# Negative Advertising and Competitive Product Positioning

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**Abstract**

Negative advertising provides information about the weaknesses of a competitor's product. We study negative advertising with a focus on how it impacts product positioning for profit-maximizing firms. We build a model of informative advertising competition, where product positioning is endogenous and consumers have rational expectations. We show that despite the informational benefits of negative advertising, permitting it (as the Federal Trade Commission in the United States does) may lead to reduced product differentiation and lower consumer welfare, even in markets where firms do not utilize negative advertising in equilibrium. We then extend our model to political competition, where a candidate's objective is to obtain a larger share of votes than the competitor. We show that political competition supports higher positional differentiation, along with more negative advertising than product competition, in line with observed high use of negative advertising in political races and their rarer use in product competition.

**Key Words:** *Negative Advertising, Product Positioning, Regulation, Political Campaigns*
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# 1 Introduction

Consider a firm entering a new market. One key strategic decision that the entrant has to make is how to position itself in the market. A major consideration in this decision is the positioning of the incumbents (Cooper and Kleinschmidt, 1987; Montoya-Weiss and Calantone, 1994). An entrant can position close to an incumbent or can carve out a position for itself by either differentiating its product design or choosing a brand image that differentiates it from the competitors (Fuchs and Diamantopoulos, 2010; Maarit Jalkala and Keränen, 2014). Advertising plays an important role for firms to inform consumers about product attributes (Meenaghan, 1995; Alden et al., 1999). The mention of an own product's strength or a competitor's weakness is known to help firms to emphasize the dimensions of differentiation to consumers (Grewal et al., 1997; Jewell and Saenger, 2014).

In response to the entrant, an incumbent may modify the design of its product (Carpenter, 1989; Ellickson et al., 2012; Seamans and Zhu, 2017), intensify positive advertising to remind consumers of its product, or tap into negative advertising to showcase the shortcomings of an entrant's product (Hauser and Shugan, 1983; Hauser and Gaskin, 1984; Kumar and Sudharshan, 1988). As modifications to product designs take longer, many firms focus on advertising as the first response strategy (Cubbin and Domberger, 1988; Thomas, 1999). Examples of incumbents utilizing negative advertising after a new entry are plenty. American Express faced abundant negative advertising from Visa and Mastercard while introducing its new card Optima, where the ads attacked its limited merchant coverage (Stevenson, 1988). The campaign was so effective that American Express downplayed its product introduction (e.g., offering it only to existing cardholders) to avoid further advertising war (Winters, 1987). After the deregulation of the Australian telephone industry, incumbent Telstra responded to the entry of Optus by running ads emphasizing how Telstra was a homegrown company compared to Optus, which was a foreign brand (Roberts, 2005). Similarly, entrants utilize negative advertising as a strategic tool accompanying their entry. The entry of Merck to the angiotensin-converting enzyme inhibitor market with Vasotec was accompanied by fierce negative advertising against Bristol-Myers Squibb's (BMS) Capoten. Merck ads emphasized Capoten's side effects, while BMS argued that studies could not confirm them. In the pain reliever market, McNeil's Tylenol faced two entrants, Datril, and Anacin, whose negative ads claimed inflammatory side effects from Tylenol (Knight, 1978; Robinson, 1988).

Negative advertising wars have been ubiquitous historically, although managers and advertisers

complain that these wars harm all affected parties and decrease market demand altogether.[1] (Beard, 2013, p.173), in his historical analysis on comparative advertising, provides a number of quotes from managers regarding the harm done by negative advertising. Bartos, a senior vice president for the agency J. Walter Thompson said regarding Coca-Cola's withdrawal from Cola Wars of the 1980s that "such strategies erode confidence in both brands in the mind of the public and that both companies would ultimately carry the soft-drink market into a commodity category" (Marketing News, 1980). McDonald's president Michael Quinlan is quoted saying regarding the Burger Wars "If you've got good [product], flaunt it, but don't tear down someone else. It's not good for the industry as a whole, and I think we ought to stop," (Hume, 1986). Following the Spaghetti Sauce Wars, an executive at Unilever proclaimed that "between [Unilever and Campbell], we're spending $60 million a year to convince consumers that our spaghetti sauce is really crappy" and "[during these wars] the category has declined every year for several years" (Neff, 1999).

There is sufficient evidence to show that industry leaders cannot avoid negative advertising wars, even though negative advertising harms all parties. Then, a rational entrant would make its product design decision considering the advertising competition down the road. As Robinson (1988) points out, "if aggressive and damaging reactions are expected, the entrant can be frightened off or choose to enter on a less ambitious scale" (p. 368). In fact, when little negative advertising is observed in an industry, this may be precisely because firms are making competitive product design choices to prevent it; and had negative advertising not been permitted, they would have made their design choices differently.

In this paper, we theoretically study how an entrant's product positioning is affected by the threat of downstream negative advertising response. To this end, we build a model of informative advertising competition where product design is endogenous. Each product in the market provides a deterministic baseline level of consumption utility to consumers; in addition, each product may

[1] Beard (2010) documents how similar sentiments were also present back in the 1920s: "Writing about George Washington Hill's war on the sweets industry, the president of the New York Coffee and Sugar Exchange Inc., observed, "Has Mr. Hill forgotten that it was only a short time ago when some of our states, on health grounds, were legislating against cigarettes and that the term 'coffin nails' was applied to them? Would it not be well for the American Tobacco Company to 'Let sleeping dogs lie?' " (Lowry, 1929). A Printers' Ink author many years after the Baking Powder War warned that this kind of damage could last for years: "Lots of people still alive and well can vividly recall the days of some years ago when they were repeatedly warned to beware of 'benzoate of soda.' ... Eventually, various manufacturers and advertisers of foods discovered the alarming effect such copy was having on their business, and they recovered their reason by stopping all such publicity" (Hanley, 1927)." He substantiates the argument by adding that "In a speech to the Advertising Club of Greater Boston, David C. Stewart, president of agency Kenyon & Eckhardt, summarized this belief: "There are certain industries and certain product areas today in which the battle of competitive advertising claims has reached the harsh crescendo of jungle warfare ... public confidence [once] shaken ... [usually exerts] a stern reaction against the industries themselves" (as cited in Overly competitive ads invite action by U.S. 1965, 68)."

have a negative and a positive attribute, whose values are randomly drawn and revealed to firms after the product launch. The design choice for the entrant is between two product positions that result in co-locating with or locating apart from the incumbent in a market where consumer preferences are horizontally differentiated. If the entrant co-locates, then it chooses a design similar to that of the incumbent, and competing products are more likely to have identical attribute values for the positive and negative attributes.

In this environment, we model advertising as a firm's choice between truthfully informing consumers about the positive aspects of its own product ("positive advertising"), or truthfully informing consumers about the negative aspects of the competitor's product ("negative advertising"); or, not advertising at all. A firm's advertising choice also allows consumers to infer its unadvertised attribute(s). Furthermore, when products are similarly positioned in the market and their positive and negative attributes are positively correlated, one firm's advertising facilitates inference about the unadvertised attributes of the other product. Given the last point, firms may avoid highlighting the negatives of a competing product when their designs are close.[2]

This simple structure, by itself, generates rich implications: (1) the presence or tone of advertising can be informative for both advertised and unadvertised products, and (2) the form of advertising competition depends on how the products are positioned in the market. We endogenize the entrant's product positioning decision which happens prior to the advertising decisions. A forward-looking entrant, therefore, considers potentially different advertising outcomes following each product positioning choice.

Our structure leads to a fundamental trade-off for the entrant. If the entrant chooses to position itself similarly to the incumbent, it cannot take advantage of the heterogeneous preferences of the consumer base. If it chooses a different product design than the incumbent, it opens itself to possible negative advertising by the incumbent. We analyze how this trade-off shapes the positioning and advertising strategies of firms.

Our first main result is that, in this setting, firms may practice negative advertising in a prisoners' dilemma setting: if they could coordinate, both would prioritize positive advertising; however, competition pushes them to deviate to negative advertising. Furthermore, the incentive to devi-

[2]For instance, if McDonald's runs a negative advertisement about Burger King, criticizing the healthiness of Burger King products, consumers may be discouraged from eating at McDonald's as well, because McDonald's products are perceived similar to those of Burger King on a healthiness scale. The survey in Dolliver (2009) documents that 38% of consumers think less of the brand that does negative advertising. Beard (2013) documents "In 1996, AnheuserBusch (A-B) executives tellingly admitted to Advertising Age that their comparative campaign criticizing craft brewers, such as the Boston Beer Company, for the questionable quality of their beer "garnered a stronger response from A-B consumers than from the non-A-B consumers they were targeting."

ate is stronger when their product positions are differentiated relative to when they are not. Our second main result is that firms have incentives to produce similar products at the product design stage to commit to not engaging in negative advertising later. If the benefit of avoiding a negative advertising war is larger than the cost of increased competition, firms choose designs that show higher similarity in equilibrium. Hence, negative advertising is more likely to be observed in markets where brands are sufficiently differentiated from each other in positioning.

Given the potential for industry-level harm, it is natural to ask if negative advertising should be allowed. Despite the warnings from managers, the Federal Trade Commission (FTC) has adopted an encouraging position on negative advertising, claiming that it "is a source of important information to consumers and assists them in making rational purchase decisions." The FTC, moreover, claims that negative advertising "encourages product improvement and innovation."[3] In contrast, the European Union had explicitly banned negative advertising until the late 1990s (Anderson and Renault, 2009). Given the opposing positions taken by the two regulatory agencies, it is essential that we investigate the welfare implications of negative advertising for consumers. Our structure is well-suited for such a welfare comparison. In our model, negative advertising has two key welfare effects on consumers: first, as claimed by the FTC, consumers see a welfare gain due to the "additional information" about the competing products. Second, counter to the FTC's claims, we show that negative advertising may incentivize "reduced product variety" in the market and lead to a welfare loss. Put differently; negative advertising can *discourage* product innovation. Our third main result is that, when prior uncertainty about the attributes of products is sufficiently low, the "reduced product variety" effect may dominate the "additional information" effect, and allowing negative advertising may result in an overall welfare loss for consumers. Furthermore, the welfare loss is higher when consumer preferences are more heterogeneous.

