ for $\delta < \bar{\delta}$ when $c_M \leq c_I < \min\{\delta, (\delta + c_M)/2\}$. Note that $c_{10} > c' \Leftrightarrow F_{15}(c_M) \geq 0$, where

$$F_{15}(c_M) \triangleq \frac{\delta(1 - \delta) + c_M(1 + 3\tilde{\delta} - 2\tilde{\delta} f) + (2c_I - c_M)\sqrt{(1 - \delta)(1 + 3\tilde{\delta})}}{c_M(\sqrt{1 + 3\tilde{\delta}} - \sqrt{1 - \delta})\sqrt{1 - \delta} - (1 - \delta) \left( \frac{1}{2} - \frac{2(\delta - c_I)}{\delta(2 - f)} \right) + \frac{(c_I - \delta - 2\tilde{\delta})\sqrt{(1 - \delta)(1 + 3\tilde{\delta})}}{\delta}.$$  

As $F_{15}(c_M)$ is decreasing in $c_M$, it suffices to show that $F_{15}(c_I) \geq 0$, since $c_M \leq c_I$.

$$F_{15}(c_I) = \frac{(c_I - \delta)\sqrt{1 - \delta} \left( \frac{\sqrt{1 - \delta} + \sqrt{1 + 3\tilde{\delta}}}{2} - \frac{2\sqrt{1 - \delta}}{2 - f} \right)}{\sqrt{1 - \delta} + \sqrt{1 + 3\tilde{\delta}} - 2\sqrt{1 - \delta} - \frac{2\sqrt{1 - \delta}}{2 - f} \leq 0 \Leftrightarrow [3(2 - f)^2 + (f + 2)^2]\delta + (2 - f)^2 - (2 + f)^2 \leq 0,$$

which is implied by $[3(2 - f)^2 + (f + 2)^2]\delta + (2 - f)^2 - (2 + f)^2 = 0$.

Third, we show that when $c_M < c_I < \min\{\delta, (\delta + c_M)/2\}$ and $\delta \geq \bar{\delta}$ and $\hat{c}_I < c_I$, we have $c' \leq 0$ if $c_I \leq \hat{c}_I$.
Table F.2  The manufacturer’s optimal pricing strategy for general $c_f$, $c_M$, $f$, $\delta$

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<thead>
<tr>
<th>Conditions</th>
<th>$\mathbf{p_h}$</th>
<th>$\mathbf{p_c^+}$</th>
<th>$\mathbf{p_c^-}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Case 1: $c_f \geq \min(\delta, (1 + c_M f)/2)$</td>
<td>$c_f \leq c_I$</td>
<td>$c_f &lt; c_I$</td>
<td>$c_f = c_I$</td>
</tr>
<tr>
<td>$c_M &lt; \delta$</td>
<td>$0 \leq c \leq c_0$</td>
<td>$[c + 1 + \delta + (2 - f)(c_f - c_M)/2] / (c + 1 + \delta + c_M f)/2$</td>
<td>$\infty$</td>
</tr>
<tr>
<td>$c_f \leq c_I$</td>
<td>$c_f &lt; c_I$</td>
<td>$c_f = c_I$</td>
<td>$\infty$</td>
</tr>
<tr>
<td>$c_f \geq c_I$</td>
<td>$0 \leq c + 1 + \delta - f c_M$</td>
<td>$[c + 1 + \delta + (2 - f)(c_f - c_M)/2] / (c + 1 + \delta + c_M f)/2$</td>
<td>$0$</td>
</tr>
<tr>
<td>$\delta &lt; \delta$</td>
<td>$c_f \leq c_I$</td>
<td>$c_f &lt; c_I$</td>
<td>$c_f = c_I$</td>
</tr>
<tr>
<td>$c_f \leq c_I$</td>
<td>$\leq c \leq c_0$</td>
<td>$c_0 \leq c_1 + 1 + \delta - \delta c_I$</td>
<td>$\infty$</td>
</tr>
<tr>
<td>$c_f \leq c_I$</td>
<td>$c_f &lt; c_I$</td>
<td>$c_f = c_I$</td>
<td>$\infty$</td>
</tr>
<tr>
<td>$c_f \geq c_I$</td>
<td>$0 \leq c + 1 + \delta - f c_M$</td>
<td>$[c + 1 + \delta + (2 - f)(c_f - c_M)/2] / (c + 1 + \delta + c_M f)/2$</td>
<td>$0$</td>
</tr>
</tbody>
</table>

Case 2: $c_f < \min(\delta, (1 + c_M f)/2)$

| $\delta < \delta$ | $c_f \leq c_I$ | $c_f < c_I$ | $c_f = c_I$ |
| $c_f \leq c_I$ | $\leq c \leq c_0$ | $c_0 \leq c_1 + 1 + \delta - \delta c_I$ | $\infty$ |
| $c_f \leq c_I$ | $c_f < c_I$ | $c_f = c_I$ | $\infty$ |
| $c_f \geq c_I$ | $0 \leq c + 1 + \delta - f c_M$ | $[c + 1 + \delta + (2 - f)(c_f - c_M)/2] / (c + 1 + \delta + c_M f)/2$ | $0$ |

Case 3: $c_f \leq c_M$

| $\delta < \delta$ | $c_f \leq c_I$ | $c_f < c_I$ | $c_f = c_I$ |
| $c_f \leq c_I$ | $\leq c \leq c_0$ | $c_0 \leq c_1 + 1 + \delta - \delta c_I$ | $\infty$ |
| $c_f \leq c_I$ | $c_f < c_I$ | $c_f = c_I$ | $\infty$ |
| $c_f \geq c_I$ | $0 \leq c + 1 + \delta - f c_M$ | $[c + 1 + \delta + (2 - f)(c_f - c_M)/2] / (c + 1 + \delta + c_M f)/2$ | $0$ |
By (F.20), we know that $c_{10} > c' \Leftrightarrow c_t > \hat{c}_t$ and it is easy to verify that $c_{10} > c'$ when $c_t > \hat{c}_t$. This is because both $c_{10}$ and $c'$ are linearly increasing in $c_t$ and they intersect at $c_t = \hat{c}_t$ and $c_{10}(\delta) - c'(\delta) = \frac{(\delta - \xi)(\sqrt{\delta + 3\delta} - \sqrt{\delta})}{2^3} > 0$ for $c_M < \delta$.

For $c_t < \hat{c}_t$, we have $c' > c_{10}$. We will derive an upper bound for $c'$ and show that upper bound is non-positive.

$$c' = \frac{2(1 - \delta)c_t}{\delta(2 - f)} + c_M(2 - f) - \frac{f(1 - \delta)}{2 - f} \leq \frac{2(1 - \delta)\hat{c}_t}{\delta(2 - f)} + c_M(2 - f) - \frac{f(1 - \delta)}{2 - f}$$

$$= -\frac{1}{\delta(2 - f)} \left( \frac{f(1 - \delta)^2}{2(1 - \delta) - (2 - f)\sqrt{(1 - \delta)(1 + 3\delta)}} - \left[ 1 + \delta(1 - f)(3 - f) \right] \right) c_M$$

$$+ \frac{1 - \delta}{2 - f} \left( 1 - f + \frac{f(1 - \delta)}{2(1 - \delta) - (2 - f)\sqrt{(1 - \delta)(1 + 3\delta)}} \right)$$

Note that

$$\frac{f(1 - \delta)^2}{2(1 - \delta) - (2 - f)\sqrt{(1 - \delta)(1 + 3\delta)}} - \left[ 1 + \delta(1 - f)(3 - f) \right] < 0$$

$$\Leftrightarrow \frac{f(1 - \delta)[2(1 - \delta) + (2 - f)\sqrt{(1 - \delta)(1 + 3\delta)}]}{4(1 - \delta) - (2 - f)^2(1 + 3\delta)} < 1 + \delta(1 - f)(3 - f),$$

which is implied by

$$4(1 - \delta) - (2 - f)^2(1 + 3\delta) = -[4 + 3(2 - f)^2]\delta + 4 - (f - 2)^2$$

$$< -[4 + 3(2 - f)^2]\delta + 4 - (f - 2)^2 = -\frac{f(2 - f)^2(f + 4)}{f^2 - 2f + 4} < 0.$$
which is implied by
\[
\frac{2f(1-\delta)}{\sqrt{1-\delta)(1+3\delta)} - 1 + \delta} - (2-f) < 0 \Leftrightarrow \sqrt{\frac{1+3\delta}{1-\delta}} > \left(\frac{2+f}{2-f}\right)^2 \Leftrightarrow [2 + f + 3(2-f)^2]\delta + (2-f)^2 - f - 2 > 0,
\]
(F.25)
since
\[
[2 + f + 3(2-f)^2]\delta + (2-f)^2 - f - 2 > [2 + f + 3(2-f)^2]\delta + (2-f)^2 - f - 2 = \frac{(1+f)(2-f)^2(2+f)}{f^2 - 2f + 4} > 0.
\]

Therefore, when \(c_M < c_1 < \min\{\delta(\delta + c_M)/2\}\) and \(\delta > \tilde{c}_1\) and \(c_1 < \tilde{c}_1\), we have \(c' < 0\) if \(c_1 < \tilde{c}_1\).

Last, for \(\tilde{c}_1 \geq \tilde{c}_1\), we define
\[
c_{12} \equiv \begin{cases} c_0, & \text{if } c_1 < \tilde{c}_1, \\ 0, & \text{if } c_1 \geq \tilde{c}_1, \end{cases}
\]
\[
c_{11} \equiv \begin{cases} c_0, & \text{if } c_1 < \tilde{c}_1, \\ \tilde{c}_1, & \text{if } c_1 \geq \tilde{c}_1. \end{cases}
\]

It is obvious that \(c_{10} > c' > c_2\) when \(\hat{c}_1 < c_1 \leq \tilde{c}_1\). One can also verify that \(c_2\) is linearly increasing in \(c_1\) and \(c_2(\tilde{c}_1) = 0\). Thus, we have \(c_{12} \geq c_{11} \geq c_2\). Consequently, Table F.2 can be reduced to Table F.3.

<table>
<thead>
<tr>
<th>Conditions</th>
<th>(p_c^*)</th>
<th>(\alpha^*)</th>
</tr>
</thead>
<tbody>
<tr>
<td>(0 \leq c_M &lt; \delta)</td>
<td>(c_1 + \delta(1-f)/2, c_1 + \delta(1-f)/2)</td>
<td>(\infty, c_M)</td>
</tr>
<tr>
<td>(\delta &gt; \tilde{c}_1)</td>
<td>(c_1 + \delta(1-f)/2, c_1 + \delta(1-f)/2)</td>
<td>(c_M, \tilde{c}_1)</td>
</tr>
<tr>
<td>(\tilde{c}_1 \geq \tilde{c}_1)</td>
<td>(c_1 + \delta(1-f)/2)</td>
<td>(\infty, \frac{2(1+\delta(1-f)/2)}{2})</td>
</tr>
<tr>
<td>(\delta &gt; \tilde{c}_1)</td>
<td>(c_1 + \delta(1-f)/2)</td>
<td>(c_M, \tilde{c}_1)</td>
</tr>
<tr>
<td>(\tilde{c}_1 \geq \tilde{c}_1)</td>
<td>(c_1 + \delta(1-f)/2)</td>
<td>(\infty, \frac{2(1+\delta(1-f)/2)}{2})</td>
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</tbody>
</table>

**Table F.3**
If we further define
\[
\begin{align*}
\hat{c}_1 & \triangleq c_2, \\
\hat{c}_2 & \triangleq \begin{cases} 
\hat{c}_1 , & \text{if } \delta < \hat{\delta} \text{ or } \delta \geq \hat{\delta}, \hat{c}_1 < c_I, \\
\hat{c}_{11}, & \text{if } \delta \geq \hat{\delta}, \hat{c}_1 \geq c_I,
\end{cases} \\
\hat{c}_3 & \triangleq \begin{cases} 
\hat{c}_{10}, & \text{if } \delta < \hat{\delta} \text{ or } \delta \geq \hat{\delta}, \hat{c}_1 < c_I, \\
\hat{c}_{12}, & \text{if } \delta \geq \hat{\delta}, \hat{c}_1 \geq c_I,
\end{cases}
\end{align*}
\]

then Table F.3 can be further reduced to Table F.4.

It is obvious that $c_{12}, c_{10}$, and $c_{10}$ are increasing in $c_I$. For $c_9$ with $c_I < \min\{\hat{c}_I, (\delta + c_M)/2\}$, we know that
\[
c_9 \propto c_I - \delta + \delta \sqrt{\frac{f(\delta - c_I)(2c_M - 4c_I + 2\delta + c_M f - c_M f)}{\delta(1 + 3\delta)}} \triangleq F_{16}(c_I)
\]
\[
d_{F_{16}(c_I)} = \frac{1 + 8c_I - 2c_M - 6\delta - 2c_M f + c_M f + \delta f}{2(1 + 3\delta)(\delta - c_I)(2c_M - 4c_I + 2\delta + c_M f - c_M f)/(f\delta)}.
\]

Note that for $c_I < \min\{\hat{c}_I, (\delta + c_M)/2\}$, we have
\[
2c_M - c_I(4 - f) + 2\delta - c_M f > 2c_M - \frac{(4 - f)(\delta + c_M)}{2} + 2\delta - c_M f = \frac{f(\delta - c_M)}{2} > 0.
\]
Thus to show $dF_{16}(c_I)/dc_I > 0$ for $c_I < \min\{\hat{c}_I, (\delta + c_M)/2\}$, it suffices to show that
\[
4(1 + 3\delta)(\delta - c_I)(2c_M - 4c_I + 2\delta + c_M f - c_M f)/(f\delta) - (8c_I - 2c_M - 6\delta - 2c_M f + c_M f + \delta f)^2 \triangleq F_{17}(c_I) > 0.
\]

One can verify that $F_{17}(c_I)$ is convex in $c_I$ and it achieves its global minimum at $c_I = \frac{\epsilon\delta(2-f)+\delta(6-f)}{2(4-f)}$. Note that $(\delta + c_M)/2 < \frac{\epsilon\delta(2-f)+\delta(6-f)}{2(4-f)} \Leftrightarrow 2(\delta - c_M) < 0$. Also, we have $F_{17}(\delta + c_M)/2 = (c_M - \delta)^2(1 - \delta)/\delta > 0$. Hence, we have $c_9(c_I)$ is increasing in $c_I$ for $c_I < \min\{\hat{c}_I, (\delta + c_M)/2\}$. This implies that $\hat{c}_1, \hat{c}_2, \hat{c}_3$ are all increasing in $c_I$. In addition, it is easy to verify that $\hat{c}$ and $\hat{\zeta}$ are decreasing in both $f$ and $\delta$.

As in this paper, we focus on Assumption 1, i.e., $c_I \geq c_M$ and $c_M < \delta$ (which implies $(\delta + c_M)/2 < \delta$), thus, we only present the Case 1 and Case 2 in Table F.4 in the main paper. But we do solve the firm’s optimal pricing strategy for general $c_I \geq 0$, $c_M \geq 0$, $f \in [0,1]$ and $\delta \in [0,1]$.

**Proof of Propositions 2, 3, 4 and 5**

In this proof, we derive the comparative statics of optimal prices ($p_u^*, p_v^*, p_w^*$), consumer surplus ($CS$), the product in use per period ($U \triangleq 1 - \theta_1$), the use of old product per period ($U^u \triangleq \theta_2 = \theta_1 = U - Q$), the new products sold per period ($Q \triangleq 1 - \theta_2$), and the per period repair quantity ($R \triangleq (1 - \theta_2)f\alpha = Qf\alpha$), as a function of $c_I$ for $c_I \geq c_M$.

The optimal prices are presented in Table F.2. As we consider Assumption 1, we only use Case 1 and 2 in Table F.2. The $\pi^*, U, Q$, and $CS$ can be derived with the optimal prices. The results are presented in Table F.5.

Below are the labels used in Table F.5.

\[
\begin{align*}
\pi_1 & = \frac{(\epsilon + \delta - f - j)^2}{4(2f - \delta - j + 3\delta)}, \\
\pi_1' & = \frac{(\delta - c_I)}{2} \left( c_I + 1 + \delta(1-f)(3-f) + 3(1-\delta)(1-f) \right), \\
\pi_4 & = \frac{(\delta + c_M f - 1) - \epsilon}{2(4 + 3\delta)}, \\
\pi_5 & = \frac{c_M f}{2} + \frac{1}{4} \left( 1 + \frac{c_M}{3} + \frac{(\epsilon + 2c_M + c_M f)^2}{4(1-\delta)} \right), \\
\pi_5' & = c_I - \frac{\delta}{2} - \frac{c_M f}{2} + \frac{(\epsilon - 2c_M + c_M f)^2}{4(1-\delta)} - \epsilon c_I(\delta - c_M) + \frac{1}{4}. \\
\theta_1(1) & = \frac{(2-f)(1+\epsilon + \delta(1-f))}{2(4-f) - 2(1-\delta)(1-f)}, \\
\theta_1(1') & = c_I/\delta, \\
\theta_1(1) & = c_I/\delta, \\
\theta_1(1') & = \frac{c_M f + \delta}{\delta}, \\
\theta_1(5) & = c_I/\delta.
\end{align*}
\]
<table>
<thead>
<tr>
<th>Case</th>
<th>Conditions</th>
<th>$p'_c$</th>
<th>$p''_c$</th>
<th>$p'''_c$</th>
<th>$\alpha^*$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Case 1: $c_I \geq \min{\delta, (\delta + c_M)/2}$</td>
<td>$0 \leq c \leq \xi$</td>
<td>$[c + 1 + \delta(1 - f)]/2$</td>
<td>$\infty$</td>
<td>$\xi(1 + \delta f)/2$</td>
<td>$0$</td>
</tr>
<tr>
<td></td>
<td>$\xi &lt; c &lt; \xi c$</td>
<td>$[c + 1 + \delta(1 - f)]/2$</td>
<td>$\infty$</td>
<td>$\xi(1 + \delta f)/2$</td>
<td>$0$</td>
</tr>
<tr>
<td></td>
<td>$\xi &lt; c &lt; 1 + \delta - c_M f$</td>
<td>$(c + 1 + \delta + c_M f)/2$</td>
<td>$0$</td>
<td>$\infty$</td>
<td>$1$</td>
</tr>
<tr>
<td>$c_M \geq \delta$</td>
<td>$0 \leq c &lt; 1 + \delta(1 - f)$</td>
<td>$[c + 1 + \delta(1 - f)]/2$</td>
<td>$\infty$</td>
<td>$\xi(1 + \delta f)/2$</td>
<td>$0$</td>
</tr>
</tbody>
</table>

Case 2: $c_M \leq c_I < \min\{\delta, (\delta + c_M)/2\}$

| $\xi < c < \xi_I$ | $[c + 1 + \delta(1 - f)]/2$ | $\infty$ | $\xi(1 + \delta f)/2$ | $0$ |
| $\xi_I < c < \xi_I c$ | $[c + 1 + \delta + c_I f(2 - f)]/2$ | $\infty$ | $\xi(1 + \delta f)/2$ | $0$ |
| $\xi_I < c < 1 + \delta - f c_M$ | $(c + 1 + \delta + c_I f)/2$ | $0$ | $\infty$ | $1$ |

Case 3: $c_I \geq \min\{\delta, (\delta + c_M)/2\}$

| $\delta < \delta$ | $0 \leq c < c_M$ | $[c + 1 + \delta + c_I f(2 - f)]/2$ | $\infty$ | $\xi(1 + \delta f)/2$ | $\alpha$ |
| $\max\{0, c_M\} \leq c < 1 + \delta - f c_I$ | $(c + 1 + \delta + c_I f)/2$ | $\infty$ | $\xi(1 + \delta f)/2$ | $\alpha$ |
| $\delta \geq \delta$ | $0 \leq c < 1 + \delta - f c_I$ | $[c + 1 + \delta - c_I f(2 - f)]/2$ | $\infty$ | $\xi(1 + \delta f)/2$ | $\alpha$ |

$\xi \leq \frac{1}{2} \frac{(1 - f)(3 - f) + 1}{c_M} + \frac{1}{2} \frac{1 - (c_M - \alpha)}{(3 - f)}$ and $\delta \leq \frac{1}{\alpha(3 - f)} + \frac{1}{2} \frac{c_M + \alpha}{3 - f}$. The manufacturer does not turn a positive profit if $c_I \geq \max\{1 + \delta - \delta c_M, 1 + \delta(1 - f)\}$. It is obvious that when $c_I \geq \min\{\delta, (\delta + c_M)/2\}$, the manufacturer’s optimal pricing strategy is independent of $c_I$ and hence all the other metrics. For $c_M < c_I < \min\{\delta, (\delta + c_M)/2\}$, we need to consider 4 cases: (1) $\delta < \delta$; (2) $\delta \geq \delta$ with $\delta_I < \delta_I$; (3) $\delta \geq \delta$ with $\delta_I > \delta_I$; (4) $\delta \geq \delta$ with $\delta_I > \delta_I$. Note that $c_M < c_I < \min\{\delta, (\delta + c_M)/2\} \Rightarrow c_M < \delta$, which implies $\min\{\delta, (\delta + c_M)/2\} = (\delta + c_M)/2$. We denote $c_I$ such that $c_2(c_I) = c$ as $c_{14} = \frac{4((1 + \delta + 1(1 - f)(2 - f)) - 2(\delta - f))}{2 + (1 + \delta - f)(1 - f)} c_I$ and $c_I$ such that $c_5(c_I) = c$ as $c_{15} = \frac{4((1 + \delta + 1(1 - f)(2 - f)) - 2(\delta - f))}{2 + (1 + \delta - f)(1 - f)} c_I$. Thus, $c_I$ such that $c_0(c_I) = c$ as $c_{16}$, $c_I$ such that $c_10(c_I) = c$ as $c_{17}$, and $c_I$ such that $c_9(c_I) = c$ as $c_{18}$.
Table F.5  Optimal Profit, Usage, New Production and Consumer Surplus

<table>
<thead>
<tr>
<th>Conditions</th>
<th>π*</th>
<th>Usage (U)</th>
<th>New Sales (Q)</th>
<th>CS</th>
</tr>
</thead>
<tbody>
<tr>
<td>Case 1: $c_I \geq \min {\delta, (\delta + c_M)/2}$</td>
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<tr>
<td>$c_M &lt; \delta$</td>
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<tr>
<td>$c_M \leq c_I &lt; \hat{c}_I$</td>
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<tr>
<td>$\hat{c}_I \leq c &lt; \hat{c}'$</td>
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<tr>
<td>$0 \leq c &lt; 1 + \delta(1-f)$</td>
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<td>$\hat{c}_I \leq c_I &lt; \hat{c}_I$</td>
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<td>$\hat{c}_I \leq c &lt; \hat{c}'$</td>
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Case 1: $\delta < \hat{\delta}$. In this case, we have $\max\{c_{10}, 0\} \geq \max\{c', 0\} \geq \max\{c_2, 0\}$ by the proof of Proposition 1. In addition, we have $c_{10}(c_M) < \hat{c}$. This is because

\[
c_{10}(c_M) = \frac{\delta(1-\delta)}{25} \left( \delta(1-\delta) + c_M(1+3\delta - 2f) - (\delta - c_M)\sqrt{(1-\delta)(1+3\delta}) \right) < \hat{c}
\]

\[
\Rightarrow \frac{1}{\delta} \left( \frac{1 - \sqrt{1-\delta}}{2} - \frac{1 - \delta}{2f} \right) \left( c_M + \frac{f}{2(2-f)} \right) < 0
\]

\[
\Rightarrow \frac{1}{\delta} \left( \frac{\sqrt{1-\delta} + \sqrt{1+3\delta}}{2} - \frac{\sqrt{1-\delta}}{2f} \right) < 0.
\]
Note that we have \(c_{10}(c_M) \geq c'_M(c_M) \geq c_2(c_M)\), where
\[
\begin{align*}
  c_2(c_M) &= \frac{2(\frac{(14)}{(1-\delta)} I(3f) - \delta I(1-\delta))}{2(1-\delta)} - 1 - \delta (1-f), \\
  c'_M(c_M) &= c_M (2-f) + (1-\delta) \left( 1 - \frac{2\delta c_M}{8f(2-f)} \right), \\
  c_{10}(c_M) &= \frac{\delta (1-\delta) + c_M (1+3\delta - 2f) + (c_M - \delta) \sqrt{1-\delta} (1+3\delta)}{2\delta}.
\end{align*}
\]

**Case 1.1:** \(c_2(c_M) > 0\). In this case, we have \(c_0 = c_{10}(c_M)\).

**Case 1.1.1:** \(0 < c < c_2(c_M)\). In this case, by Table F.2 we know that \(p_u^*, p_r^*\) and \(p_u^*\) are independent of \(c_l\) for \(c_l > c_M\). By Table F.5, we know that \(\pi, CS, U, Q, U^u = U - Q\), and SW are all independent of \(c_l\).

**Case 1.1.2:** \(c_2(c_M) \leq c < c'_M(c_M)\). In this case, by Case 1.1.1, we know that \(p_u^*, p_r^*, p_u^*\), the manufacturer’s profit, consumer surplus, usage and new product sales are all independent of \(c_l\) for \(c_l \geq c_t\). When \(c_l < c_t\), by Table F.2, it is clear that \(p_u^*\) and \(p_u^*\) increase with \(c_l\) and \(p_u^* = \infty\) is independent of \(c_l\). By Table F.5, the manufacturer’s profit \(\pi = \pi'\). It is clear that \(\pi'\) is concave and quadratic in \(c_l\). One can verify that \(\\pi'\) achieves its maximum at \(c_l = c_t\). Thus, \(\\pi'\) increases with \(c_l\).

For social welfare, we have
\[
SW = \frac{(\delta - c_l))}{\delta(2-f)} \left( c_l \frac{1+\delta(1-f)}{\delta(2-f)} - 1 - \frac{(\delta - c_l)^2}{2\delta} \frac{(1+\frac{1}{\delta(2-f)^2})}{2} \right) + \frac{(\delta - c_l)^2}{2\delta} \left( 1 + \frac{1}{\delta(2-f)^2} \right),
\]
which is concave and quadratic in \(c_l\) and achieves its maximum at \(c_l = \frac{\delta(2c + \delta - 1) - f(c + \delta - 1)}{1+\delta(1-f)}\). Note that \(\frac{\delta(2c + \delta - 1) - f(c + \delta - 1)}{1+\delta(1-f)}\) increases with \(c\) and when \(c = c^*_M\), we have
\[
\frac{\delta[(2\delta + \delta - 1) - f(c + \delta - 1)]}{1+\delta(1-f)}(3-f) - c_M = 0. \tag{F.26}
\]
Hence, SW decreases with \(c_l\) for \(c_M < c_l < c_t\). One can verify that \(p_u^*, p_r^*, p_u^*, \pi, CS, SW, U, U^u, R\) and \(Q\) are continuous at \(c_l = c_t\). Hence, in this case, \(p_u^*, p_r^*, p_u^*\), and \(\pi\) increase with \(c_l\) but \(CS, SW, U, U^u, R,\) and \(Q\) decrease with \(c_l\) for \(c_l > c_M\).