In the benchmark model, we abstract away from modeling price competition to deliver the intuition without added analytical complexity. In an extension, we examine the impact of endogenizing price competition in our setting, considering a model *a lá* Diamond (1971) and Kuksov (2004). Our fourth main result states that price competition reduces the incentives to co-locate to avoid negative advertising, but it does not entirely remove them as long as the heterogeneity in product preferences is sufficiently small.

In a second extension, we discuss a market where negative advertising is widespread — political competition. Our fifth main result is, negative advertising is more likely to be observed in political

[3] See the FTC statement on comparative advertising here https://www.ftc.gov/public-statements/1979/08/statement-policy-regarding-comparative-advertising

competition. This is because the objective of a politician is to win by plurality (i.e., receiving more votes than the competitor, rather than maximizing own vote count). This subtle modification to the objective function implies that a decrease in the overall voter base is not inherently bad for a politician; thus, the damage from running negative advertising is smaller for the candidates. We indeed observe that negative advertising is abundant in political competition (Ansolabehere et al., 1994; Gandhi et al., 2016). These results are robust to simultaneous rather than sequential entry.

Our paper contributes to three different strands of the literature. The first strand is on the theory of informative advertising competition. The literature starts with seminal papers by Grossman and Shapiro (1984), Austen-Smith (1987), and Meurer and Stahl II (1994) who analyze the role of information provision about product characteristics for firm competition in models where firms choose how much advertising to do.[4] These papers either do not explicitly consider consumers' inference from advertising or assume consumers do not have rational expectations. Coate (2004) introduces Bayesian voters who make inference on unadvertised candidates, using the equilibrium advertising choices. Schultz (2007) introduces the ability to advertise about the opponent's type as well; however, a perfect unraveling result leads firms to advertise both types or neither and prevents advertising tone from being a meaningful choice. The closest papers to ours are by Singh and Iyer (2020) and Polborn and David (2004). Both have a meaningful decision between positive and negative advertising in a Bayesian framework. We extend the framework of these papers in two directions, which are novel contributions to this literature, to the best of our knowledge. First, we model the correlation between competing product attributes, which allows advertising to have spillover effects beyond the advertised product. Second, we model product positioning and advertising tone decisions jointly as equilibrium outcomes. The two extensions allow our model to generate: (1) a prisoners' dilemma type negative advertising war, (2) reduced product variety by firms to avoid negative advertising wars and (3) more candidate polarization in political competition relative to product competition.[5]

The second strand of the literature that we contribute to analyzes the role of negative advertising

[4]See LeBlanc (1998) and Amaldoss and He (2010) for examples of advertising competition where advertising informs about prices instead of product attributes. See Anderson and Renault (2009) and Emons and Fluet (2012) for examples of "comparative advertising" models, where firms disclose information about both firms, in relation to one another.

[5]See Skaperdas and Grofman (1995), Harrington Jr and Hess (1996), Bass et al. (2005), and Chen et al. (2009) for examples of persuasive, as opposed to informative, models of advertising competition through advertising tone. Bass et al. (2005) introduce brand and generic advertising, which are very close to our definitions of negative and positive advertising, respectively. They show that firms prefer generic advertising to enlarge the market in the short run and brand advertising to steal consumers from competitors in the long run. The model in Chen et al. (2009) yields a prisoners' dilemma similar to as ours, however, through intensified price competition. Authors show that increasing the cost of advertising can help firms by preventing a pricing war that results in an advertising war.

in political competition. The core mechanism that discourages firms from negative advertising in our case is the shrinking consumer base due to negative advertising. The literature on political competition does not have a consensus on the effects of negative advertising on voter turnout.[6] One puzzle is why politicians utilize negative advertising, despite the ambiguous effect of the negative tone of advertising on their voter base. We contribute to this literature by demonstrating that, even in an environment in which negative advertising demobilizes one's own voters, political candidates may use more negative advertising relative to firms.

Finally, our paper is also related to the literature on product positioning. Gavish et al. (1983), Moorthy (1988) and Horsky and Nelson (1992) are among the large body of papers that study positioning decisions in the spirit of Hotelling (1929). Kuksov (2004) and Thomadsen (2007) show how classical results in these models can be reversed once consumer search costs and asymmetric competitors, respectively, are taken into account. These papers study positioning in the absence of advertising decisions. We model the positioning decision jointly with advertising decisions and show how the availability of negative advertising may lead to inefficiently low product differentiation.

The rest of the paper is organized as follows. In Section 2, we present the focal model in which competing entities have to make positioning and advertising decisions. In Section 3, we analyze the model for the case of firms selling products that have the objective of maximizing profits and obtain our key insights on positioning and advertising. In Section 4, we conduct the welfare analysis. In Section 5, we analyze the model for the case of political candidates aiming to win by plurality and obtain results on positioning and advertising that provide a contrast to the case of firm competition. In Section 6, we conclude. Analysis details and all proofs are provided in an online appendix to the paper.

## 2 Model

We build a microfounded model of informative advertising for competing but substitute products. We use a parsimonious specification and assume two competing products, each of which are defined by a product position, as well as a positive and a negative attribute. Consumers use information from advertising to infer the attributes of products before they make a purchase decision. We focus on the interdependence of product design and advertising by assuming that product attributes are similar for similarly designed products, which determines the firms' positioning and advertising

[6] See Lau et al. (2007) and Arceneaux and Nickerson (2010) for no effect, Niven (2006) and Barton et al. (2016) for increased voter turnout, and Ansolabehere et al. (1994) for decreased voter turnout.

strategies. In this section, we explain how we model each of the above components in detail. We frame the main analysis focusing on product competition, and in Section 5.2 we discuss the implications for political competition.

## 2.1 Setup

**Consumer Preferences** There is a mass 1 of risk-neutral consumers whose ideal product is positioned at point $L$, and a mass 1 whose ideal product is positioned at point $R$. Consumers can either consume nothing for a utility of 0 or one of the available products in the market. The utility that consumer $j$ at position $\chi_j \in \{L, R\}$ derives from consuming product $i$ positioned at $x_i$ is

$$U_{ij} = \gamma_j - |x_i - \chi_j| + A_i \tag{1}$$

where $\gamma_j$ denotes the reservation value of consuming a product for consumer $j$, $A_i$ represents the properties of product $i$ that can only be revealed with advertising, and $|x_i - \chi_j|$ represents the positional distance between consumer $j$'s ideal product ($\chi_j$) and the position of product $i$ ($x_i$). We consider a horizontal differentiation model wheere consumers prefer products that are closer to their ideal product and incur a disutility from any deviation from this point, proportional to the distance between $x_i$ and $\chi_j$. The reservation value is heterogeneous across consumers according to the *cdf* $\gamma_j \sim \Gamma : [\underline{\gamma}, \overline{\gamma}] \to [0, 1]$. Throughout, we assume $\Gamma$ is the uniform *cdf* for derivations.

**Firms and Products** Two ex-ante identical firms (Firm 1 and Firm 2) produce substitute goods, and each firm chooses its product design and advertising strategy. The possible product designs are represented by the two consumer mass locations, $L$ and $R$, where each position has an associated product design.

While consumers have zero distance to a product whose design is at their own location, the distance to the product at the opposite location is denoted by $\delta$, which is a measure of heterogeneity in consumer tastes. A higher $\delta$ indicates that consumers are more dissimilar in their tastes, or the product in the opposite location has a lower match value. In this setting, positioning the two products at an identical location vs. differentiated locations can be interpreted as lower versus higher product variety in the market. We summarize the locations of the consumers and products in Figure 1.[7]

[7] Choosing a *continuous distribution of consumers* would lead to qualitatively similar insights; however, that would make the derivations intractable. Previous studies have also used a discrete consumer distribution when product positioning is a decision (e.g., Kuksov 2004).

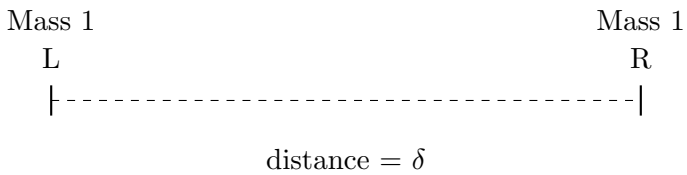


Figure 1: Consumer and Feasible Product Positions

We assume that each product in the market is associated with two attributes, where one increases and the other decreases consumer utility. Conceptually, these two attributes can be considered as "strong" and "weak" attributes of any product. For the exposition, we refer to the attributes of product $i$ as positive and negative, and denote them with $P_i$ and $N_i$.[8] There is uncertainty regarding the values of the positive and negative attributes, which we represent by using Bernoulli distributions:

$$P_i = \begin{cases} \Pi, & \text{w.p. } \sigma_\Pi \\ 0, & \text{o.w.} \end{cases} \quad \text{and } N_i = \begin{cases} -\beta, & \text{w.p. } \sigma_\beta \\ 0, & \text{o.w.} \end{cases} \tag{2}$$

In the above formulation, $-\beta$ and $\Pi$ indicate the valuation of the product attributes while $\sigma_\beta$ and $\sigma_\Pi$ indicate the uncertainty around the valuation of products. The attributes $P_i$ and $N_i$ can be thought of as the outcomes of firms' experimentation when designing products. The realized values of these attributes to the consumers are unknown to the firms and to the consumers before the entrant makes the product location choice. Firms learn the attribute values for both products after they are designed but before any advertising. Consumers can only learn the values of the attributes through advertising. Throughout the paper, we use a convention where we say the positive (negative) attribute is "present" if $P_i = \Pi$ ($N_i = -\beta$). $A_i$ in (1) then becomes the combined effect of the two attributes, i.e., the sum of $N_i$ and $P_i$. We assume, without loss of generality, $\sigma_\Pi \Pi = \sigma_\beta \beta$, i.e. $E[A_i] = 0$.