**Case 1.1.3:** \(c'_M(c_M) \leq c < c_{10}(c_M)\). In this case, by Case 1.1.2, we know that \(p_u^*, p_r^*, p_u^*, \pi, Q\) increase with \(c_l\) but \(CS, SW\) (by (F.26)) and \(U\) decrease with \(c_l\) for \(c_l > c_t\). When \(c_l < c_l < c_t\), by Table F.2 we know that \(p_u^*, p_r^*, \) and \(p_u^*\) increase with \(c_l\). By Table F.5, the manufacturer’s profit \(\pi = \pi'_t\). It is clear that \(\pi'_t\) is concave and quadratic in \(c_l\). One can verify that \(\pi'_t\) achieves its maximum at \(c_l = \frac{(c_M + \delta)}{2}\). Note that
\[
\frac{(c_M + \delta)}{2} - c_t = \frac{1}{2} \left( 1 + \frac{\delta}{1-\delta} \right) \frac{(c_M + \delta)}{2} - \frac{\delta(2-f)c}{2(1-\delta)} + \frac{\delta(1-f)}{2} > 0
\]
\[
\frac{(c_M + \delta)}{2} - c_t = \frac{1}{2} \left( 1 + \frac{\delta(2-f)^2}{\delta(2-f)} \right) c_M - \frac{\delta(2-f)c}{2(1-\delta)} + \frac{\delta(1-f)}{2} > 0
\]
\[
\frac{(c_M + \delta)(2-f)}{4\sqrt{1-\delta}} > 0.
\]
Thus, \(\pi'_t\) increases with \(c_l\). The consumer surplus \(CS = CS'_t\). It is clear that \(CS'_t\) is convex and quadratic in \(c_l\). One can verify that \(CS'_t\) achieves its minimum at \(c_l = \delta\). As \(c_l < \delta\), \(CS'_t\) decreases with \(c_l\). For social welfare, we have
\[
SW'_t = c_l - \frac{c}{4} - \frac{\delta}{4} c_{M} f + \frac{(c-2c_{M} + c_{M} f)^2}{4(1-\delta)} - \frac{c_l(c_l-c_{M})}{\delta} + \frac{1}{4}
\]
\[
+ \frac{c_{M} - c}{4} + \frac{3\delta}{8} c_{M} f + \frac{1}{8} + \frac{c_l^2}{2\delta} + \frac{(c-2c_{M} + c_{M} f)^2}{8(1-\delta)},
\]
which is concave and quadratic in $c_I$ and achieves its maximum at $c_I = c_M$. Thus, $SW$ decreases with $c_I$ since $c_I \geq c_M$. It is clear that $U_5' = 1 - \theta^{(5)}_1 = 1 - c_I/d$ decreases with $c_I$ and $Q'_5 = 1 - \theta^{(5)}_2 = \frac{1}{2} \left( 1 - \frac{c - c_M (2-f)}{1 - \delta} \right)$ is independent of $c_I$. Then it is clearly that $U^n = U - Q$ is decreasing in $c_I$. For $R'_5 = Q'_5 f a'_5 = \frac{f}{2} \left( 1 - \frac{c - c_M (2-f)}{1 - \delta} \right) a'_5 = \frac{f}{2} - \frac{c - c_M (2-f)(c-2c_M + c_M f)}{2(1-\delta)}$, which is decreasing in $c_I$. One can verify that $p''_t, p^*_t, p^*_u, \pi, CS, SW, U, U^n, R$ and $Q$ are continuous at $c_I = c_{I_3}$. Hence, in this case, $p''_t, p^*_t, p^*_u, \pi$ increase with $c_I$ but $U, U^n, R, CS$ and $SW$ decrease with $c_I$ for $c_I > c_M$.

**Case 1.1.4:** $c_{I_0} (c_M) \leq c < c_I$. In this case, by Case 1.1.3, we know that $p''_t, p^*_t, p^*_u, \pi$ increase with $c_I$ but $Q, U, CS$ and $SW$ decrease with $c_I$ for $c_I > c_{I_3}$. When $c_M < c_I < c_{I_3}$ by Table F.2 we know that $p''_t, p^*_t$ and $p^*_u$ are independent of $c_I$. However, $p''_t, p^*_t$ and $p^*_u$ are discontinuous at $c_{I_3}$. In particular, we have

$$\lim_{c_I \downarrow c_{I_3}} p''_t(c_I) = 0 \leq \lim_{c_I \downarrow c_{I_3}} p^*_t(c_I) = c_I, \lim_{c_I \downarrow c_{I_3}} p''_u(c_I) > \lim_{c_I \downarrow c_{I_3}} p^*_u(c_I) > \lim_{c_I \downarrow c_{I_3}} p^*_u(c_I).$$

This is because

$$\lim_{c_I \downarrow c_{I_3}} p''_u(c_I) - c_{I_3} = \frac{[(3\delta - 2\delta f + 1)c_M + \delta(1 - 2e - \delta)](\sqrt{1 + 3\delta} - \sqrt{1 - \delta})}{2(1 - \delta)(1 + 3\delta)} = F_{18}(c),$$

which decreases with $c$ and $F_{18}(c) = 0$.

By Table F.5, we know that $\pi = \pi_4$, which is independent of $c_I$. One can verify that $\pi$ is continuous at $c_I = c_{I_3}$, $CS = CS_4, SW = CS_4 + \pi_4, U = U_4, Q = Q_4, U^n = U - Q$, and $F = Q f \alpha = Q f$, which are independent of $c_I$ and discontinuous at $c_I = c_{I_3}$. In particular, $CS_4 < \lim_{c_I \downarrow c_{I_3}} CS'_4(c_I)$. To show $CS_4 < \lim_{c_I \downarrow c_{I_3}} CS'_4(c_I)$, it suffices to show $CS_4 < CS_1$, since by Case 1.1.3, we know that $CS$ decreases with $c_I$ for $c_I > c_{I_3}$. One can verify that $CS_4 = \pi_4/2$ and $CS_1 = \pi_1/2$. This together with the fact that $\pi$ increases with $c_I$ ensures that $CS_4 < CS_1$ and hence, $CS_4 < \lim_{c_I \downarrow c_{I_3}} CS'_4(c_I)$. Because $\pi$ is continuous in $c_I$ for all $c_I > c_M$, thus, $SW_4 < \lim_{c_I \downarrow c_{I_3}} SW'_4(c_I)$.

We have $U_4 < \lim_{c_I \downarrow c_{I_3}} U'_4(c_I)$, as

$$\lim_{c_I \downarrow c_{I_3}} U'_4(c_I) = \frac{\sqrt{1 + 3\delta} - 1}{2\delta(1 + 3\delta)} \left[ (3\delta - 2\delta f + 1)c_M + \delta(1 - 2c - \delta) \right] > 0,$$

and

$$Q_4 - Q_1 = \frac{c}{2} \left( \frac{1}{1 + \delta(1 - f)(3 - f)} - \frac{1}{1 + 3\delta} \right) + \frac{1 + \delta(1 - f) + 2\delta(1 - f)(2 - f)}{2(1 + \delta(1 - f)(3 - f))} - \frac{1 + c M f + 5\delta}{2(1 + 3\delta)}$$

$$< \frac{c}{2} \left( \frac{1}{1 + \delta(1 - f)(3 - f)} - \frac{1}{1 + 3\delta} \right) + \frac{1 + \delta(1 - f) + 2\delta(1 - f)(2 - f)}{2(1 + \delta(1 - f)(3 - f))} - \frac{1 + c M f + 5\delta}{2(1 + 3\delta)}$$

$$= \frac{f(\delta - c_M)(2 - f)}{4(1 + \delta(1 - f)(3 - f))} < 0.$$
Finally,
\[ U_4 - \lim_{c \to c_i} U_4^c(c) = U_4 - Q_4 - \lim_{c \to c_i} [U_4^c(c) - Q_4^c(c)] \]
\[ \propto \left(1 + \delta - \sqrt{(1 - \delta)(1 + 3\delta)}\right)\left[c_M(1 + 3\delta - 2\delta f) + (1 - 2c - \delta)\right] \]
\[ \propto c_M(1 + 3\delta - 2\delta f) + (1 - 2c - \delta) \Rightarrow \frac{f(\delta - c_M)(1 - \delta)}{2 - f} > 0. \] (F.30)

Hence, in this case, for \( c_i > c_M, p_i^*, \) and \( \pi \) increase with \( c_i; U^u \) and \( R \) decrease with \( c_i \) and have a downward jump at \( c_i = c_{i3}; p_i^* \) and \( p_i^u_* \) have a downward jump at \( c_i = c_{i3}; CS, SW, U \) and \( Q \) have an upward jump at \( c_i = c_{i3} \) and then \( p_i^*, \) and \( p_i^u_* \) increase with \( c_i \) for \( c_i \geq c_{i3}; Q, U, CS \) and SW decrease with \( c_i \) for \( c_i \geq c_{i3}. \)

**Case 1.1.5:** \( c \leq c < \bar{c}. \) In this case, by Table F.2 we know that when \( c_i \geq (\delta + c_M)/2, \) \( p_i^*, \) \( p_i^u_* \) and \( p_i^u \) are independent of \( c_i. \) Similarly, \( \pi, CS, SW, U, U^u, Q = \frac{1}{2} \left(1 + \frac{c - c_M(2 - \delta)}{1 - \delta}\right), \) and \( R = \frac{1}{2} - \frac{c_M}{2\delta} + \frac{(2 - (\pi - 2c_M - c_M\pi)))}{2(1 - \delta)} \) are all independent of \( c_i. \) When \( c_i < c_i \leq (\delta + c_M)/2, \) by Case 1.1.4, we know that \( p_i^*, p_i^u, p_i^u_* \) and \( \pi \) increase with \( c_i; CS, SW, U^u, R, \) and \( Q \) decrease with \( c_i; Q = \frac{1}{2} \left(1 + \frac{c - c_M(2 - \delta)}{1 - \delta}\right) \) is independent of \( c_i. \) One can verify that at \( c_i = (\delta + c_M)/2, p_i^*, p_i^u, p_i^u_* \) and \( \pi, CS, SW, U^u, R, \) and \( Q \) are independent of \( c_i \) and except for \( \pi \) that is continuous at \( c_i = c_{i3}, \) all of the other metrics we study are discontinuous at \( c_{i3}. \) In particular, \( \lim_{c_i \to c_{i3}} p_i^*(c_i) = 0 < \lim_{c_i \to c_{i3}} p_i^u(c_i) = c_i, \) \( \lim_{c_i \to c_{i3}} p_i^u(c_i) > \lim_{c_i \to c_{i3}} p_i^u_*\) and \( \lim_{c_i \to c_{i3}} p_i^u\) \( \lim_{c_i \to c_{i3}} CS_3(c_i) < \lim_{c_i \to c_{i3}} CS_3^u(c_i), SW_4 < \lim_{c_i \to c_{i3}} SW_4^u(c_i), U_4 < \lim_{c_i \to c_{i3}} U_4^u(c_i), U_4^u > \lim_{c_i \to c_{i3}} U_4^u(c_i), R_4 > \lim_{c_i \to c_{i3}} R_3^u(c_i), \) and \( Q_4 < \lim_{c_i \to c_{i3}} Q_4^u(c_i) \) by the same argument in Case 1.1.4. Hence, in this case, for \( c_i < c_M, p_i^*, Q, \) and \( \pi \) increase with \( c_i \) while \( p_i^u, \) \( p_i^u, \) \( U^u, \) and \( R \) have a downward jump at \( c_i = c_{i3} \) but \( CS, SW, U, Q \) have an upward jump at \( c_i = c_{i3} \) and then \( p_i^*, \) and \( p_i^u \) increase with \( c_i \) for \( c_i \geq c_{i3}; U, SW, U^u, R, \) and \( Q \) decrease with \( c_i \) for \( c_i \geq c_{i3}. \)

**Case 1.1.6:** \( c \geq \bar{c}. \) In this case, by Table F.2 and Table F.5 we know that for \( c_i > c_M, p_i^*, p_i^u, p_i^u_* \), \( \pi, CS, SW, U^u, R, \) and \( Q \) are all independent of \( c_i. \)

As a summary, in Case 1.1, we have \( p_i^*, \) and \( \pi \) always increase (weakly) with \( c_i; U^u \) and \( R \) always decrease (weakly) with \( c_i \) for \( c_i > c_M. \) For \( 0 \leq c < c_2(c_M) \) or \( c \geq \bar{c}, \) all metrics are independent of \( c_i. \) For \( c_2(c_M) \leq c < c_{10}(c_M), \) \( p_i^* \) and \( p_i^u \) increase with \( c_i; Q, U, SW \) and CS decrease with \( c_i. \) For \( c_{10}(c_M) \leq c < c, p_i^*, p_i^u, U^u, \) and \( R \) have a downward jump at \( c_i = c_{i3} \) but \( CS, SW, U \) and \( Q \) have an upward jump at \( c_i = c_{i3} \) and then \( p_i^*, \) and \( p_i^u \) increase with \( c_i \) when \( c_i \geq c_{i3}; Q, U, SW, \) and CS decrease with \( c_i \) when \( c_i \geq c_{i3}. \) For \( c \leq c < \bar{c}, Q \) is (weakly) increasing in \( c_i; p_i^*, p_i^u, U^u, \) and \( R \) have a downward jump at \( c_i = c_{i3} \) but \( CS, SW, U \) and \( Q \) have an upward jump at \( c_i = c_{i3} \) and then \( p_i^*, \) and \( p_i^u \) increase with \( c_i \) when \( c_i \geq c_{i3}; U, SW, \) and CS decrease with \( c_i \) when \( c_i \geq c_{i3}. \)

**Case 1.2:** \( c_2(c_M) \leq 0 < e_1^u(c_M). \) In this case, we have \( c_{0} = c_{10}(c_M). \) The analysis is the same as that in Case 1.1. except without case Case 1.1.1. As a summary, in Case 1.2, we have \( p_i^*, \) and \( \pi \) always increase (weakly) with \( c_i; U^u \) and \( R \) always decrease (weakly) with \( c_i \) for \( c_i > c_M. \) For \( c \geq \bar{c}, \) all metrics are independent of \( c_i. \) For \( 0 \leq c < c_{10}(c_M), \) \( p_i^* \) and \( p_i^u \) increase with \( c_i; Q, U, SW, \) and CS decrease with \( c_i. \) For \( c_{10}(c_M) \leq c < c, p_i^*, p_i^u, U^u, \) and \( R \) have a downward jump at \( c_i = c_{i3} \) but \( CS, SW, U \) and \( Q \) have an upward jump at \( c_i = c_{i3} \) and then \( p_i^*, \) and \( p_i^u \) increase with \( c_i \) when \( c_i \geq c_{i3}; Q, U, SW, \) and CS decrease with \( c_i \) when \( c_i \geq c_{i3}. \)
\(c_I \geq c_M\). For \(c \leq \rho < \bar{c}\), \(Q\) is (weakly) increasing in \(c_I\); \(p_n^*, p_u^*, U^u\), and \(R\) have a downward jump at \(c_I = c_{I_3}\) but \(CS, SW, U\) and \(Q\) have an upward jump at \(c_I = c_{I_3}\) and then \(p_n^*\) and \(p_u^*\) increase with \(c_I\) when \(c_I \geq c_{I_3}\); \(U, SW,\) and \(CS\) decrease with \(c_I\) when \(c_I \geq c_{I_3}\).

**Case 1.3: \(c'_M(c_M) \leq 0 < c_{10}(c_M)\).** In this case, we have \(\hat{c}_0 = c_{10}(c_M)\). The analysis is the same as that in Case 1.1, except without case Case 1.1.1 and Case 1.1.2. As a summary, in Case 1.3, we have \(p_n^*\) and \(\pi\) always increase (weakly) with \(c_I\); \(U^u\) and \(R\) always decrease (weakly) with \(c_I\) for \(c_I > c_M\). For \(c \geq \bar{c}\), all metrics are independent of \(c_I\). For \(0 \leq c < c_{10}(c_M)\), \(p_n^*\) and \(p_u^*\) increase with \(c_I\); \(Q, SW, U\) and \(CS\) decrease with \(c_I\). For \(c_{10}(c_M) < c < \bar{c}\), \(p_n^*, p_u^*, U^u\), and \(R\) have a downward jump at \(c_I = c_{I_3}\) but \(CS, SW, U\) and \(Q\) have an upward jump at \(c_I = c_{I_3}\) and then \(p_n^*\) and \(p_u^*\) increase with \(c_I\) when \(c_I \geq c_{I_3}\); \(Q, U, SW,\) and \(CS\) decrease with \(c_I\) when \(c_I \geq c_{I_3}\). For \(c \leq \rho < \bar{c}\), \(Q\) is (weakly) increasing in \(c_I\); \(p_n^*, p_u^*, U^u\), and \(R\) have a downward jump at \(c_I = c_{I_3}\) but \(CS, SW, U\) and \(Q\) have an upward jump at \(c_I = c_{I_3}\) and then \(p_n^*\) and \(p_u^*\) increase with \(c_I\) when \(c_I \geq c_{I_3}\); \(U, SW,\) and \(CS\) decrease with \(c_I\) when \(c_I \geq c_{I_3}\).

**Case 1.4: \(c_{10}(c_M) \leq 0\).** In this case, we have \(\hat{c}_0 = 0\). The analysis is the same as that in Case 1.1, except without Case 1.1.1, Case 1.1.2 and Case 1.1.3. As a summary, in Case 1.4, we have \(p_n^*\) and \(\pi\) always increase (weakly) with \(c_I\); \(U^u\) and \(R\) always decrease (weakly) with \(c_I\) for \(c_I > c_M\). For \(c \geq \bar{c}\), all metrics are independent of \(c_I\). For \(0 \leq c < c_{10}(c_M)\), \(p_n^*\) and \(p_u^*\) increase with \(c_I\); \(Q, SW, U\) and \(CS\) decrease with \(c_I\). For \(c_{10}(c_M) < c < \bar{c}\), \(p_n^*, p_u^*, U^u\), and \(R\) have a downward jump at \(c_I = c_{I_3}\) but \(CS, SW, U\) and \(Q\) have an upward jump at \(c_I = c_{I_3}\) and then \(p_n^*\) and \(p_u^*\) increase with \(c_I\) when \(c_I \geq c_{I_3}\); \(Q, U, SW,\) and \(CS\) decrease with \(c_I\) when \(c_I \geq c_{I_3}\).

As a summary, in Case 1. we have \(p_n^*\) and \(\pi\) always increase (weakly) with \(c_I\); \(U^u\) and \(R\) always decrease (weakly) with \(c_I\) for \(c_I > c_M\). For \(c \geq \bar{c}\) or \(0 \leq c < c_{10}(c_M)\), all metrics are independent of \(c_I\). For \(c \leq \rho < \bar{c}\), \(Q, SW, U\) and \(CS\) decrease with \(c_I\). For \(0 \leq c < c_{10}(c_M)\), \(p_n^*\) and \(p_u^*\) increase with \(c_I\); \(Q, SW, U\) and \(CS\) decrease with \(c_I\). For \(c_{10}(c_M) < c < \bar{c}\), \(p_n^*, p_u^*, U^u\), and \(R\) have a downward jump at \(c_I = c_{I_3}\) but \(CS, SW, U\) and \(Q\) have an upward jump at \(c_I = c_{I_3}\) and then \(p_n^*\) and \(p_u^*\) increase with \(c_I\) when \(c_I \geq c_{I_3}\); \(Q, U, SW,\) and \(CS\) decrease with \(c_I\) when \(c_I \geq c_{I_3}\).

**Case 2: \(\delta \geq \bar{\delta}\) and \(\hat{c}_t < \underline{c}_t\).** In this case, we show that \(\hat{c}_t > c_M\). Note that
\[
\hat{c}_t > c_M \Leftrightarrow \frac{1}{2} \left( c_M \right) = \frac{\left( c_M \right) + \frac{f((\delta - c_M)\sqrt{1 - \delta})}{2\sqrt{1 - \delta} - (2 - f)\sqrt{1 + 3\delta}} > c_M \right.
\]
\[
\Leftrightarrow \delta - c_M \geq \frac{f((\delta - c_M)\sqrt{1 - \delta})}{2\sqrt{1 - \delta} - (2 - f)\sqrt{1 + 3\delta}} > 0 \Leftrightarrow 0 \Rightarrow 0 \Leftrightarrow 2 + \frac{f}{2 - f} < \sqrt{\frac{1 + 3\delta}{1 - \delta}}.
\]
Note that \(2\sqrt{1 - \delta} - (2 - f)\sqrt{1 + 3\delta} < 0\) for \(\delta \geq \bar{\delta}\) by (F.23). Then
\[
1 + \frac{f\sqrt{1 - \delta}}{2\sqrt{1 - \delta} - (2 - f)\sqrt{1 + 3\delta}} > 0 \Leftrightarrow (2 + f)\sqrt{1 - \delta} - (2 - f)\sqrt{1 + 3\delta} < 0 \Leftrightarrow \frac{2 + f}{2 - f} < \sqrt{\frac{1 + 3\delta}{1 - \delta}},
\]
which is implied by (F.25).
Therefore, we have $\hat{c}_t > c_{M}$. In the proof of Proposition 1, we have shown that under $c_{M} < c_{I} < \min\{\delta, (\delta + c_{M})/2\}$, $\delta \geq \bar{c}$ and $\hat{c}_t < c_{I}$, we have $\varphi' \leq 0$ if $c_{I} \leq \hat{c}_t$. As a result, when $c_{M} < c_{I} < \hat{c}_t$, we have $c_{10}(c_{I}) < \varphi(c_{I}) < 0$ by (F.21). This implies that $\max\{c_{10}, 0\} \geq \max\{\varphi', 0\} \geq \max\{c_{2}, 0\}$ and we have $\hat{c}_0 = \max\{0, c_{10}(c_{M})\}$.

As a summary, in Case 2, we have $p_{n}^{*}$ and $\pi$ always increase (weakly) with $c_{I}$; $U^{*}$ and $R$ always decrease (weakly) with $c_{I}$ for $c_{I} > c_{M}$. For $c \geq \bar{c}$, all metrics are independent of $c_{I}$. For $0 \leq c < \bar{c}$, $p_{n}^{*}$, $R$, $U^{*}$, and $R$ have a downward jump at $c_{I} = c_{I_{a}}$ but $CS$, $SW$, $U$ and $Q$ have an upward jump at $c_{I} = c_{I_{a}}$ and then $p_{n}^{*}$ and $p_{u}^{*}$ increase with $c_{I}$ when $c_{I} \geq c_{I_{a}}$; $Q$, $SW$, $U$ and $CS$ (weakly) decrease with $c_{I}$ when $c_{I} \geq c_{I_{a}}$.

**Case 3:** $\delta > \bar{c}$ and $c_{I} \leq \hat{c}_t < c_{I}$. In this case, for $c_{I} > \hat{c}_t$, we have $c_{10} > \varphi > c_{2}$ by (F.21) and (F.22). We also have $c_{10}(\hat{c}_t) < \hat{c}_t$ since

$$c_{10}(\hat{c}_t) - \bar{c} = \frac{f(\delta - c_{M})(1 - \delta)^{2}}{\delta(2 - f)[2(1 - \delta) - (2 - f)\sqrt{(1 - \delta)(1 + 3\delta)}]} < 0 \Leftrightarrow 2(1 - \delta) - (2 - f)\sqrt{(1 - \delta)(1 + 3\delta)} < 0,$$

which is proved in (F.23). Note that in Case 3, we have $\hat{c}_0 = \max\{0, c_{0}(c_{M})\}$.

The only thing that is different from the previous analysis is when $c_{I}$ increases from $c_{I_{a}}$ to $c_{I_{a}}^{+}$, provided that $c_{I_{a}} > c_{M}$. By definition of $c_{I_{a}}$ and $c_{0}$, we know that when $c_{I} \in [c_{M}, c_{I_{a}}]$, all consumers seek firm repair when their products fail, as the firm provides free repair service; when $c_{I} \in [c_{I_{a}}^{+}, +\infty]$, no consumer will repair their failed products. It is obvious that $p_{n}^{*}$ and $\pi$ (weakly) increase with $c_{I}$. One can verify that $p_{n}^{*}$ and $p_{u}^{*}$ jump downwardly at $c_{I} = c_{I_{a}}$. Note that $p_{n}^{*}$ is discontinuous at $c_{I_{a}}$ and is (weakly) increasing in $c_{I}$ for $c_{I} \in [c_{I_{a}}^{+}, +\infty]$. Thus, to show $\lim_{c_{I} \nearrow c_{I_{a}}} p_{n}^{*}(c_{I}) > \lim_{c_{I} \searrow c_{I_{a}}} p_{n}^{*}(c_{I})$, it suffices to show that $\lim_{c_{I} \nearrow c_{I_{a}}} p_{n}^{*}(c_{I}) > \lim_{c_{I} \searrow c_{I_{a}}} p_{n}^{*}(c_{I})$, which is implied by Table F.2, i.e.,

$$\lim_{c_{I} \nearrow c_{I_{a}}} p_{n}^{*}(c_{I}) = (c + 1 + \delta + c_{M}f)/2 > (c + 1 + \delta(1 - f))/2 = \lim_{c_{I} \rightarrow +\infty} p_{n}^{*}(c_{I}).$$

Now we show $\lim_{c_{I} \nearrow c_{I_{a}}} p_{n}^{*}(c_{I}) > \lim_{c_{I} \searrow c_{I_{a}}} p_{n}^{*}(c_{I})$. Note that $\lim_{c_{I} \rightarrow c_{I_{a}}} p_{n}^{*}(c_{I}) = \frac{24 + c + c_{M}f}{4 + 3\delta}$ and by definition of $c_{9}$ (see (F.18)), $c_{I_{a}}$ is the solution to

$$F_{8}(c_{I}) = \frac{\delta - c_{I}}{2(2 - f)} \left( \left(1 + \delta(1 - f)(3 - f)\right) \right) + \frac{\delta(1 - \delta)(1 - f)}{2(2 - f)} - c = \frac{(1 - c + \delta - c_{M}f)^{2}}{4(1 + 3\delta)} = 0.$$

It is obvious that $F_{8}(c_{I})$ is concave in $c_{I}$ and $F_{8}(\hat{c}_t) = 0$ has two roots denoted by $c'_{I_{a}}$ and $c'_{I_{a}}$. Without loss of generality, we assume $c'_{I_{a}} \leq c'_{I_{a}}$. We now show $c_{I_{a}} = c'_{I_{a}}$.