The positioning choices of the products indicate an overlap between attributes such that, for products that are co-located, $cor(P_1, P_2) = cor(N_1, N_2) = \rho > 0$.[9] Hence, if product $i$ has a negative

[8] We assume each product has a positive and negative attribute to focus on the more interesting case where a brand faces a trade-off and a competitive threat of negative advertising from the competing brand.

[9] In some industries such as pharmaceuticals, the ingredients used in drugs may be fairly common, for instance, when a patent expires, implying a high $\rho$ between the competing drugs in the market. Therefore, the presence of a side effect in a branded drug may likely indicate the presence of similar side effects in generic drugs. In other markets where firms rely on trade secrets in the design of a product, such as in perfumery, $\rho$ is expected to be low. A perfume having certain base notes tells little about what to expect from other perfumes that share similar top notes.

attribute, i.e., $N_i = -\beta$, then the competitor's product is more likely to have the negative attribute as well, if its location is the same. We assume $cor(P_1, P_2) = cor(N_1, N_2) = 0$ when product are located apart. There are two implicit assumptions that are not necessary for the results but simplify the exposition. First, all consumers value these attributes equally, regardless of their ideal position or reservation value. Second, the values of the attributes are identically distributed regardless of product location ($L$ and $R$).

**Advertising** In this setting, advertising serves to inform consumers of the realized values of the product attributes. In particular, a firm can either advertise the presence of the positive attribute of its own product ($P_i = \Pi$) or the presence of the negative attribute ($N_i = -\beta$) of the competing product, but not both.[10] We will refer to these choices as "positive" and "negative" advertising, respectively. Firms can also choose not to advertise; however, consumers make rational inferences from this choice. We assume that firms know the attributes of both products at the advertising stage and advertise truthfully.[11]

The firms and consumers have a common prior, which is identical to the distribution of products' attribute values given in (2). Consumers make rational inferences about the attributes through the advertising choices of the firms. Specifically, advertising can influence consumer valuation of a product positively or negatively. For example, if firm $i$ announces $P_i = \Pi$, then consumers' posterior belief will put probability 1 on $P_i = \Pi$, increasing the expected utility from product $i$. On the other hand, if firm $-i$ announces $N_i = -\beta$, then consumers' posterior belief will put probability 1 on $N_i = -\beta$, lowering the expected utility from purchasing product $i$.

Advertising of a single attribute, in our model, can have three distinct informative effects in equilibrium. First, it changes the consumer beliefs to a degenerate distribution about the advertised attribute. We refer to this as the "direct effect" of advertising. Second, it changes the consumer beliefs about the attribute(s) that are not advertised. Since advertising is a choice, the fact that a certain attribute is not advertised may inform consumers as well. We refer to this latter effect as the "inference effect" of advertising. Third, when products are co-located, and hence the realizations

[10] This assumption is made for analytical tractability, however, either a budget constraint for firms or a limited attention assumption for consumers can deliver one type of advertising.

[11] It is reasonable to question if the truth in advertising assumption that we are making is a reasonable one. Regulations in the United States and European Union (EU) both prevent firms from "untruthful" advertising – that is, if a firm is advertising that its product has characteristics that it does not have or provides benefits to consumers that it does not provide in reality, these ads are legally required to be removed. Therefore, even if firms could engage in untruthful ads in the short term, this cannot be a viable long term strategy. Similarly, comparative advertising regulations in the EU require that a firm cannot claim a negative attribute for a competitor if the competitor does not really have this negative property. The guidelines of the Federal Trade Commission (FTC) of the US can be found at https://www.ftc.gov/news-events/media-resources/truth-advertising.

of their attributes are correlated, advertising may influence consumer beliefs about the competing product. We refer to this effect as the "spillover effect" of advertising.

The two firms' advertising may have rich combinations; however, our structure allows us to eliminate some. In particular, when all three effects are present for a given attribute, the "direct effect" always dominates the other two because the announcements are always truthful (as discussed in Footnote 11). The "inference effect," when present, dominates the "spillover effect" in any pure strategy equilibrium because, as will become clear shortly, information about the competing product is (infinitely) more precise under the former effect relative to the latter. We will discuss these effects in detail in subsection "Inference from Advertising" in Section 3.2.

**Consumers' and Firms' Problems** Consumers choose between buying product 1, buying product 2, and not buying anything by comparing the expected utilities of each. If two products offer the same expected utility, we assume that consumers choose each good with equal probability. Consumers form their expectations using their posterior beliefs constructed from their common priors and the firms' advertising. The utility of the outside option is normalized to 0.

We first consider a sequential entry scenario where one of the firms ("incumbent") is already located at $L$, and the other ("entrant") makes the location choice.[12] Without loss of generality, let Firm 1 be the incumbent firm and Firm 2 be the entrant and let the incumbent be located at the left end of the spectrum of consumer tastes (as provided in Figure 1), i.e., $x_1 = L$.[13] Firms maximize revenue by choosing their product positioning, $x_i \in \{L, R\}$, and advertising strategy, $a_i \in \{P_i, N_{-i}, \varnothing\}$.

We assume that the market is competitive and firms are price takers where the price is normalized to 1. This is done purposefully to keep our core model applicable to various cases of competition, such as product as well as political competition. This simplification makes it easier to comprehend advertising-related forces at play and how they interact with positioning. However, for the product competition case, we do include *price* as a decision variable in an extension of the model in Section 5. The main tension is that price competition is higher with co-location than with differentiation; we build this tension into the model in Section 5 and show that our key results remain unchanged as long as price competition is not too high.

[12] In the online appendix Section A.1.1 we also consider the case of simultaneous entry and show that the results are robust.

[13] Throughout the manuscript, we will use superscripts to denote policy functions and subscripts to denote realized decisions.

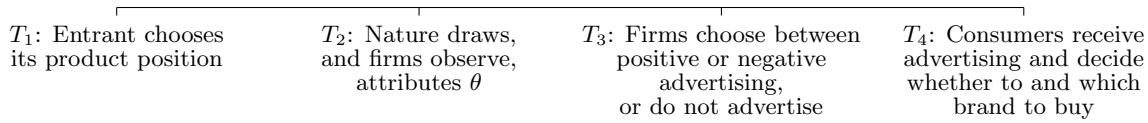


Figure 2: Timeline of the Incumbent-Entrant Game

**Timeline of the Game** The timing of decisions is given in Figure 2. First, Firm $k$, the entrant, chooses its product position to co-locate or differentiate itself from the position of the incumbent. Then, nature draws the values of the attributes $\theta = \{P_1, N_1, P_2, N_2\}$ for both products. At this stage, both firms observe $\theta$, but consumers do not. Following this stage, the firms simultaneously choose their advertising strategies — they decide whether to carry out positive advertising, negative advertising, or choose not to advertise. Finally, consumers receive advertising, update their beliefs about the products, and make their product purchase decisions.

### 2.2 Equilibrium Definition

We will next characterize the equilibrium definition that we will utilize.

**Definition** Let $\theta_i = \{P_i, N_i\}$ denote the type of firm $i$ and $\theta = \{\theta_1, \theta_2\}$. A Perfect Bayesian Equilibrium (PBE) in this game is a positioning decision for the entrant $x^2 \in \{L, R\}$, advertising decisions $a^2(x_2, \theta) \in \{P_2, N_1, \varnothing\}$, $a^1(x_2, \theta) \in \{P_1, N_2, \varnothing\}$ of firms, beliefs of consumers over firms types $\mathcal{F} : \theta_1 \times \theta_2 \to [0, 1]$, and purchase decisions of consumers $\{g^j(x_2, a_1, a_2)\}_j \in \{i, k, o\}$ such that:

1. Consumers choices are sequentially rational, i.e. $\{g^j(.)\}_j$ maximizes $E[U_{ij}|\mathcal{F}]$.
2. $a^2(.)$ and $a^1(.)$ constitute a Nash Equilibrium (NE) of the advertising sub-game given $\{g^j(.)\}_j$.
3. The location choice $x^2 \in \{L, R\}$ maximizes firm 2's profits given $a^2(.)$, $a^1(.)$, and $\{g^j(.)\}_j$.
4. The consumers' beliefs $\mathcal{F}$ are updated based on $a^2(.)$ and $a^1(.)$ according to the Bayes' Rule.

## 3 Product Positioning and Advertising Strategies

In this section, we characterize the solution to the model described in Section 2. We start by discussing the informative role of advertising, which sets the stage for the algebra of belief updating in equilibrium. Second, to provide a benchmark, we characterize the firm strategy in an environment

where negative advertising is not allowed. Third, we characterize the solution to the full model, where both positive and negative advertising is permissible.

In the analysis below, we assume that some consumers will always buy a product and some consumer who will not buy either. Assumption 1 ensures that trivial cases where advertising is ineffective are eliminated.

**Assumption 1.** $\underline{\gamma} < \Pi$ *and* $\overline{\gamma} > \delta + \sigma_\Pi \Pi + (1 - \sigma_\beta)\beta$

We will use backward induction to derive the equilibrium: first, characterize the equilibrium of the advertising sub-game, and then characterize the positioning strategy in the complete game.

### 3.1 Benchmark: Negative Advertising Not Permitted

We start the analysis with the consideration of a benchmark case where negative advertising is not permitted. For instance, until the 1990s, negative advertising was not permitted in numerous countries in the European Union (Anderson and Renault, 2009). Thus, the only options for a firm are to run positive advertising or not run any advertising. Furthermore, since advertising is truthful, firm $i$ can run positive advertising only if $P_i = \Pi$. Proposition 1 characterizes the equilibrium under this case.