When $c_{0}(c_{I})$ is defined, for $\max\{0, c_{0}(c_{M})\} \leq c \leq c_{0}(\hat{c}_t)$, it is obvious that as $c_{I}$ increases from $c_{M}$, there exists $c_{I_{a}}$ such that $c = c_{0}(c_{I_{a}})$. To show $c_{I_{a}} = c'_{I_{a}}$, by concavity of $F_{8}(c_{I})$, it suffices to show that $F_{8}(c_{M}) < 0$ for $c > \max\{0, c_{0}(c_{M})\}$. We have

$$F_{8}(c_{M}) = -\frac{c^{2}}{4(1 + 3\delta)} + \frac{2(1 + \delta - c_{M}f)}{4(1 + 3\delta)} + \frac{c_{M} - \delta}{2(2 - f)} c - \frac{(1 + \delta - c_{M}f)^{2}}{4(1 + 3\delta)} - \frac{(c_{M} - \delta)[c_{I_{a}}[1 + \delta(1 - f)(3 - f)]] + \delta(1 - \delta)(1 - f)]}{\delta^{2}(2 - f)^{2}} \leq F_{24}(c),$$

which is concave in $c$. We denote $F_{24}(c)$’s global maximum as $c_{24}$. One can verify that

$$c_{24} - c_{0}(c_{M}) = \frac{2(\delta - c_{M})\sqrt{2f(1 + 3\delta)}}{(2 - f)\sqrt{\delta}} < 0.$$

Hence, $F_{24}(c)$ is decreasing in $c$ for $c > \max\{0, c_{0}(c_{M})\}$. It is easy to verify that $F_{24}(c_{0}(c_{M})) = 0$. Hence, $F_{8}(c_{M}) < 0$ for $c > \max\{0, c_{0}(c_{M})\}$. Thus, we have $c_{I_{a}} = c'_{I_{a}}$. 
Next, we show that \( F_8 \left( \frac{\delta(2\delta + c + c_Mf)}{1 + 3\delta} \right) \geq 0 \). Note that

\[
F_8 \left( \frac{\delta(2\delta + c + c_Mf)}{1 + 3\delta} \right) = \frac{f(\delta(16 - 7f) - f)c^2}{4(2 - f)^2(1 + 3\delta)^2} - \left[ \frac{f[(21f^2 - 66f + 36)c_M + 28 - 16f]}{18(2 - f)^2(1 + 3\delta)} + \frac{f(4 - f)}{9(2 - f)^2} \left( \frac{1}{2} + 2(3c_Mf - 2)^2 \right) \right] c
+ \frac{\delta^2}{\delta(2 - f)^2} \left( 1 - \frac{28 + c_Mf}{4} \right) \left( 1 - \delta \right) \left( 1 - f \right) + \frac{\delta^2 + c_Mf[(1 + \delta)(1 - f)]}{1 + 3\delta} - \frac{(1 - c_Mf + 1)^2}{4(1 + 3\delta)} \leq F_{22}(c),
\]

which is convex in \( c \), as \( \delta(16 - 7f) - f > \frac{f(2 - f)(f + 14)}{f^2 - 2f + 4} > 0 \). Let \( c_{22} \) be the global minimum of \( F_{22}(c) \). Note that \( c < c_L \). One can verify that

\[
c_{22} - c = \frac{\delta - c_Mf)(8f^2 - 29f + 24c^2 + (8f - 2f)\delta - f}{\delta(2 - f)^2(16 - 7f) - f} > 0.
\]

This is because \( 8f^2 - 29f + 24 > 0 \), \( \forall f \in [0, 1] \) obviously, and \( (8f^2 - 29f + 24)\delta^2 + (8f - 2f)\delta - f \) achieves its global minimum at \( \delta = -\frac{4f}{8f^2 - 29f + 24} < 0 \), which implies that \( (8f^2 - 29f + 24)\delta^2 + (8f - 2f)\delta - f \) is increasing in \( \delta \) for \( \delta \in [\delta, 1] \). One can verify that at \( \delta = \tilde{\delta} \), \( (8f^2 - 29f + 24)\delta^2 + (8f - 2f)\delta - f = \frac{(2f - f(12 + 28f - f^2))}{(4f - 2f + f^2)^2} > 0 \).

Thus, \( F_{22}(c) \) is decreasing in \( c \) for \( 0 \leq c < c_L \).

By (F.21), we know that \( c_0(c_1) = c_{10}(c_1) = c_L(c_1) \) at \( c = \tilde{c}_1 \) and \( c_0 \) is defined for \( c_1 \leq \tilde{c}_1 \). We denote \( c_{23} \triangleq c_L(c_1) \). We have \( c_L \leq c_{23} \). Thus,

\[
F_8(c_{23}) = \frac{f(\delta - c_M)(8f^2 - 29f + 24c^2 + (8f - 2f)\delta - f)}{\delta(2 - f)^2(16 - 7f) - f} > 0.
\]

One can show that \( \delta - 1 + \frac{[64 - 88f + 45f^2 - 8f^3]\delta + 2f(f - 4)\delta + f^2}{8\delta - 4f + 12\delta^2 + (4f - 2)\sqrt{(1 - \delta)(1 + 3\delta)} + 8f^2(1 + 3\delta)} > 0 \). Hence, for \( \delta \in [\tilde{\delta}, 1] \), we have \( F_8(c_{23}) > 0 \). This implies that \( F_8 \left( \frac{\delta(2\delta + c + c_Mf)}{1 + 3\delta} \right) \geq 0 \).

Therefore, we have \( \lim_{c_1 \rightarrow c_{14}} p_a^{(1)}(c_1) > \lim_{c_1 \rightarrow c_{14}} p_a^{(2)}(c_1) \).

For \( CS \), it is obvious that \( CS \) is independent of \( c_1 \) for \( c_1 \in [c_M, c_{14}) \); is decreasing in \( c_1 \) for \( c_1 \in (c_{14}, c_M + \frac{\delta}{2}] \); is independent of \( c_1 \) for \( c_1 \in ([c_M + \frac{\delta}{2}, +\infty) \). To show \( \lim_{c_1 \rightarrow c_{14}} CS(c_1) < \lim_{c_1 \rightarrow c_{14}} CS(c_1) \), it suffices to show that \( \lim_{c_1 \rightarrow c_{14}} CS(c_1) < \lim_{c_1 \rightarrow c_{14}} CS(c_1) \). We have

\[
\begin{align*}
\lim_{c_1 \rightarrow c_{14}} CS(c_1) &= \frac{c - c_Mf + 1}{8(1 + 3\delta)}, \\
\lim_{c_1 \rightarrow c_{14}} CS(c_1) &= \frac{c - c_Mf - 1}{8(1 + 3\delta)}, \quad \lim_{c_1 \rightarrow c_{14}} CS(c_1) \leq \lim_{c_1 \rightarrow c_{14}} CS(c_1) \Leftrightarrow \\
F_{19}(c) &= \frac{3[(1 - f)(3 - f)]c^2 + 2(1 + \delta - c_Mf)[1 + \delta(1 - f)(3 - f)] - (1 + 3\delta)[1 + \delta(1 - f)]c}{(1 - f)(3 - f)} \geq 0, \quad \text{(F.31)}
\end{align*}
\]

One can compute \( F_{19}(c) \) is convex and achieves its global minimum at \( c = 1 + \delta - c_Mf + \frac{(1 + 3\delta)c_Mf}{\delta(4 - f)} \). As we have \( c \leq c_L \) by definition of \( c_0 \) and \( c_M < \delta \), one can show that \( 1 + \delta - c_Mf + \frac{(1 + 3\delta)c_Mf}{\delta(4 - f)} - c \geq 2[1 + \delta(1 - f)(3 - f)](\delta - c_Mf) > 0 \). Hence, \( F_{19}(c) \) is decreasing in \( c \) for \( c \in (0, c_L) \). Then it is easy to see that

\[
F_{19}(c_L) = \frac{f^2(\delta - c_Mf)^2(1 - \delta)[1 + \delta(1 - f)(3 - f)]}{\delta(2 - f)^2} \geq 0.
\]

For \( SW \), as \( SW = CS + \pi \) and we know \( \pi \) is increasing in \( c_1 \). Hence, based on the analysis of \( CS \), we know that \( \lim_{c_1 \rightarrow c_{14}} SW(c_1) < \lim_{c_1 \rightarrow c_{14}} SW(c_1) \). In addition, by (F.26), we know that \( \pi_{14} + CS_{14} \) is decreasing in \( c_1 \) for \( c_1 \geq c_M \).
Both $Q$ and $U$ jump upwardly at $c_I = c_{I_k}$, provided that $c_{I_k} > c_M$. It is obvious that $Q$ is decreasing in $c_I$ for $c_I > c_{I_k}$. Hence, to show $\lim_{c_I \nearrow c_{I_k}} Q(c_I) < \lim_{c_I \nearrow c_{I_k}} Q(c_I)$, it suffices to show $\lim_{c_I \nearrow c_{I_k}} Q(c_I) < \lim_{c_I \rightarrow \infty} Q(c_I)$. We have

$$\lim_{c_I \nearrow c_{I_k}} Q(c_I) = 1 - \theta_2^{(4)}$$

$$\lim_{c_I \rightarrow \infty} Q(c_I) = 1 - \theta_2^{(1)}$$

$$\Leftrightarrow F_{20}(c) = \frac{f(\delta - c_M)}{(1 + 3\delta)(2 - f)} \geq 0.$$

As $R = Qf\alpha$ and we know that $\alpha = 1$ for $c_I \in [c_M, c_{I_k})$ and $\alpha = 0$ for $c_I \in (c_{I_k}, +\infty)$, hence, we have $R$ is decreasing in $c_I$.

As $U = 1 - \theta_1$ and $p_u^* = \delta \theta_1$ for $c_I \in \{c_{I_k}, c_{I_k}^+\}$, it is clear that $U$ jumps upwardly at $c_I = c_{I_k}$ given the analysis of $p_u^*$.

As $U^* = U - Q$, we know that for $c_I \in [c_M, c_{I_k})$, $U^*$ is independent of $c_I$; for $c_I \in (c_{I_k}, (\delta + c_M)/2)$, $U^* = \theta_2^{(1)} - \theta_1^{(1)} = \frac{(1-f)(\delta-c_I)}{\delta(2-f)}$, which is decreasing in $c_I$; for $c_I \in ((\delta + c_M)/2, +\infty)$, $U^*$ is independent of $c_I$.

However, at $c_I = c_{I_k}$, $U^*$ can jump upwardly or downwardly, depending on the jump size of $Q$ and $U$.

**Case 4: $\delta \geq \delta$ and $\delta \geq c_I$.** In this case, we have

$$c_{10}(\delta_I) > 0 \Leftrightarrow \left(2 - f + \frac{(\delta - 1)(\sqrt{1 + 3\delta} - \sqrt{1 - \delta})}{\delta[2\sqrt{1 - \delta} - (2 - f)\sqrt{1 + 3\delta}]}ight) c_M + \frac{1 - \delta}{2}\left(1 + \frac{f\sqrt{1 + 3\delta}}{2\sqrt{1 - \delta} - (2 - f)\sqrt{1 + 3\delta}}\right) \geq 0.$$

By (F.23), we know that $2\sqrt{1 - \delta} - (2 - f)\sqrt{1 + 3\delta} < 0$ for $\delta \geq \delta$. Hence, $F_{16}(c_M)$ is increasing in $c_M$. Note that

$$\hat{c}_I \geq c_{I_k} \Leftrightarrow c_M \geq \frac{(1 - \delta)\delta}{1 + \delta(1 - f)(3 - f)} \left(\frac{f[1 + \delta(1 - f)]}{\sqrt{1 - \delta}(1 + 3\delta) - (1 - \delta)} - (1 - f)\right) \geq c_{M_2}.$$

Hence

$$F_{16}(c_{I_k}) > F_{16}(c_{M_2}) = \frac{\delta f(1 - \delta)(2 - f)[1 + \delta(1 - f)]}{[1 + \delta(1 - f)(3 - f)][\sqrt{1 + 3\delta} - \sqrt{1 - \delta}]\sqrt{1 - \delta}} > 0.$$

Note that in Case 4, we have $\hat{c}_0 = \max\{0, \gamma/c_M\}$. The analysis of Case 4 is the same as that in Case 2 and Case 3. □.

**Proof of Corollary 1**

It is obvious that (i) and (iii) hold by Proposition 5, since $E = Q\gamma_q + U^u\gamma_{u_2} + R\gamma_r$. For (ii), when $\gamma_{u_2} = \gamma_r = 0$, we have $E \propto Q$, which implies that $E$ first (weakly) increases and then decreases (as $c_I$ decreases). Hence, by continuity, $E$ first (weakly) increases and then decreases (as $c_I$ decreases) for sufficiently large $\gamma_q$.

Similarly, when $\gamma_q = \gamma_r = 0$, we have $E \propto U^u$ and by Proposition 5 we know that $U^u$ is increasing (as $c_I$ decreases) if $\delta < \frac{2f}{(1-f)^2 + 3}$. Hence, by continuity, $E$ increases (as $c_I$ decreases) for sufficiently large $\gamma_{u_2}$. □.
Proof of Proposition 6

Note that the environmental impact is defined as \( \mathcal{E} \triangleq \gamma_{q}Q + \gamma_{u2}U^{u} + \gamma_{r}R \). Based on Proposition 4 and Proposition 5, it is obvious that (i) holds, i.e., when \( c \geq \bar{c} \), the RTR has no impact on consumer surplus and the environment, as all of the metrics are independent of \( c_{l} \).

When \( c < \bar{c}_{0} \), it is clear that (ii) holds, i.e., RTR always improves (at least weakly) consumer surplus and increase the waste (and hence, makes the environment worse off) for any weight vector \((\gamma_{q}, \gamma_{u2}, \gamma_{r})\), since \( Q \), \( U^{u} \), and \( R \) are all decreasing in \( c_{l} \).

When \( \bar{c}_{0} \leq c < \bar{c} \), by the proof of Proposition 4, we know that there exists \( \bar{c}_{l} \) such that the consumer surplus \( CS(c_{l}) \) is independent of \( c_{l} \) for \( c_{l} \in [c_{M}, \bar{c}_{l}] \) and jumps upwardly at \( c_{l} = \bar{c}_{l} \), and then decreases with \( c_{l} \) for \( c_{l} \in (\bar{c}_{l}, +\infty) \). This \( \bar{c}_{l} \) is when the equilibrium outcome described in Proposition 1 switches from “full repair” region to “partial repair” region or to “no repair” region. Thus, only if when \( c_{l} > \bar{c}_{l} \) after RTR (assuming that RTR will not increase \( c_{l} \)), the consumer surplus can be improved due to RTR. Note that \( CS(c_{M}) < \lim_{c_{l} \rightarrow \infty} CS(c_{l}) \) is proved by (F.31). However, in this case, i.e., \( c_{l} > \bar{c}_{l} \) after RTR, \( Q \) and \( U^{u} \) are all decreasing in \( c_{l} \) by the proof of Proposition 5, which implies that RTR can only increase the waste and hence, makes the environment worse off. Therefore, this implies (iii-1), i.e., if \( c_{l} > \bar{c}_{l} \), then RTR makes the consumer better off but the environment worse off simultaneously. As a result, combining all of the previous analysis, RTR can never make consumers and the environment strictly better off simultaneously.

For (iii-2), when \( \bar{c}_{0} \leq c < \bar{c} \) and \( c_{l} < \bar{c}_{l} \), RTR makes the consumer worse off. If the weight \( \gamma_{q} \) is sufficiently high, then the environmental impact is primarily generated by \( Q \) and we have \( Q(c_{M}) < \lim_{c_{l} \rightarrow \infty} Q(c_{l}) \) by (F.29). This implies that RTR will make the environment better off. If the weight \( \gamma_{u2} \) is sufficiently high, then the environmental impact is primarily generated by \( U^{u} \) and we have \( U^{u}(c_{l}) \) is decreasing in \( c_{l} \) for \( c_{l} \geq c_{M} \) by (F.30) under \( \delta < \delta \). Hence, in this case, RTR makes the consumers and the environment worse off simultaneously. □

Lemma 2. When the transaction cost is sufficiently high, and the quality of the independent repair is strictly inferior to that of the manufacturer, there exist \( \theta_{1}, \theta_{2}, \theta_{3}, \theta_{4} \) with \( 0 \leq \theta_{1} \leq \theta_{2} \leq \theta_{3} \leq \theta_{4} \leq 1 \) such that: consumers with \( \theta \in [0, \theta_{1}) \) are inactive; consumers with \( \theta \in [\theta_{1}, \theta_{2}) \) seek independent repair upon product failure and hold on to a used product otherwise; consumers with \( \theta \in [\theta_{2}, \theta_{3}) \) seek manufacturer repair upon product failure and hold on to a used product otherwise; consumers with \( \theta \in [\theta_{3}, \theta_{4}) \) buy new upon product failure and hold on to a used product otherwise; consumers with \( \theta \in [\theta_{4}, 1) \) buy a new product every period, regardless of whether the current product is still functional.

Proof of Lemma 2. We have

\[
V(\theta) = [\theta - p_{n} + \rho \{(1 - f)\max\{\delta \theta + \rho V(\theta), V(\theta)\}\} + \rho \max\{V(\theta), \delta \theta - p_{r} + \rho V(\theta), \mu \theta - c_{l} + \rho V(\theta)\}]^{+}.
\]

(F.32)

Case 1: Consumers with \( \theta \geq \frac{p_{r} - c_{l}}{3 - \mu} \). In this case, these consumers will not perform independent repair. Thus (F.32) for these consumers reduces to

\[
V(\theta) = [\theta - p_{n} + \rho \{(1 - f)\max\{\delta \theta + \rho V(\theta), V(\theta)\}\} + \rho \max\{V(\theta), \delta \theta - p_{r} + \rho V(\theta)\}]^{+}.
\]
Case 1.1: $V(\theta) \geq \delta \theta + \rho V(\theta)$. In this case, we have $V(\theta) = [\theta - p_n + \rho V(\theta)]^+$, which implies $V(\theta) = \frac{\theta - p_n}{1 - \rho^e}$ and we require
\[
\begin{align*}
V(\theta) &\geq 0 \\ V(\theta) &
\end{align*}
\]
\[
\begin{align*}
\Leftrightarrow & \quad \theta \geq p_n, \\
\Rightarrow & \quad \theta \geq \frac{p_n}{1 - \delta}.
\end{align*}
\]
Case 1.2: $\delta \theta - p_r + \rho V(\theta) \leq V(\theta) < \delta \theta + \rho V(\theta)$. In this case, we have $V(\theta) = [\theta - p_n + \rho \{\delta \theta + \rho V(\theta)\} + fV(\theta)]^+$, which implies $V(\theta) = \frac{\theta [1 + \rho (1 - f)] - p_n}{(1 - \rho) [1 + \rho (1 - f)]}$ and we require
\[
\begin{align*}
V(\theta) &\geq 0 \\
V(\theta) &< \delta \theta + \rho V(\theta) \\
V(\theta) &\geq \delta \theta - p_r + \rho V(\theta)
\end{align*}
\]
\[
\begin{align*}
\Leftrightarrow & \quad \theta \geq \frac{p_n}{1 + \rho (1 - f)}, \\
\Leftrightarrow & \quad \theta < \frac{p_n}{1 - \delta}, \\
\Leftrightarrow & \quad \theta \geq \frac{p_n - p_r [1 + \rho (1 - f)]}{1 - \delta}
\end{align*}
\]
\[
\Rightarrow \max \left\{ \frac{p_n}{1 + \rho (1 - f)}, \frac{p_n - p_r [1 + \rho (1 - f)]}{1 - \delta} \right\} < \frac{p_n}{1 - \delta}.
\]
Case 1.3: $V(\theta) < \delta \theta - p_r + \rho V(\theta))$. In this case, we have $V(\theta) = [\theta - p_n + \rho \{\delta \theta + \rho V(\theta) - fp_r\}]$, which implies $V(\theta) = \frac{\theta [1 + \rho (1 - f)] - p_n}{(1 - \rho) [1 + \rho (1 - f)]}$ and we require
\[
\begin{align*}
V(\theta) &\geq 0 \\
V(\theta) &< \delta \theta - p_r + \rho V(\theta)
\end{align*}
\]
\[
\begin{align*}
\Leftrightarrow & \quad \theta \geq \frac{p_n + fp_r}{1 + \rho (1 - f)}, \\
\Leftrightarrow & \quad \theta < \frac{p_n - p_r [1 + \rho (1 - f)]}{1 - \delta}, \\
\Rightarrow & \quad \theta \geq \frac{p_n - p_r [1 + \rho (1 - f)]}{1 - \delta}
\end{align*}
\]
\[
\frac{p_n + fp_r}{1 + \rho (1 - f)} < \frac{p_n - p_r [1 + \rho (1 - f)]}{1 - \delta} \Leftrightarrow \frac{p_n}{1 - \delta} < \frac{\delta p_n}{1 + \rho (1 - f)}.
\]
Note that
\[
\frac{p_n + fp_r}{1 + \rho (1 - f)} < \frac{p_n - p_r [1 + \rho (1 - f)]}{1 - \delta} \Leftrightarrow \frac{p_n}{1 - \delta} < \frac{\delta p_n}{1 + \rho (1 - f)}.
\]
Therefore, we can define
\[
(\bar{\theta}_1, \bar{\theta}_2, \bar{\theta}_3) = \left\{ \frac{p_n + fp_r}{1 + \rho (1 - f)}, \frac{p_n - p_r [1 + \rho (1 - f)]}{1 - \delta}, \frac{p_n}{1 - \delta} \right\}, \quad \text{if } p_r < \frac{\delta p_n}{1 + \rho (1 - f)};
\]
\[
\left\{ \frac{p_n}{1 - \delta}, \frac{p_n}{1 + \rho (1 - f)} \right\}, \quad \text{if } p_r \geq \frac{\delta p_n}{1 + \rho (1 - f)}.
\]

- **When $p_r \leq c_f$**, consumers with $\theta \in [\bar{\theta}_3, 1]$ will buy new products in every period regardless of the functionality of the used product; consumers with $\theta \in [\bar{\theta}_2, \bar{\theta}_3]$ will buy new products only if the products reach end-of-life or fail, and hold onto to use if the used products are still functional; consumers with $\theta \in [\bar{\theta}_1, \bar{\theta}_2]$ will buy new products if the products reach end-of-life and these consumers will seek firm repair if the products fail.

- **When $p_r > c_f$ and $p_r \leq \frac{\delta p_n}{1 + \rho (1 - f)}$**, if $\frac{p_n}{1 - \delta} < \frac{p_n - c_f}{\delta - \mu}$, then consumers with $\theta \in \left[ \frac{p_n - c_f}{\delta - \mu}, 1 \right]$ will buy new products in every period regardless of the functionality of the used product. If $\frac{p_n - p_r [1 + \rho (1 - f)]}{1 - \delta} < \frac{p_n - c_f}{\delta - \mu} \leq \frac{p_n}{1 - \delta}$, then consumers with $\theta \in \left[ \frac{p_n}{1 - \delta}, 1 \right]$ will buy new products in every period regardless of the functionality of the used product; consumers with $\theta \in \left[ \frac{p_n - p_r [1 + \rho (1 - f)]}{1 - \delta}, \frac{p_n}{1 - \delta} \right]$ will buy new products only if the products reach end-of-life or fail, and hold onto to use if the used products are still functional. If $\frac{p_n + fp_r}{1 + \rho (1 - f)} < \frac{p_n - c_f}{\delta - \mu} \leq \frac{p_n - p_r [1 + \rho (1 - f)]}{1 - \delta}$, then consumers with $\theta \in \left[ \frac{p_n}{1 + \rho (1 - f)} \right]$ will buy new products in every period regardless of the functionality of the used product; consumers with $\theta \in \left[ \frac{p_n - p_r [1 + \rho (1 - f)]}{1 - \delta}, \frac{p_n}{1 - \delta} \right]$ will buy new products only if the products reach end-of-life or fail, and hold onto to use if the used products are still functional; consumers with $\theta \in \left[ \frac{p_n - p_r [1 + \rho (1 - f)]}{1 - \delta}, \frac{p_n}{1 + \rho (1 - f)} \right]$
will buy new products only if the products reach end-of-life or fail, and hold onto to use if the used products are still functional; consumers with \( \theta \in \left[ \frac{p_n - \rho f c_1}{1 + \delta (1 - f)}, \frac{p_n + \rho f c_1}{1 + \rho (1 - f)} \right] \) will buy new products if the products reach end-of-life and these consumers will seek firm repair if the products fail; consumers with \( \theta \in \left[ \frac{p_n - c_l}{1 + \mu (1 - f)}, \frac{p_n + \rho f c_1}{1 + \rho (1 - f)} \right] \) will forgo use.

- **When** \( p_r > c_l \) and \( p_r \geq \frac{\delta p_n}{1 + \delta \rho (1 - f)} \), if \( \frac{p_n}{1 + \delta} < \frac{p_n - c_l}{\delta - \mu} \), then consumers with \( \theta \in \left[ \frac{p_n - c_l}{\delta - \mu}, \frac{p_n}{1 + \delta} \right] \) will buy new products in every period regardless of the functionality of the used product. If \( \frac{p_n}{1 + \delta} < \frac{p_n - c_l}{\delta - \mu} < \frac{p_n}{1 + \delta} \), then consumers with \( \theta \in \left[ \frac{p_n}{1 + \delta}, \frac{p_n}{1 + \delta} \right] \) will buy new products in every period regardless of the functionality of the used product; consumers with \( \theta \in \left[ \frac{p_n - c_l}{\delta - \mu}, \frac{p_n}{1 + \delta} \right] \) will buy new products only if the products reach end-of-life or fail, and hold onto to use if the used products are still functional. If \( \frac{p_n - c_l}{\delta - \mu} \leq \frac{p_n}{1 + \delta} \), then consumers with \( \theta \in \left[ \frac{p_n}{1 + \delta}, \frac{p_n}{1 + \delta} \right] \) will buy new products in every period regardless of the functionality of the used product; consumers with \( \theta \in \left[ \frac{p_n - c_l}{\delta - \mu}, \frac{p_n}{1 + \delta} \right] \) will buy new products only if the products reach end-of-life or fail, and hold onto to use if the used products are still functional; consumers with \( \theta \in \left[ \frac{p_n - c_l}{\delta - \mu}, \frac{p_n}{1 + \delta} \right] \) will forgo use.