**Proposition 1. (Negative Advertising Not Permitted)** *When negative advertising is not permitted, there exists a unique Perfect Bayesian Equilibrium in which the entrant locates its product apart from the incumbent, and only firms with positive attributes engage in advertising. Consumers are fully informed about positive attributes in this equilibrium but not informed about the negative attributes.*

The proposition suggests that, when negative advertising is not permitted, firms advertise only when they can, that is, only when they have a positive attribute to announce ($P_i = \Pi$), and do not advertise otherwise. Consumers learn about the positive attribute of a product if the firm engages in positive advertising (direct effect of advertising) and infer that the positive attribute is missing (i.e., has a magnitude of 0) when it is not advertised (inference effect of advertising). Consumers cannot learn about the negative attributes of the products ($N_i$) since negative advertising is not allowed and firms, by assumption, cannot advertise their own negative attributes, or lack thereof.

Recall that the common prior is $E[A_i] = E[P_i + N_i] = \sigma_\Pi \Pi - \sigma_\beta \beta$, which we normalize to 0 by assuming $\Omega \equiv \sigma_\Pi \Pi = \sigma_\beta \beta$. On the one hand, if firm $i$ runs positive advertising, consumers update their beliefs for product $i$ positively, i.e., $E[A_i | a_i = P_i] = \Pi - \sigma_\beta \beta = (1 - \sigma_\Pi)\Pi$. On

the other hand, running no advertising leads consumers to update their beliefs negatively, i.e., $E[A_i|a_i = \varnothing] = 0 - \sigma_\beta \beta = -\sigma_\Pi \Pi$. Hence, regardless of the positioning of the products, the firms are weakly better off running positive advertising when they can. Lastly, the "spillover effect" is not present in this equilibrium because consumers can infer a product's positive attribute perfectly from the advertising decision.

Next, let us consider the location choice of the entrant, which has to be made before the attribute values are realized. The entrant compares the expected payoff from choosing each location, where the expectation is taken over the potential realizations of the attributes of each product. Since negative advertising is forbidden, the realization of $N_i$ becomes irrelevant.

When the firms are co-located, and one firm has the positive attribute ($P_i = \Pi$), but the other firm is missing it ($P_i = 0$), the former will capture the entire market. Because the environment is symmetric across locations and no one is buying from the opponent, the location decision does not matter for the total demand. When firms are symmetric in attributes, i.e., $P_1 = P_2 = \Pi$ or $P_1 = P_2 = 0$ they share the market equally. If they are co-located ($x_2 = L$), then consumers located at $R$ will have to do with a product that is not at their favorite position. If firms are located apart ($x_2 = R$), then all consumers have access to a product in their favorite position. The aggregate demand will be larger in the latter scenario, leading to a higher payoff for both firms. Hence, the expected payoff of locating apart is higher than that of co-locating. Proposition 1 formalizes the reasoning.

When negative advertising is not permitted, consumers are not fully informed about product characteristics and may make decisions they regret ex-post. Proposition 1 suggests, however, that in the unique equilibrium, products are differentiated, which allows more consumers to buy a product matching their preferences. In the next subsection, we argue that allowing negative advertising leads to more informed consumers yet may also reduce product differentiation in the market.

### 3.2 Full Model with Positive and Negative Advertising

Next, we analyze a model where negative and positive advertising is allowed; hence, the decision-set of the firm is to run positive ads, run negative ads, or not advertise. Notice that firm $i$ can run positive advertising only if it has a positive attribute ($P_i = \Pi$) and run negative advertising only if its competitor has a negative attribute ($N_{-i} = -\beta$).

The trade-off for a firm between positive and negative advertising is as follows. Positive advertising by firm $i$ increases the expected utility of consumers from consuming product $i$, and it

hence allows stealing consumers from its competitor while at the same time expanding the market size. Negative advertising reduces the expected utility from consuming product $-i$, thus helps to steal the competitor's consumers, but it may at the same time shrink the overall market. So, a firm would only find it attractive to engage in negative advertising if the gain due to stealing customers exceeds the loss due to the shrinking market size. This trade-off can result in a *prisoners' dilemma* outcome in advertising strategies: If they could choose, both firms would benefit from positive advertising. However, due to competition, they may find themselves in a negative advertising equilibrium, where they each experience losses due to the reduced market size. The presence of a "spillover effect" can help to prevent a prisoners' dilemma outcome by penalizing a firm engaging in negative advertising when the competitor shares similar design features (i.e., has highly correlated attributes).[14] In anticipation of this spillover effect, the entrant may choose its product positioning to avoid an advertising war down the road. Proposition 2 formalizes this positioning strategy in anticipation of a subsequent advertising competition.

**Proposition 2. (Co-location and Positive Advertising)** *When negative advertising is permitted, there exists a Perfect Bayesian Equilibrium in which the entrant co-locates and both firms prioritize positive advertising. In the off-the-equilibrium path where the entrant locates apart, firms engage in negative advertising wars even when positive advertising is available.*

In Proposition 1 we showed that when negative advertising is not permitted, the unique equilibrium outcome is positive advertising and firms choosing to locate apart. Proposition 2 points to the possibility of another equilibrium with a counterintuitive outcome when negative advertising is allowed: The entrant may choose to co-locate with the incumbent to activate spillover effects and reduce the chances of a downstream negative advertising attack on itself. This strategy, in turn, pushes the incumbent and the entrant to engage in positive advertising. On the one hand, with no negative advertising, firms prevent the shrinkage of total market demand. On the other hand, co-location reduces the variety of products offered to consumers. Put differently; there is a direct relationship between negative advertising wars and product differentiation.

[14] Beard (2013) documents several examples of how the spillover effect discourages negative advertising in practice: "Referring to comparative advertising that targeted prescription drugs Seldane and Alegra on behalf of Claritin, ad agency executive Lorraine Pastore tellingly told Advertising Age that they would not respond: "That would damage the category as a whole; its not a strategy we would be comfortable with." (Wilke, 1997)." Another example is by Microsoft's vice president of systems strategy, Jonathan Lazarus, who argues that negative advertising is "bad business. I don't think there's ever been a study that shows that negative advertising sells products. In our high-tech industry, people have a fear of the computer. They are worried about losing data and that it's complicated. So if I suddenly paint a competitor's products as complicated, I'm overall feeding those arguments that things will be tough to deal with" (Jaben, 1992).

The existence of the equilibrium outcome described above requires a set of conditions to hold. We next discuss these conditions and the consumer beliefs about firm strategies that are consistent with them.[15]

**Inference from Advertising** To simplify the discussion that follows, we will distinguish between firms based on their ability to advertise. We will refer to firm $i$ as the "weak" opponent if $P_i = 0$ and $N_{-i} = 0$, i.e. if it cannot run any advertising. Otherwise, we will refer to the firm as the "strong" opponent. We will show that the optimal advertising strategy (and consequently the consumers' beliefs about the optimal strategy) against a weak opponent is different from that against a strong opponent. Under the Perfect Bayesian Equilibrium in Proposition 2, the consumers correctly believe that the firms prioritize (1) negative advertising under locating apart and positive advertising under co-location against strong opponents and (2) positive advertising in either location outcome against weak opponents. Given these beliefs, the posterior expectations about unobserved attributes ($A_i$) are as given in Table 1.

| | | Locating Apart | | Co-location | |
|---|---|---|---|---|---|
| $a_i$ | $a_{-i}$ | $E[A_i]$ | $E[A_{-i}]$ | $E[A_i]$ | $E[A_{-i}]$ |
| $P_i$ | $P_{-i}$ | $(1-\sigma_\Pi)\Pi + \sigma_\beta\beta$ | $(1-\sigma_\Pi)\Pi + \sigma_\beta\beta$ | $(1-\sigma_\Pi)\Pi$ | $(1-\sigma_\Pi)\Pi$ |
| $P_i$ | $N_i$ | $(1-\sigma_\Pi)\Pi - (1-\sigma_\beta)\beta$ | $\sigma_\beta\beta$ | $(1-\sigma_\Pi)\Pi - (1-\sigma_\beta)\beta$ | $-\sigma_\Pi\Pi - \rho(1-\sigma_\beta)\beta$ |
| $N_{-i}$ | $N_i$ | $-(1-\sigma_\beta)\beta$ | $-(1-\sigma_\beta)\beta$ | $-\sigma_\Pi\Pi - (1-\sigma_\beta)\beta$ | $-\sigma_\Pi\Pi - (1-\sigma_\beta)\beta$ |
| $P_i$ | $\varnothing$ | $(1-\sigma_\Pi)\Pi + \sigma_\beta\beta$ | $-\sigma_\Pi\Pi$ | $(1-\sigma_\Pi)\Pi + \sigma_\beta\beta$ | $-\sigma_\Pi\Pi + \rho\sigma_\beta\beta$ |
| $N_{-i}$ | $\varnothing$ | $\sigma_\beta\beta - \sigma_\Pi\Pi$ | $-\sigma_\Pi\Pi - (1-\sigma_\beta)\beta$ | $-\sigma_\Pi\Pi + \sigma_\beta\beta$ | $-\sigma_\Pi\Pi - (1-\sigma_\beta)\beta$ |
| $\varnothing$ | $\varnothing$ | $-\sigma_\Pi\Pi + \sigma_\beta\beta$ | $-\sigma_\Pi\Pi + \sigma_\beta\beta$ | $-\sigma_\Pi\Pi + \sigma_\beta\beta$ | $-\sigma_\Pi\Pi + \sigma_\beta\beta$ |

Table 1: Posterior Beliefs Given Advertising Outcomes

The table demonstrates how the three effects of informative advertising choices by the firms influence consumer utility. First, the "direct effect" of advertising results in consumers updating their beliefs about the benefits from using the product based on the advertised attribute, regardless of the positioning choice of the entrant in the market. In particular, the prior beliefs put $P(P_i = \Pi) = \sigma_\Pi$, and positive advertising by firm $i$ ensures that product $i$ has the positive attribute with certainty, resulting in the posterior belief $P(P_i = \Pi) = 1$. This increases the product attributes' expected value $E[A_i]$ by $(1-\sigma_\Pi)\Pi$. Second, the prior belief that product $i$ has a negative attribute is $P(N_i = -\beta) = \sigma_\beta$. Negative advertising by the competitor firm $-i$ ensures that the product has the negative attribute with certainty, resulting in the posterior belief $P(N_i = -\beta) = 1$ and decreasing $E[A_i]$ by $(1-\sigma_\beta)\beta$.[16]

[15] We discuss the possibility of multiple equilibria in the Appendix Section A.1.2.