**Case 2:** Consumers with \( \theta < \frac{p_n - c_l}{\delta - \mu} \). In this case, these consumers will not perform firm repair. Thus (F.32) for these consumers reduces to

\[
V(\theta) = [\theta - p_n + \rho \{(1 - f)\max\{\delta \theta + \rho V(\theta), V(\theta)\} + f \max\{V(\theta), \mu \theta - c_l + \rho V(\theta)\}\}]^+.
\]

**Case 2.1:** \( V(\theta) \geq \delta \theta + \rho V(\theta) \). In this case, we have \( V(\theta) = [\theta - p_n + \rho \{(1 - f)\max\{\delta \theta + \rho V(\theta), V(\theta)\}\}]^+ \), which implies \( V(\theta) = \frac{\theta - p_n}{1 - \delta} \) and we require

\[
\begin{cases}
V(\theta) \geq 0 \\
V(\theta) \geq \delta \theta + \rho V(\theta) \Rightarrow \theta \geq \frac{p_n}{1 - \delta}.
\end{cases}
\]

**Case 2.2:** \( \mu \theta - c_l + \rho V(\theta) \leq V(\theta) < \delta \theta + \rho V(\theta) \). In this case, we have \( V(\theta) = [\theta - p_n + \rho \{(1 - f)\max\{\delta \theta + \rho V(\theta), V(\theta)\}\}]^+ \), which implies \( V(\theta) = \frac{\theta + \rho \{(1 - f)\delta \theta + \rho V(\theta)\}}{(1 - \mu)\{(1 - f)\delta \theta + \rho V(\theta)\}} \) and we require

\[
\begin{cases}
V(\theta) \geq 0 \\
V(\theta) \leq \mu \theta - c_l + \rho V(\theta) \Rightarrow \theta \geq \frac{p_n - c_l}{\{(1 - \mu)\{(1 - f)\delta \theta + \rho V(\theta)\\}} \leq \theta \leq \frac{p_n}{1 - \delta}.
\end{cases}
\]

**Case 2.3:** \( V(\theta) < \mu \theta - c_l + \rho V(\theta) \). In this case, we have \( V(\theta) = [\theta - p_n + \rho \{(1 - f)\max\{\delta \theta + \rho V(\theta), V(\theta)\}\}]^+ \), which implies \( V(\theta) = \frac{\theta + \rho \{(1 - f)\delta \theta + \rho V(\theta)\}}{(1 - \mu)\{(1 - f)\delta \theta + \rho V(\theta)\}} \) and we require

\[
\begin{cases}
V(\theta) \geq 0 \\
V(\theta) \leq \mu \theta - c_l + \rho V(\theta) \Rightarrow \theta \geq \frac{p_n + \rho f c_l}{\{(1 + \rho f \mu + \delta (1 - f))\delta \theta + \rho V(\theta)\}} \leq \theta \leq \frac{p_n - c_l}{\{(1 - \mu)\{(1 - f)\delta \theta + \rho V(\theta)\\}} \leq \frac{p_n}{1 - \delta}.
\end{cases}
\]

Note that

\[
\frac{p_n + \rho f c_l}{1 + \delta \rho (1 - f)} \leq \frac{p_n - c_l}{\{(1 - \mu)\{(1 - f)\delta \theta + \rho V(\theta)\\}} + 1 - \delta \Leftrightarrow \frac{p_n}{1 + \delta \rho (1 - f)} \leq \frac{p_n - c_l}{\{(1 - \mu)\{(1 - f)\delta \theta + \rho V(\theta)\\}} + 1 - \delta \Leftrightarrow c_l \leq \frac{\mu p_n}{1 + \delta \rho (1 - f)}.
\]
Therefore, we can define

$$\left( \hat{\theta}_1, \hat{\theta}_2, \tilde{\theta}_0 \right) \triangleq \begin{cases} 
\left( \frac{p_n + p_c \delta}{1 + p_c \delta (1 - f)}, \frac{p_n + p_c (1 + \delta - f)}{1 + p_c \delta (1 - f)}, 1 - \delta \right), & \text{if } c_I \leq \frac{\mu p_n}{1 + \theta_0 (1 - f)}, \\
\left( \frac{p_n}{1 + \theta_0 (1 - f)}, \frac{p_n}{1 + p_c \delta (1 - f)}, \frac{p_n}{1 + p_c \delta (1 - f)} \right), & \text{if } c_I \geq \frac{\mu p_n}{1 + \theta_0 (1 - f)}.
\end{cases}$$

- When $p_r \leq c_I$, Case 2 does not exist.
- When $p_r > c_I$ and $c_I < \frac{\mu p_n}{1 + \theta_0 (1 - f)}$, if $\frac{p_n}{1 + \theta_0 (1 - f)} < \frac{p_r - c_I}{\delta - \mu}$, then consumers with $\theta \in \left[ \frac{p_n}{1 + \theta_0 (1 - f)}, \frac{p_r - c_I}{\delta - \mu} \right]$ will buy new products in every period regardless of the functionality of the used product; consumers with $\theta \in \left[ \frac{p_n}{1 + \theta_0 (1 - f)}, \frac{p_r - c_I}{\delta - \mu} \right]$ will buy new products only if the products reach end-of-life or fail, and hold onto to use if the used products are still functional; consumers with $\theta \in \left[ \frac{p_n}{1 + \theta_0 (1 - f)}, \frac{p_r - c_I}{\delta - \mu} \right]$ will buy new products if the products reach end-of-life and these consumers will perform independent repair if the products fail; consumers with $\theta \in \left[ 0, \frac{p_n + p_c \delta (1 + \delta - f)}{1 + p_c \delta (1 - f)} \right]$ will forgo use. If $\frac{p_n}{1 + \theta_0 (1 - f)} < \frac{p_r - c_I}{\delta - \mu}$, then consumers with $\theta \in \left[ \frac{p_n}{1 + \theta_0 (1 - f)}, \frac{p_r - c_I}{\delta - \mu} \right]$ will buy new products only if the products reach end-of-life, and these consumers will perform independent repair if the products fail; consumers with $\theta \in \left[ 0, \frac{p_n + p_c \delta (1 + \delta - f)}{1 + p_c \delta (1 - f)} \right]$ will forgo use. If $\frac{p_n}{1 + \theta_0 (1 - f)} < \frac{p_n + p_c \delta (1 + \delta - f)}{1 + p_c \delta (1 - f)}$, then consumers with $\theta \in \left[ 0, \frac{p_n + p_c \delta (1 + \delta - f)}{1 + p_c \delta (1 - f)} \right]$ will forgo use. If $\frac{p_n}{1 + \theta_0 (1 - f)} < \frac{p_n + p_c \delta (1 + \delta - f)}{1 + p_c \delta (1 - f)}$, then consumers with $\theta \in \left[ 0, \frac{p_n + p_c \delta (1 + \delta - f)}{1 + p_c \delta (1 - f)} \right]$ will forgo use.

- When $p_r > c_I$ and $c_I \geq \frac{\mu p_n}{1 + \theta_0 (1 - f)}$, if $\frac{p_n}{1 + \theta_0 (1 - f)} < \frac{p_r - c_I}{\delta - \mu}$, then consumers with $\theta \in \left[ \frac{p_n}{1 + \theta_0 (1 - f)}, \frac{p_r - c_I}{\delta - \mu} \right]$ will buy new products in every period regardless of the functionality of the used product; consumers with $\theta \in \left[ \frac{p_n}{1 + \theta_0 (1 - f)}, \frac{p_r - c_I}{\delta - \mu} \right]$ will buy new products only if the products reach end-of-life or fail, and hold onto to use if the used products are still functional; consumers with $\theta \in \left[ 0, \frac{p_n + p_c \delta (1 + \delta - f)}{1 + p_c \delta (1 - f)} \right]$ will forgo use. If $\frac{p_n}{1 + \theta_0 (1 - f)} < \frac{p_n + p_c \delta (1 + \delta - f)}{1 + p_c \delta (1 - f)}$, then consumers with $\theta \in \left[ 0, \frac{p_n + p_c \delta (1 + \delta - f)}{1 + p_c \delta (1 - f)} \right]$ will forgo use. If $\frac{p_n}{1 + \theta_0 (1 - f)} < \frac{p_n + p_c \delta (1 + \delta - f)}{1 + p_c \delta (1 - f)}$, then consumers with $\theta \in \left[ 0, \frac{p_n + p_c \delta (1 + \delta - f)}{1 + p_c \delta (1 - f)} \right]$ will forgo use.

Combining Case 1 and Case 2 together, and assume $\rho = 1$ we have:

(1) $p_r \leq c_I$: consumers with $\theta \in [\tilde{\theta}_0, 1]$ will buy new products in every period regardless of the functionality of the used product; consumers with $\theta \in [\hat{\theta}_2, \tilde{\theta}_0]$ will buy new products only if the products reach end-of-life or fail, and hold onto to use if the used products are still functional; consumers with $\theta \in [\tilde{\theta}_1, \hat{\theta}_2]$ will buy new products if the products reach end-of-life and these consumers will seek firm repair if the products fail.

(2) $p_r > c_I$:

(2-1) $p_r < \frac{\delta p_n}{1 + \theta_0 (1 - f)}$: In this case, we have

$$\left\{ \begin{array}{c}
p_n + p_c \delta (1 + \delta - f) < \frac{p_n + p_c (2 - f)}{1 + p_c \delta (1 - f)} < \frac{p_n + p_c (2 - f)}{1 + p_c \delta (1 - f)}, \\
\frac{p_n + p_c \delta (1 + \delta - f)}{1 + p_c \delta (1 - f)} < \frac{p_n + p_c (2 - f)}{1 + p_c \delta (1 - f)} < \frac{p_n + p_c (2 - f)}{1 + p_c \delta (1 - f)}. \end{array} \right. \Rightarrow \left\{ \begin{array}{c}
p_n + p_c \delta (1 + \delta - f) < \frac{p_n + p_c (2 - f)}{1 + p_c \delta (1 - f)} \Rightarrow p_r < \frac{c_I (1 + \delta) + p_n (\delta - \mu)}{1 + \delta - f (\delta - \mu)}, \\
\frac{p_n + p_c (2 - f)}{1 + p_c \delta (1 - f)} > \frac{p_n + p_c \delta (1 + \delta - f)}{1 + p_c \delta (1 - f)} \Rightarrow p_r < \frac{c_I (1 + \delta) + p_n (\delta - \mu)}{1 + \delta - f (\delta - \mu)}. \end{array} \right.$$

One can verify that, in this case,

$$c_I < \frac{c_I (1 + \delta) + p_n (\delta - \mu)}{1 + \delta - f (\delta - \mu)} < \frac{c_I (1 - \delta) + p_n (\delta - \mu)}{1 + \delta - 2\mu - f (\delta - \mu)} < \frac{\delta p_n}{1 + \delta (1 - f)}. \quad (F.33)$$
This is because
\[
\begin{align*}
\frac{c_f(1+\delta) + p_n(\delta - \mu)}{1+\delta - 2\mu - f(\delta - \mu)} - c_f &= \frac{(1-\mu)(p_n + c_f)}{1+\delta - 1} > 0, \\
\frac{c_f(1+\delta) + p_n(\delta - \mu)}{1+\delta - 2\mu - f(\delta - \mu)} - \frac{c_f(1+\delta) + p_n(\delta - \mu)}{1+\delta - 2\mu - f(\delta - \mu)} > 0 \Leftrightarrow c_f < \frac{\mu p_n}{1+\delta(1-\delta)}.
\end{align*}
\]

Also, note that
\[
\begin{align*}
\frac{p_n + f p_r}{1+\delta} < \frac{p_n - c_f(2-f)}{(\delta - \mu)(2-f) + 1-\delta} \Leftrightarrow p_r \begin{align*}
&< \frac{1}{1+\delta} \left( \frac{p_n - c_f(2-f)}{1-\delta + (\delta - \mu)(2-f) - p_n} \right) \\
&= \frac{1}{1+\delta} \left( \frac{p_n - c_f(2-f)}{1-\delta + (\delta - \mu)(2-f) - p_n} \right) - \frac{\delta p_n}{1+\delta(1-\delta)} \approx F_1(c_I),
\end{align*}
\]
where \( F_1(c_I) = \frac{(2-f)(1+\delta c_I)}{f[1-\delta + (\delta - \mu)(2-f)]} - \frac{\delta p_n}{1+\delta(1-\delta)} - \frac{(1+\delta)p_n}{f(1-\delta) + (\delta - \mu)(1+\delta(1-\delta))}, \)
which is decreasing in \( c_I \) and one can verify that \( F_1 \left( \frac{\mu p_n}{1+\delta(1-\delta)} \right) = 0. \) Hence, \( \frac{p_n + f p_r}{1+\delta} < \frac{p_n - c_f(2-f)}{(\delta - \mu)(2-f) + 1-\delta}. \) Similarly, we have
\[
\begin{align*}
\frac{p_n + f c_I}{1+\delta} < \frac{p_n - c_f(2-f)}{1+\delta} \Leftrightarrow p_r \begin{align*}
&< \frac{1}{1+\delta} \left( \frac{p_n - c_f(2-f)}{2-f} \right) \\
&= \frac{1}{1+\delta} \left( \frac{p_n - c_f(2-f)}{1-\delta + (\delta - \mu)(2-f) - p_n} \right) - \frac{\delta p_n}{1+\delta(1-\delta)} \approx F_2(c_I),
\end{align*}
\]
where \( F_2(c_I) = \frac{-c_f(2-f)[\delta(1-f) + 1+\delta f]}{2-f[1+\delta(1-\delta)]} + \frac{p_n(1-\delta)}{2-f[1+\delta(1-\delta)]}, \)
which is decreasing in \( c_I \) and one can verify that \( F_2 \left( \frac{\mu p_n}{1+\delta(1-\delta)} \right) = 0. \) Hence, \( \frac{p_n + f c_I}{1+\delta} < \frac{p_n - p_r(2-f)}{1+\delta}. \)

\( (2-1-1) \) \( c_I < p_r < \frac{c_f(1+\delta) + p_n(\delta - \mu)}{1+\delta - 2\mu - f(\delta - \mu)}, \)

in this case, we have
\[
\begin{align*}
\frac{p_n + f c_I}{1+\delta} < \frac{p_n + f p_r}{1+\delta} < \frac{p_n - c_f(2-f)}{1+\delta} < \frac{p_n - p_r(2-f)}{1+\delta}.
\end{align*}
\]
This implies
\[
\begin{align*}
\frac{p_n - c_f}{\delta - \mu} < \frac{c_f(1+\delta) + p_n(\delta - \mu)}{1+\delta - 2\mu - f(\delta - \mu)} - c_f = \frac{\mu p_n + f c_I}{1+\delta - 1}.
\end{align*}
\]
Hence, consumers with \( \theta \in \left[ \frac{p_n}{\mu}, 1 \right] \) will buy new products in every period regardless of the functionality of the used product; consumers with \( \theta \in \left[ \frac{p_n - p_r(2-f)}{1+\delta}, \frac{p_n}{\mu} \right] \) will buy new products only if the products reach end-of-life or fail, and hold onto to use if the used products are still functional; consumers with \( \theta \in \left[ \frac{p_n + f p_r}{1+\delta}, \frac{p_n - p_r(2-f)}{1+\delta} \right] \) will buy new products if the products reach end-of-life and these consumers will seek firm repair if the products fail; consumers with \( \theta \in \left[ 0, \frac{p_n + f p_r}{1+\delta} \right] \) will forgone use.