[16] Remember that the prior expectation equals 0 due to the normalization $\sigma_\Pi\Pi = \sigma_\beta\beta$.

Second, the "inference effect" of advertising allows consumers to update beliefs about the un-advertised attributes of products in two different cases. In case 1, if firm $i$ runs no advertising, then consumers infer that product $i$ must lack the positive attribute, resulting in the posterior $P(P_i = \Pi) = 0$ and decreasing $E[A_i]$ by $\sigma_\Pi \Pi$. Moreover, consumers should also infer that the competitor must lack the negative attribute, updating their posterior to $P(N_{-i} = -\beta) = 0$ and increasing $E[A_{-i}]$ by $\sigma_\beta \beta$. In case 2, consumer inference depends on the positioning choice of the entrant and the type of opponent (weak vs. strong). In particular, if the entrant locates apart, consumers believe negative advertising to be the prioritized strategy against strong opponents. Hence, if firm $-i$ runs positive advertising, consumers would infer that product $i$ does not have the negative attribute, i.e., $P(N_i = -\beta) = 0$, increasing $E[A_i]$ by $\sigma_\beta \beta$. Otherwise, if the opponent is weak or the entrant co-locates, again per Perfect Bayesian Equilibrium described in Proposition 2, consumers believe positive advertising to be the prioritized strategy. Hence, if firm $i$ runs negative advertising, consumers infer that firm $i$ does not have the positive attribute, i.e. $P(P_i = \Pi) = 0$, decreasing $E[A_i]$ by $\sigma_\Pi \Pi$.

Lastly, the "spillover effect" of advertising is only present if the entrant chooses to co-locate. On the one hand, when firm $i$ runs negative and firm $-i$ runs positive advertising, consumers cannot learn about $N_i$ through direct or inference effects.[17] However, consumers learn about the negative attribute of product $-i$ and update to $P(N_{-i} = -\beta) = 1$ due to firm $i$'s negative advertising. Because the values of $N_i$ and $N_{-i}$ are correlated when firms co-locate, consumers take the correlation between the product attributes into consideration, updating posterior beliefs to $P(N_i = -\beta) = (\sigma_\beta + \rho(1 - \sigma_\beta))$, which decreases $E[A_i]$ by $\rho(1 - \sigma_\beta)\beta$. In this case running negative advertising hurts the firm because consumers can deduce that the firm's own product is also likely to have the negative attribute. On the other hand, when firm $i$ runs positive advertising and firm $-i$ does not advertise, consumers do not learn about $N_{-i}$ through direct or inference effects. However, they update to $P(N_i = -\beta) = 0$ because of the absence of firm $-i$'s negative advertising. Again, because $N_i$ and $N_{-i}$ are correlated, consumers update their posterior beliefs to $P(N_i = -\beta) = \sigma_\beta - \rho\sigma_\beta$, which increases $E[A_{-i}]$ by $\rho\sigma_\beta\beta$. In this second case, avoiding negative advertising helps firm $i$ because consumers can deduce that the product $i$ is also likely to lack the negative attribute. In both cases, the presence of correlated attributes under co-location discourages negative advertising.

[17] This is because in the Perfect Bayesian Equilibrium described in Proposition 2, firm $-i$ is expected to prioritize positive advertising regardless of the value of $N_i$. Therefore, consumers cannot make an inference.

### 3.2.1 Advertising Strategies

After establishing the three effects of advertising, we next move on to analyzing advertising strategies under different positioning decisions of the entrant. Through these three effects, advertising can help firms to compete with the competitor and the outside option. In particular, advertising can help firms to steal consumers from their opponent, protect own consumers from their opponent, and expand their market by reaching out to consumers with lower reservation values ($\gamma_j$). While positive advertising helps to improve its standing against the competitor and the outside option, negative advertising helps to improve the standing against the competitor but hurts the standing relative to the outside option. Firms will prefer to do negative advertising only if the number of consumers gained from the competitor exceeds those lost to the outside option.

We move on to characterizing the parameter set where negative advertising is prioritized and do so according to both the positioning of the firms as well as the characteristics of the opponent. In particular, in what comes next, given the belief set in Table 1, we will describe the conditions under which the equilibrium in Proposition 2 would hold based on a $2 \times 2$ environment description: locating apart vs. co-locating and competing against a strong vs. a weak opponent. We will see that the prevalence of a negative equilibrium may vary depending on the environment.

**Entrant Locates Apart** If the entrant locates apart ($x_2 = R$), the consumers at $R$ buy from the entrant and the consumers at $L$ buy from the incumbent in the absence of advertising. In this case, to steal consumers, the advantage of advertising must be sufficiently large to surpass the disutility from buying a product that does not match one's preferences, measured by distance $\delta$. Firm $i$ may prioritize negative advertising only if $E[A_i] - E[A_{-i}] > \delta$ holds under negative advertising, but not under positive advertising. In any other scenario, when $E[A_i] - E[A_{-i}] \leq \delta$, positive advertising is prioritized. This is because while running positive advertising increases $E[A_i]$ and expands the market, negative advertising cannot steal more consumers from the opponent than positive advertising does.[18]

We start with the discussion of the conditions required to run negative advertising against a weak opponent in Lemma 1.

**Lemma 1. (Locating Apart, Weak Opponent)** *When firms are located apart, a firm running against a weak opponent prioritizes negative advertising over positive advertising if and only if*

[18] The model has the property that a firm either steals no consumers from the competitor or steals all consumers. Hence, once firm $i$ beats its competitor enough to overcome the preference heterogeneity ($E[A_i] - E[A_{-i}] > \delta$), the additional margin of victory is irrelevant.

$\beta > \delta > \Pi + \sigma_\beta \beta$. *Positive advertising is prioritized otherwise.*

Recall that a weak opponent $-i$, by definition, cannot advertise (i.e., $P_{-i} = 0$ and $N_i = 0$). Then, when firm $i$ runs either positive or negative advertising, the expected attributes of firm $i$ are more desirable, i.e., $E[A_i] > E[A_{-i}]$. When firm $i$ runs negative advertising and $-i$ runs no advertising, $E[A_i] = \sigma_\beta \beta - \sigma_\Pi \Pi$, and $E[A_{-i}] = -(1-\sigma_\beta)\beta - \sigma_\Pi \Pi$. Hence, $E[A_i] - E[A_{-i}] = \beta$, so the first inequality in Lemma 1 ensures negative advertising allows stealing consumers from the opposite location. When firm $i$ runs positive advertising instead and $-i$ runs no advertising, then $E[A_i] = (1-\sigma_\Pi)\Pi + (1-\sigma_\beta)\beta$, $E[A_{-i}] = -\sigma_\Pi \Pi$. Hence, $E[A_i] - E[A_{-i}] = \Pi + \sigma_\beta \beta$, so the second inequality ($\delta > \Pi + \sigma_\beta \beta$) ensures that positive advertising does not allow stealing consumers from the incumbent's location. The set of parameters for which negative advertising is prioritized against weak opponents does not satisfy the remaining inequalities, i.e., (A1a)-(A1d), for the equilibrium in Proposition 2 to exist. Therefore, in the equilibrium described in Proposition 2, positive advertising is prioritized against weak opponents when firms are locating apart.

Lemma 2 describes the conditions for a firm running against a strong opponent to use negative advertising if firms locate apart.

**Lemma 2. (Locating Apart, Strong Opponent)** *When firms are located apart, a firm running against a strong opponent prioritizes negative advertising over positive advertising if and only if (i)* $\beta > (1-\sigma_\Pi)\Pi + \delta$ *and (ii)* $\bar{\gamma} + \sigma_\beta \beta > (1-\sigma_\Pi)\Pi + \delta$. *Positive advertising is prioritized otherwise.*

Recall that a strong opponent $-i$ uses either positive or negative advertising. So for firm $i$ to prioritize negative advertising against $-i$, two conditions must be satisfied. First, if the opponent runs positive advertising, firm $i$ runs negative advertising only if it can steal the opponent's consumers. More specifically, condition (i) requires $E[A_i | a_i = N, a_{-i} = P] > E[A_{-i} | a_i = N, a_{-i} = P] + \delta$. Condition (i) is necessary and sufficient to ensure that negative advertising is the best response to an opponent who runs negative advertising. While running positive advertising leads to 0 demand because it would allow the opponent to steal consumers, running negative advertising allows sharing the market equally with the opponent.

Second, although condition (i) is necessary to ensure negative advertising is the best response to an opponent who runs negative advertising, it is not sufficient: running negative advertising may help the firm to steal consumers from the opponent, but the number of these consumers may be fewer than those lost to the outside option. For negative advertising to be the best response to an opponent who runs negative advertising, firm $i$ should acquire more consumers from firm $-i$ than it would have acquired had it used positive advertising and expanded the overall market demand.