\( (2-1-2) \) \( c_f(1+\delta) + p_n(\delta - \mu) \leq p_r < \frac{c_f(1+\delta) + p_n(\delta - \mu)}{1+\delta - 2\mu - f(\delta - \mu)}, \)

in this case, we have
\[
\begin{align*}
\frac{p_n + f c_I}{1+\delta} < \frac{p_n + f p_r}{1+\delta} < \frac{p_n - c_f(2-f)}{1+\delta} < \frac{p_n - p_r(2-f)}{1+\delta}.
\end{align*}
\]
This implies
\[
\begin{align*}
\frac{p_n + f p_r}{1+\delta} < \frac{p_r - c_f}{\delta - \mu} < \frac{c_f(1+\delta) + p_n(\delta - \mu)}{1+\delta - 2\mu - f(\delta - \mu)} - c_f = \frac{\mu p_n + f p_r}{1+\delta - 1}.
\end{align*}
\]
Hence, if \( \frac{p_n + f_p c_f}{1+\delta} \leq \frac{p_n - c_f (2-f)}{(\delta - \mu)(2-f) + 1-\delta} \), then consumers with \( \theta \in \left[ \frac{p_n}{1+\delta}, 1 \right] \) will buy new products in every period regardless of the functionality of the used product; consumers with \( \theta \in \left[ \frac{p_n - p_r (2-f)}{1+\delta}, \frac{p_n}{1+\delta} \right] \) will buy new products only if the products reach end-of-life or fail, and hold onto to use if the used products are still functional; consumers with \( \theta \in \left[ \frac{p_n - c_f (2-f)}{(\delta - \mu)(2-f) + 1-\delta}, \frac{p_n}{1+\delta} \right] \) will buy new products if the products reach end-of-life and these consumers will seek firm repair if the products fail; consumers with \( \theta \in \left[ \frac{p_n + f_p c_f}{1+\delta}, \frac{p_n - c_f (2-f)}{(\delta - \mu)(2-f) + 1-\delta} \right] \) will buy new products if the products reach end-of-life and these consumers will perform independent repair if the products fail; consumers with \( \theta \in \left[ 0, \frac{p_n + f_p c_f}{1+\delta} \right) \) will forgo use.

(2-1-3) \( \frac{c_f (1-\delta) + p_n (\delta - \mu)}{1+\delta} \leq p_r < \frac{c_f + \delta p_n}{1+\delta (1-f)} \): In this case, we have

\[
\frac{p_n + f_p c_f}{1+\delta + \delta (1-f)} < \frac{p_n + f_p c_f}{1+\delta} < \frac{p_n - p_r (2-f)}{1-\delta} < \frac{p_n - c_f (2-f)}{(\delta - \mu)(2-f) + 1-\delta}.
\]

This implies

\[
\frac{p_r - c_f}{\delta - \mu} > \frac{p_n - c_f (2-f)}{(\delta - \mu)(2-f) + 1-\delta}.
\]

Consumers with \( \theta \in \left[ \frac{p_n}{1+\delta}, 1 \right] \) will buy new products in every period regardless of the functionality of the used product; consumers with \( \theta \in \left[ \frac{p_n - c_f (2-f)}{(\delta - \mu)(2-f) + 1-\delta}, \frac{p_n}{1+\delta} \right] \) will buy new products only if the products reach end-of-life or fail, and hold onto to use if the used products are still functional; consumers with \( \theta \in \left[ \frac{p_n + f_p c_f}{1+\delta + \delta (1-f)}, \frac{p_n - c_f (2-f)}{(\delta - \mu)(2-f) + 1-\delta} \right] \) will buy new products if the products reach end-of-life and these consumers will perform independent repair if the products fail; consumers with \( \theta \in \left[ 0, \frac{p_n + f_p c_f}{1+\delta} \right) \) will forgo use.

(2-2) \( p_r < \frac{c_f + \delta p_n}{1+\delta (1-f)}, c_f \geq \frac{\mu p_n}{1+\delta (1-f)} \): In this case, we have \( \frac{p_n - c_f (1-\delta) + \delta p_n}{1+\delta (1-f)} < \frac{p_n + f_p c_f}{1+\delta} \). This is because,

\[
\begin{align*}
\left\{ \begin{array}{l}
p_r < \frac{c_f + \delta p_n}{1+\delta (1-f)}, \\
c_f \geq \frac{\mu p_n}{1+\delta (1-f)},
\end{array} \right. \Rightarrow p_r - c_f < \frac{(\delta - \mu) p_n}{1+\delta (1-f)} \Rightarrow \frac{p_r - c_f}{\delta - \mu} < \frac{p_n}{1+\delta (1-f)}.
\end{align*}
\]

Also, we have \( \frac{p_n - c_f (1-\delta) + \delta p_n}{1+\delta (1-f)} \Leftrightarrow p_r < \frac{c_f (1-\delta) + \delta p_n}{1+\delta (1-f)} \). Hence, we have: consumers with \( \theta \in \left[ \frac{p_n}{1+\delta}, 1 \right] \) will buy new products in every period regardless of the functionality of the used product; consumers with \( \theta \in \left[ \frac{p_n - p_r (2-f)}{1+\delta}, \frac{p_n}{1+\delta} \right] \) will buy new products only if the products reach end-of-life or fail, and hold onto to use if the used products are still functional; consumers with \( \theta \in \left[ \frac{p_n + f_p c_f}{1+\delta}, \frac{p_n - c_f (2-f)}{(\delta - \mu)(2-f) + 1-\delta} \right] \) will buy new products if the products reach end-of-life and these consumers will seek firm repair if the products fail; consumers with \( \theta \in \left[ 0, \frac{p_n + f_p c_f}{1+\delta} \right) \) will forgo use.

(2-3) \( p_r \geq \frac{c_f + \delta p_n}{1+\delta (1-f)}, c_f \geq \frac{\mu p_n}{1+\delta (1-f)} \): In this case, consumers with \( \theta \in \left[ \frac{p_n}{1+\delta}, 1 \right] \) will buy new products in every period regardless of the functionality of the used product; consumers with \( \theta \in \left[ \frac{p_n - p_r (2-f)}{1+\delta}, \frac{p_n}{1+\delta} \right] \) will buy new products only if the products reach end-of-life or fail, and hold onto to use if the used products are still functional; consumers with \( \theta \in \left[ 0, \frac{p_n + f_p c_f}{1+\delta} \right] \) will forgo use.

(2-4) \( p_r \geq \frac{c_f + \delta p_n}{1+\delta (1-f)}, c_f \geq \frac{\mu p_n}{1+\delta (1-f)} \): In this case, by (F.34), we know that \( \frac{p_r - c_f}{\delta - \mu} > \frac{p_n}{1+\delta (1-f)} \). Also, we have \( \frac{p_n - c_f (1-\delta) + \delta p_n}{1+\delta (1-f)} \Leftrightarrow p_r < \frac{c_f (1-\delta) + \delta p_n}{1+\delta (1-f)} \). By (F.33), we know that \( \frac{p_r - c_f (1-\delta) + \delta p_n}{1+\delta (1-f)} \Leftrightarrow \frac{c_f (1-\delta) + \delta p_n}{1+\delta (1-f)} \). Hence, consumers with \( \theta \in \left[ \frac{p_n}{1+\delta}, 1 \right] \) will buy new products in every period regardless of the functionality of the used product; consumers with \( \theta \in \left[ \frac{p_n - c_f (2-f)}{(\delta - \mu)(2-f) + 1-\delta}, \frac{p_n}{1+\delta} \right] \) will buy new products only if the products reach end-of-life or fail, and hold onto to use if the used products are still functional; consumers with
\( \theta \in \left[ \frac{p_n + fc_I}{1 + f \mu + \delta (1 - f)} , \frac{p_n - c_I (2 - f)}{(\delta - \mu)(2 - f) + 1 - \delta} \right] \) will buy new products if the products reach end-of-life and these consumers will perform independent repair if the products fail; consumers with \( \theta \in \left[ 0, \frac{p_n + fc_I}{1 + f \mu + \delta (1 - f)} \right] \) will forgone use.

Therefore, there exists \( 0 \leq \theta_1 \leq \theta_2 \leq \theta_3 \leq \theta_4 \leq 1 \) such that consumers with \( \theta \in [\theta_4, 1] \) will buy new products in every period regardless of the functionality of the used product; consumers with \( \theta \in [\theta_3, \theta_4] \) will buy new products only if the products reach end-of-life or fail, and hold onto to use if the used products are still functional; consumers with \( \theta \in [\theta_2, \theta_3] \) will buy new products if the products reach end-of-life and these consumers will seek firm repair if the products fail; consumers with \( \theta \in [\theta_1, \theta_2] \) will buy new products if the products reach end-of-life and these consumers will perform independent repair if the products fail; consumers with \( \theta \in [0, \theta_1] \) will forgone use.

Note that \( \theta_4 = \frac{p_n}{1 - \delta} \wedge 1 \).

When \( p_r \leq c_I \), we have
\[
(\theta_1, \theta_2, \theta_3) = \begin{cases} 
\left( \frac{p_n + fc_n}{1 + \delta} \wedge 1, \frac{p_n + fp_n}{1 + \delta} \wedge 1, \frac{p_n - pr_c(2 - f)}{1 - \delta} \wedge 1 \right), & \text{if } p_r < \frac{\delta p_n}{1 + \delta(1 - f)}, \\
\left( \frac{p_n + fc_I}{1 + f \mu + \delta (1 - f)} \wedge 1, \frac{p_n + fp_n}{1 + \delta} \wedge 1, \frac{p_n - pr_c(2 - f)}{1 - \delta} \wedge 1 \right), & \text{if } p_r \geq \frac{\delta p_n}{1 + \delta(1 - f)}. 
\end{cases}
\]

When \( p_r > c_I \), we have
\[
(\theta_1, \theta_2, \theta_3) = \begin{cases} 
\left( \frac{p_n + fc_I}{1 + \delta} \wedge 1, \frac{p_n + fp_n}{1 + \delta} \wedge 1, \frac{p_n - pr_c(2 - f)}{1 - \delta} \wedge 1 \right), & \text{if } p_r < \frac{c_I (1 + \delta) + p_n (\delta - \mu)}{1 + \delta - f (\delta - \mu)}, \\
\left( \frac{p_n + fc_I}{1 + f \mu + \delta (1 - f)} \wedge 1, \frac{p_n + fc_I}{1 + \delta} \wedge 1, \frac{p_n - pr_c(2 - f)}{1 - \delta} \wedge 1 \right), & \text{if } \frac{c_I (1 + \delta) + p_n (\delta - \mu)}{1 + \delta - f (\delta - \mu)} \leq p_r < \frac{c_I (1 + \delta) + p_n (\delta - \mu)}{1 + \delta - 2 \mu - f (\delta - \mu)}, \\
\left( \frac{p_n + fc_I}{1 + \delta} \wedge 1, \frac{p_n - pr_c(2 - f)}{(\delta - \mu)(2 - f) + 1 - \delta} \wedge 1, \frac{p_n - c_I (2 - f)}{(\delta - \mu)(2 - f) + 1 - \delta} \wedge 1 \right), & \text{if } p_r \geq \frac{c_I (1 + \delta) + p_n (\delta - \mu)}{1 + \delta - 2 \mu - f (\delta - \mu)}. 
\end{cases}
\]

When \( c_I \geq \frac{\mu p_n}{1 + \delta (1 - f)} \), \( (\theta_1, \theta_2, \theta_3) = \begin{cases} 
\left( \frac{p_n}{1 + \delta (1 - f)} \wedge 1, \frac{p_n + fp_n}{1 + \delta} \wedge 1, \frac{p_n - pr_c(2 - f)}{1 - \delta} \wedge 1 \right), & \text{if } p_r < \frac{\delta p_n}{1 + \delta (1 - f)}, \\
\left( \frac{p_n}{1 + \delta (1 - f)} \wedge 1, \frac{p_n}{1 + \delta (1 - f)} \wedge 1, \frac{p_n}{1 + \delta (1 - f)} \wedge 1 \right), & \text{if } p_r \geq \frac{\delta p_n}{1 + \delta (1 - f)}. 
\end{cases}
\]

Based on (F.35), we can formulate the simultaneous pricing game in the repair service market between the manufacturer and a monopolistic independent repair shop.

The manufacturer sets \( p_n \geq 0, p_r \geq 0 \) and the independent repair shop sets \( p'_r \geq c_I \) simultaneously to maximize their expected profit.

\[
\max_{p_n \geq 0, p_r \geq 0} \pi(p_n, p_r) = \left( p_n - c \right) (1 - \theta_4) + \frac{(p_n - c)(\theta_4 - \theta_3)}{2 - f} + \frac{|p_n - c + (p_r - c_M)(\theta_3 - \theta_2)|}{2} + \frac{(p_n - c)(\theta_2 - \theta_1)}{2},
\]
\[
\max_{p'_r \geq c_I} \pi'(p'_r) = \left( p'_r - c_I \right) (\theta_2 - \theta_1),
\]
where \( (\theta_1, \theta_2, \theta_3, \theta_4) \) are defined in (F.35) with \( c_I \) being replaced by \( p'_r \). This simultaneous pricing game can be solved numerically.