Formally, $D_i(a_i = N, a_{-i} = N) > D_i(a_i = P, a_{-i} = N)$ must hold, where $D_i$ is the total demand for firm $i$. Jointly, conditions (i) and (ii) are sufficient to ensure that the number of consumers stolen from the opponent more than makes up for those lost to the outside option.

**Entrant Co-locates** In the absence of advertising, if the entrant co-locates ($x_2 = L$), all consumers are indifferent between the two products. Similar to the previous section, we will next describe the advertising strategies of firms when competing against a weak or a strong opponent.

**Lemma 3. (Co-location, Weak Opponent)** *When firms co-locate, a firm running against a weak opponent always prioritizes positive advertising over negative advertising.*

Since firms are otherwise symmetric, all that is required to steal an opponent's consumers is some advertising advantage (i.e., $E[A_i] - E[A_{-i}] > 0$). Because both positive and negative advertising can generate such an advantage against a weak opponent, firms always prioritize positive advertising against weak opponents under co-location as positive advertising expands the overall market demand. This is summarized in Lemma 3.

**Lemma 4. (Co-location, Strong Opponent)** *When firms co-locate, a firm running against a strong opponent always prioritizes positive advertising if and only if* $\Pi > (1-\rho)(1-\sigma_\beta)\beta$. *Otherwise, negative advertising may be prioritized.*

For a firm to prioritize positive advertising against a strong opponent in the unique sub-game equilibrium under co-location, positive advertising should generate an advantage over negative advertising. This can only happen when the stolen consumers with negative advertising (if any) are fewer than the consumers lost due to the shrinking market. This is the condition stated in Lemma 4. Notice that because a "spillover effect" is at work under co-location, the condition is more likely to be satisfied with a large correlation between the attributes ($\rho$). Put differently, when firms position similarly, and their attributes overlap more, the presence of a negative spillover makes negative advertising less desirable and curbs firms' desire to use this strategy.

### 3.2.2 Entrant's Positioning Choice

The discussion until now allowed us to describe the advertising choices of firms under both co-location and locating apart. Next, we discuss the entrant's positioning decision, which boils down to comparing the expected payoff from each position given firms' advertising strategies and consumer beliefs. Since product attributes only realize after the entrant makes a positioning decision, the

entrant takes an expectation over all possible attribute realizations to calculate the expected payoff from locating apart and co-locating with the incumbent.

In the Perfect Bayesian Equilibrium given in Proposition 2, the entrant has three considerations over the two location choices. First, locating apart allows the firms to serve a greater share of the consumers in the market. When the entrant co-locates with the incumbent ($x_2 = L$), consumers in $R$ will incur a disutility of $\delta$ when buying from the firms located at $L$. Hence, a smaller fraction of consumers at $R$ would make a purchase. Second, locating apart makes it more likely for the entrant to face a negative advertising attack because the spillover effect under co-location discourages negative advertising. Third, realizations where firms have similar attributes are more likely under co-location due to the correlation between product attributes. While this last consideration does not necessarily lead to a difference in payoffs, it can amplify or mitigate the magnitude of the previous two considerations. The main trade-off faced by the entrant is thus between the gains from serving a differentiated product and the losses from a negative advertising attack.

**Lemma 5. (Location Choice)** *Let the outcomes of the advertising sub-games be as given in Lemmas 1-4. The entrant co-locates if and only if*

$$\begin{aligned}(1-\sigma_\Pi)\sigma_\Pi\big((1-\rho)-(1-\sigma_\beta)(1-3\sigma_\beta)\big)\Pi >&\sigma_\beta(1-\sigma_\beta)\big(1-(1-\sigma_\Pi)(1-\rho)(1-\rho+\rho\sigma_\Pi)\big)\beta\\ &+\big(0.5-(1-\sigma_\beta)(\sigma_\beta+(1-\sigma_\beta)\sigma_\Pi(1-\sigma_\Pi))\big)\delta\end{aligned}$$

The condition stated in Lemma 5 compares the expected profits for the entrant following co-location and locating apart. The condition suggests the entrant co-locates when (1) negative attributes are more likely to be present (high $\sigma_\beta$), (2) positive attribute is more valuable (high $\Pi$), and (3) consumer preferences are less heterogeneous (low $\delta$). As $\sigma_\beta$ grows, the negative advertising attacks become more likely; hence the entrant has a stronger incentive to avoid locating apart where negative advertising is prioritized. As $\Pi$ grows, positive advertising leads to a bigger boost in demand; hence the entrant has a stronger incentive to co-locate where positive advertising is prioritized. The condition also suggests that as consumers' preference heterogeneity $\delta$ becomes larger, the advantage of serving a differentiated product becomes larger, and the entrant finds locating apart more desirable. Put differently, when consumers care about buying a product closer to their tastes, care more about the positive aspects of a product, and when products are more likely to have negative attributes, in equilibrium, product offerings are less differentiated, which is less desirable from a consumer's perspective, as will be highlighted in Section 4.

### 3.3 Comparative Statics

We next discuss the characteristics of markets that are more likely to observe the equilibrium described in Proposition 2. Our analysis in this subsection takes the beliefs in Table 1 as given. We focus on $\{\delta, \rho, \sigma_\beta, \sigma_\Pi\}$: the dispersion in consumer tastes, the degree of correlation between the attributes of co-located products, and the prior beliefs about attributes, respectively. The comparative statics on these parameters can map to various market conditions and shed light on when to expect negative and positive advertising to be more likely.

The parameter $\delta$ gives a simple measure of how dispersed the consumers are in their preferences. In some sectors, consumers have strict preferences over what type of product they demand, while in others, consumers readily switch between different characteristics. Then, we might want to ask how comparative advertising affects firm behavior change for sectors with varying consumer taste heterogeneity.

**Corollary 1. (Degree of Heterogeneity in Consumer Tastes)** *There exists a threshold $\bar{\delta}$ for the consumer taste heterogeneity such that the entrant co-locates only if $\delta < \bar{\delta}$. The entrant always locates apart for $\delta > \bar{\delta}$.*

When consumer taste heterogeneity is low, negative advertising becomes more attractive for firms located apart, as it allows stealing customers located at the other end of the line. To avoid a negative advertising war, firms have stronger incentives to co-locate. Johnson and Myatt (2006) also concluded that a smaller taste dispersion would lead to a more generic product design because firms want to be able to market to a larger audience. Our analysis adds one more mechanism in line with this result: the incentive to avoid a negative advertising war.

The parameter $\rho$ measures the likelihood that the attributes that are revealed through advertising will be similar for co-located products. We might want to ask how does the effect of comparative advertising on firm behavior varies between markets with high and low attribute correlations.

**Corollary 2. (Degree of Correlation)** *There exist thresholds $\bar{\rho}$ and $\underline{\rho}$ for the correlation between product attributes such that co-located firms prioritize positive advertising only if $\rho > \underline{\rho}$. The entrant co-locates only if $\underline{\rho} < \rho < \bar{\rho}$.*

The first part of Proposition 2 indicates that when the correlation among attributes is high, it acts as a deterrent to negative advertising when firms are co-located. The second part indicates that an entrant is less likely to co-locate when $\rho$ is too large or too small. If the correlation is too small ($\rho < \underline{\rho}$), then co-location is followed by firms prioritizing negative advertising; hence there

are no incentives to co-locate. If the correlation is too high ($\rho > \bar{\rho}$), then it becomes less likely that only one firm will have the positive attribute and have the whole market to itself. This reduces the expected payoff from co-locating and hence reduces the incentives to do so.

Finally, parameters $\sigma_\Pi$ and $\sigma_\beta$ indicate the prior probability that the products have positive and negative attributes, respectively. On the one hand, as $\sigma_\Pi$ ($\sigma_\beta$) grows, positive (negative) advertising becomes less effective since it leads to only a marginal update in consumer beliefs—or, the "direct effect" of advertising is small. On the other hand, when $\sigma_\Pi$ ($\sigma_\beta$) is high, the absence of a positive (negative) attribute results in a large update in consumer beliefs—or, the "inference effect" of advertising is large. Corollary 3 summarizes how changes in $\sigma_\Pi$ and $\sigma_\beta$ impact advertising decisions.

**Corollary 3. (Prior Beliefs about Product Attributes)** *There exist thresholds $\underline{\sigma_\Pi}$ and $\underline{\sigma_\beta}$, such that when the presence of an attribute is more likely ($\sigma_\Pi > \underline{\sigma_\Pi}$ and $\sigma_\beta > \underline{\sigma_\beta}$), competing firms are more likely to engage in positive (negative) advertising under co-location (locating apart).*

The intuition of Corollary 3 follows from the fact that as $\sigma_\Pi$ and $\sigma_\beta$ grow, the "direct effect" of advertising shrinks while the "inference effect" grows. With a large "inference effect", consumer's beliefs can be self-fulfilling: firms may find a strategy to be optimal because consumers believe that only a firm whose product has desirable attributes would take the associated action. Therefore, the higher $\sigma_\Pi$ and $\sigma_\beta$, the more likely are firms to run advertising strategies in accordance with consumer beliefs given in Table 1.

## 4 Welfare Analysis

We next turn our attention to the welfare gains for consumers from negative and positive advertising. On the one hand, negative advertising allows consumers to learn about the attributes of own and competitor's products, resulting in "information gain." On the other hand, the threat of negative advertising reduces the assortment of products in a market, therefore resulting in a "loss in product match." We compare consumer welfare from the benchmark equilibrium when negative advertising is not permitted (in Proposition 1) to the equilibrium when negative advertising is feasible (in Proposition 2). We present the results in Lemmas 6 and 7.

**Lemma 6. (Welfare Gain due to Information)** *Permitting negative advertising leads to no additional information revealed to consumers when both products have the positive attribute. If one or both products lack the positive attribute, however, consumers have welfare gains due to additional*

*information from negative advertising. The associated welfare gain increases with the uncertainty around the negative attribute.*

First, Lemma 6 follows from the fact that in a market with negative advertising, both the presence and the absence of negative advertising can be informative to a consumer. When firm $i$ engages in negative advertising, it informs the consumer of the competing product's (direct effect) or its own product's weak characteristics (spillover effect). Similarly, if firm $i$ does *not* engage in negative advertising, it informs the consumer about the absence of the competing product's weak characteristics (inference effect) or the absence of the own product's weak characteristics (spillover effect). In both cases, the presence of negative advertising may increase consumer welfare by convincing consumers to buy a product with high valuation, by helping consumers avoid buying a product with low valuation, or by helping them find the product that better fits their taste. The only situation where the absence of negative advertising does not inform the consumers is when both products have the positive attribute. This is because, then, consumers expect firms to prioritize positive advertising and hence consumers do not make an inference on the negative attribute.

**Lemma 7. (Welfare Loss due to Reduced Product Differentiation)** *If only one product has the positive attribute, permitting negative advertising leads to no change in welfare through product variety. Otherwise, permitting negative advertising leads to welfare loss through reduced product variety. The associated welfare loss increases with the heterogeneity in consumer preferences ($\delta$).*

Second, Lemma 7 presents an outcome indicating a possible welfare loss due to the reduced variety of products in a market when negative advertising is allowed. In Proposition 2, we highlighted that due to the threat of a negative advertising war down the road, an entrant coming to a market chooses to co-locate with an incumbent, reducing product variety in the marketplace. This reduced variety implies that, for some consumers, there will be a loss of welfare due to either purchasing a product that is different from their ideal product or due to not buying any products at all. Unlike the welfare gains due to information, the welfare loss from reduced product variety discussed here is not a direct outcome of consumers learning about products from the attributes advertised, but rather an indirect outcome of the product design choices made by firms which anticipate negative advertising possibility.

Third, allowing negative advertising leads to co-location in Proposition 2, where the joint distribution of attributes is different than the one for locating apart due to the presence of correlation ($\rho$) between attributes. Hence, there is a "correction" term which reflects the welfare change because

of the change in the joint distribution of attributes. This term can either be positive or negative, and it is proportional to $\rho$, hence disappears as $\rho \to 0$.

Finally, in the proposition that follows, we evaluate the net change in welfare from permitting negative advertising.

**Proposition 3.** ***(Consumer Welfare)*** *Permitting negative advertising leads to a reduction in consumer welfare when the prior uncertainty around the negative attribute is sufficiently small. The reduction in welfare grows as $\delta$ grows.*

Proposition 3 states that, in markets where sufficient information about negative attributes is readily available, permitting negative advertising leads to a welfare loss. Furthermore, as consumers care more about purchasing a product that is close to their ideal one, permitting negative advertising leads to a bigger reduction in welfare. While consumers make a more-informed product choice, the pressure from advertising reduces the variety of the products that they can buy. In particular, the products that consumers purchase may match their preferences poorly when negative advertising is allowed. This finding emphasizes the adverse welfare consequences of unregulated advertising for consumers. To our knowledge, while some negative effects from advertising competition have been documented (Fruchter, 1999; Stegeman, 1991), the effects of advertising on product variety and product design choices have not been investigated, despite the critical consequences for consumers.

# 5 Extensions

In this section, we extend our model to two new settings. First, in the main model, we assume that firms are price-takers and here, relaxing this assumption, we consider the implications of price competition. Second, we consider the application of negative advertising to an alternate environment: political marketing.

## 5.1 Pricing with Consumer Search

In the main model, to improve tractability and exposition, we assumed that competing firms in the market are price-takers. This assumption may not be without loss of generality when introducing dissimilar products, as it may alleviate price competition between firms. Then, pricing might act as a counter-balancing force to the colocation-inducing effect of negative advertising. Our aim in this section is to show that even though colocation leads to higher price competition, if this competition is not extreme, then the results from our main model hold qualitatively.

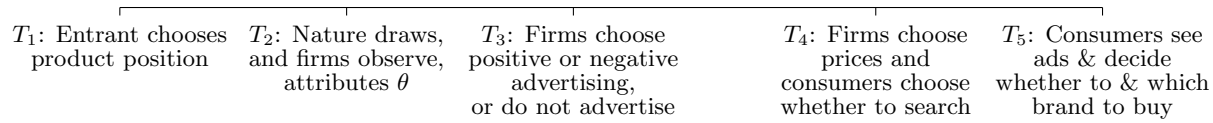


Figure 3: Timeline of the Incumbent-Entrant Game with Pricing

Adding pricing is not straightforward, since price competition *à la* Bertrand would lead to a zero price under co-location. To soften competition, we introduce a search cost for consumers, using a similar approach to that in Diamond (1971) and Kuksov (2004). In this model, consumers search for price quotes about competing products and can only purchase a product after receiving a price quote. Each consumer can access the first price quote for free, but has to pay an additional cost $\kappa$ to receive a second price quote. Firms pricing decisions and consumers' search decisions are made simultaneously.

With this model, we make three modeling modifications to incorporate prices. First, the (indirect) utility function of consumers is now

$$U_{ij}^{p} = \gamma_j + A_i - |x_i - \chi_j| - p_i, \tag{3}$$

where $p_i$ denotes the price of good $i$. Second, consumers face search frictions as in Diamond (1971). When two products are otherwise symmetric for consumer $j$, i.e, $E[A_i] - |x_i - \chi_j| = E[A_{-i}] - |x_{-i} - \chi_j|$, then consumers are equally likely to receive the quote of either product for free.[19] When $E[A_i] - |x_i - \chi_j| > E[A_{-i}] - |x_{-i} - \chi_j|$, then consumers receive the price quote for product $i$ for free. This structure resembles the setting in Kuksov (2004) and fits well to a market where consumers have easier access to advertising and information about product characteristics relative to price information.[20] Third, with the modifications given above, we assume that firms engage in price competition to maximize revenues after deciding on their product positioning and advertising strategies. Consumers decide whether to search for the second price quote, whether to buy a product, and if they do, which product to buy to maximize their indirect utility. The updated timing of the game is reflected in Figure 3. To solve the model under price competition, we update the equilibrium definition given on page 12, as follows.

**Definition** A Perfect Bayesian Equilibrium (Perfect Bayesian Equilibrium) with pricing is the

[19] Kuksov (2004) follows a similar structure and provides a detailed description of the implications of assuming search costs for prices.

[20] See Christou and Vettas (2008) for a model where advertising informs consumers about prices instead.

positioning decision of the entrant $x^2 \in \{L, R\}$, advertising $a^2(x_2, \theta) \in \{P_2, N_1, \varnothing\}$, $a^1(x_2, \theta) \in \{P_1, N_2, \varnothing\}$ and pricing decisions $p^2(x_2, a_1, a_2, \theta) \in R^+$, $p^1(x_2, a_1, a_2, \theta) \in R^+$ of firms, beliefs of consumers over firms types $\mathcal{F} : \theta_1 \times \theta_2 \to [0, 1]$, search $s^j(x_2, a_1, a_2, \theta) \in \{search, not\} \in R^+$ and purchase decisions of consumers $\{g^j(x_2, a_1, a_2, p_1, p_2, s^j)\}_j \in \{i, k, o\}$ such that

1. Consumers' purchase decisions are sequentially rational, i.e. $\{g^j(.)\}_j$ maximizes $E[U^p_{ij}|\mathcal{F}]$.
2. $p^2(.)$, $p^1(.)$, and $s^j(.)$ constitute a NE of the pricing-search sub-game given $\{g^j(.)\}_j$.
3. $a^2(.)$ and $a^1(.)$ constitute a NE of the advertising sub-game given $p^2(.)$, $p^1(.)$, $s^j(.)$, and $\{g^j(.)\}_j$.
4. The positioning decision $x^2 \in \{L, R\}$ maximizes firm 2's profits given $a^2(.)$, $a^1(.)$, $p^2(.)$, $p^1(.)$, $s^j(.)$, and $\{g^j(.)\}_j$.
5. The consumers' beliefs $\mathcal{F}$ are updated based on $a^2(.)$ and $a^1(.)$ according to the Bayes' Rule.

**Lemma 8.** ***(Pricing Strategies)*** *Consumers do not search for a second quote in the advertising sub-games. Under locating apart, and when the advertising outcomes are symmetric ($a_1 = a_2$), firms charge $p^i = \frac{E[A_i]+\overline{\gamma}}{2}$. Otherwise, there exists a $\bar{\delta}$ such that for $\delta \leq \bar{\delta}$, firms charge $p^i = \frac{E[A_i]+\overline{\gamma}}{2} - \frac{\delta}{4}$.*

Lemma 8 suggests that in a game with pricing, independent of firms' location choices, consumers do not search for a second price quote. As a result, the prices in the market resemble the prices under a monopoly, as if there was no competition.[21]

The associated price depends on the location and advertising outcomes. When firms locate apart and $E[A_1] = E[A_2]$, the incumbent sells to consumers at position $L$ and the entrant sells to consumers at position $R$. In this case, the firms price as "local" monopolies. In any other scenario, the firm(s) sell to both locations, hence price as a "global" monopoly. Since the global monopoly has to overcome the taste heterogeneity $\delta$ to reach to consumers at the more distant location, the associated profit maximizing price is lower. Hence, the price is weakly lower under co-location relative to locating apart. Furthermore, better advertising outcomes (high $E[A_i]$) increase the profit maximizing prices and the difference between the prices is proportional to the consumer taste heterogeneity ($\delta$).

[21] This is the well-known paradox of Diamond (1971): regardless of how small the search cost is, as long as it is positive, the market behaves like a monopoly.

**Proposition 4. (Positioning and Advertising with Pricing)** *Pricing competition makes co-locating less desirable and less likely. However, for a sufficiently small $\delta$, the equilibrium described in Proposition 2 still exists.*

Proposition 4 delivers the key point that there still exists a Perfect Bayesian Equilibrium such that the entrant co-locates and both firms prioritize positive advertising and in the off-equilibrium path where the entrant locates apart, firms engage in a negative advertising war, even when positive advertising is an available option, as described in Proposition 2. In other words, our results are robust to the inclusion of a price competition *a là* Diamond, and the entrant may still choose to co-locate with the incumbent to avoid a negative advertising war. However, as anticipated, co-locating puts a downward pressure on prices, and relative to the benchmark scenario where firms were price-takers, co-locating is less likely.

## 5.2 Political Positioning and Advertising

Our results so far suggest that firms have incentives to avoid negative advertising competition, and this might even induce them to avoid differentiating. The reader may agree that, indeed, many firms do not engage in negative advertising, even though it is legal. In political competition, however, negative advertising is utilized frequently (Ansolabehere et al., 1994). As these campaigns are among the most sophisticated marketing campaigns in the US (Petrova et al., 2021), in this section, we extend our model to study negative advertising in politics. This section will demonstrate that since political candidates do not care about a drop in voter turnout if both candidates lose voters proportionally, they tend to run negative advertising more often than firms do.[22]

We set the political competition as close to our setting for the commercial firms as possible. Most of the model components are identical, except for the interpretation and the labels—consumers are replaced by "voters" and firms are replaced by "political candidates". There is an incumbent politician and an entrant (challenger) running for the office. While the position (e.g., policy stance) of the incumbent is already set, the entrant can choose her position. The only difference we introduce here compared to the benchmark model in Section 2 is regarding the objective of the candidate: to maximize the *winning probability* instead of the number of votes.[23] We demonstrate below that this simple difference in the objective function goes a long way to account for the tone

[22] Results are provided for a two-candidate political race. Empirical studies show that in the U.S., indeed, most elections are bipartisan (Garcia-Jimeno and Yildirim, 2017).

[23] We assume that when the candidates have equal number of votes, the winning probability equals 0.5 for both candidates.

differences in political advertising. Since the only difference is in the objective function, the action sets and equilibrium definitions are identical to those in the product competition described in the benchmark.

We make an assumption equivalent to Assumption 1 for candidates, where only the interpretation changes. In words, if both politicians have the same policy position, then some voters with different preferences would not vote for either candidate. Moreover, there are always some voters willing to vote, regardless of the position and advertisement strategies of the candidates. In the next section, we discuss how the incentives for negative advertising change under political competition.

#### 5.2.1 Advertising Strategies in Political Competition

Changing the objective from maximizing the number of votes to maximizing the winning probability has a direct effect on candidates' advertising incentives. In particular, in product competition, advertising that reduces the demand for both firms would not be used, but it can be used in political competition as long as it reduces the opponent's vote count more than it reduces the own vote count. Thus, the main disadvantage of negative advertising under product competition—that it shrinks the overall market—is not a disadvantage in political competition. Proposition 5 shows how the change in the objective function can explain the prevalence of negative advertising in political competition.

**Proposition 5. (Political Advertising)** *The set of parameters for which negative advertising is prioritized in product competition is a strict subset of the set of parameters for which negative advertising is prioritized in political competition. Put differently, negative advertising is weakly more likely in political competition than product competition.*

Proposition 5 indicates that, the incentive to run negative advertising is higher compared to political competition. To see this, first consider why the incentives to run negative advertising against a weak opponent are stronger. Facing a weak opponent, both positive and negative advertising result in a higher advertising impact $E[A_i]$ than the opponent. In product competition, this results in positive advertising being prioritized, regardless of consumer beliefs: if consumers won't buy from the competitor, the firm should only focus on increasing its own $E[A_i]$. In political competition, however, negative advertising may still be prioritized as long as its negative effect on the competing candidate is sufficiently stronger than the effect of positive advertising on the candidate. In product competition, under locating apart, a necessary condition for negative advertising to be prioritized is when only negative advertising allows stealing consumers from the competitor,

i.e., $E[A_i - A_{-i}|a_i = N] > \delta > E[A_i - A_{-i}|a_i = P]$. In political competition, negative advertising may be prioritized even when it does not result in stealing votes. The only requirement to utilize it is that, it leads to a higher winning probability, i.e., $E[A_i - A_{-i}|a_i = N] > 0$, which is a weaker condition than the one in product competition.

Second, consider the incentives to run negative advertising against strong opponents. In product competition, under both co-location and locating apart, two conditions need to be satisfied for negative advertising to be prioritized: A firm utilizing negative advertising should be able to steal consumers from a positive advertiser, and the number of consumers stolen should make up for those who are lost to the outside option. In political competition, the latter condition is not necessary because a decline in the own vote count is not damaging as long as the opponent's vote count declines more. Furthermore, a weaker version of the former condition is sufficient: $E[A_i - A_{-i}|a_i = N] > 0$ may be desirable even when it doesn't allow stealing voters. We discuss the implications of these altered incentives for the positioning decision of a political entrant next.

### 5.2.2 Positioning Strategies in Political Competition

Although the model has sharp predictions about the advertising strategies for political competition, it has less to say about the positioning choices of new candidates. However, this result changes sharply if we assume that a mass $\eta$ of voters at point $L$ ("core supporters") slightly prefer the incumbent. The presence of supporters could be explained by the well-documented incumbency advantage literature (Petrova et al., 2021).[24] In political competition, this tweak would immediately pin down the equilibrium. A newcomer would locate apart as co-location leads to a defeat even after symmetric advertising decisions. In product competition, however, an entrant might still choose to co-locate to escape a negative advertising war when $\eta$ is sufficiently small, as demonstrated in Proposition 6.

**Proposition 6. (Positioning in Political Competition)** *In political competition:*
*(i) When $\eta = 0$, the entrant is always indifferent between co-locating and locating apart, regardless of the advertising sub-game outcome.*
*(ii) When an arbitrarily small mass $\eta$ of voters at $L$ prefer the incumbent, the entrant always chooses to locate apart.*

*In product competition, the entrant firm will still co-locate for $\eta$ small enough.*

[24] $U_{ij} = \gamma_j - |x_i - \chi_j| + A_i + 1_{\{\chi_j = L\}}\epsilon$ where $\epsilon > 0$ and $i$ is the incumbent.

Part (i) of the proposition suggests that, in a political competition where voters are symmetrically distributed in their preferences, either location choice by the incumbent results in an (expected) winning probability of $\frac{1}{2}$. Hence, the rich feedback from advertising incentives to positioning that exist in product competition are missing in political competition. Therefore, in this case, advertising is inconsequential for the entering politician's positioning choice. However, with a small (and intuitive) tweak in voter preferences, we get much sharper predictions about positioning in political competition.

The intuition behind part (ii) of the proposition is that, while a few consumers have little effect on the outcome of product competition, a few voters can determine the winner in political competition when candidate attributes are similar. In product markets, the fear of negative advertising wars might push an entrant into a product choice where the consumer base is already favoring the incumbent. In politics, negative advertising wars are nothing to be afraid of for candidates, yet a few votes can change the outcome when the candidates are similar. This result corroborates the findings from studies suggesting significant polarization in political races (Gentzkow et al. 2019).

This section emphasizes how changing objectives in product and political advertising leads to two very different outcomes. Because candidates, as opposed to firms, are not necessarily hurt by a shrinking voter base, they are more likely to take the risk and engage in negative advertising.

# 6 Conclusions and Discussion

Negative advertising is a form of advertising that informs and persuades consumers about the weaknesses of a competitor's product, and while doing so, highlights the relative strengths of one's own brand. While negative advertising is commonly utilized, its implications are little understood. This study focuses on the competition between two substitute products and their product design strategies in anticipation of a negative advertising war. The firms face a tradeoff between choosing sufficiently differentiated designs that allow matching consumer heterogeneity and products with common characteristics that pre-empt negative advertising.

In this setting, we find that the threat of negative advertising can motivate firms to co-locate, reducing product differentiation in the marketplace. While co-locating reduces the likelihood of a negative advertising attack down the road, it also leads to under-utilization of the full consumer demand. Moreover, permitting negative advertising may result in an overall welfare loss for consumers when the welfare loss from reduced product differentiation exceeds the gain from additional information that consumers receive.

In an extension, we then study the case of political competition, where the objective of competitors is to win by plurality (and not profit maximization). We show that this change in the objective function can explain the relatively widespread use of negative advertising in electoral races, as competitors care less about an overall reduction in voter turnout.

Our findings have important implications for managers and policymakers alike. For managers, our study highlights the close relationship between product design and advertising wars. As explained in detail in the introduction, negative advertising wars reduce demand for all involved parties. Therefore, a new firm entering the market may want to consider if its design features will risk negative advertising. While regulators shunned negative advertising in various parts of Europe until the 1990s,[25] in the United States, the FTC's Regulatory Overboard of Advertising has been actively encouraging firms to name their competitors and draw comparisons about pricing and product attributes.[26] The FTC statement regarding comparative advertising argued that it could "... assist (consumers) in making rational purchase decisions through direct comparisons of brands" and "... encourages product improvement and innovation, and can lead to lower prices in the marketplace." For negative advertisements to inform consumer decision-making and incentivize firm innovation, firms should be actually using it in practice. Despite the FTC's encouragement, only about one-third of commercial brands ever engage in comparative ads (Grewal et al., 1997). Our results show that, in some sectors, firms might be avoiding product designs that are too different from the existing ones to avoid a subsequent negative advertising war. Therefore, in contrast to the FTC's claims, product improvement and innovation are discouraged in these sectors due to the threat of negative advertising. In such cases, welfare can be improved by restricting negative advertising.

While, to our knowledge, this paper is the first to study negative advertising in the context of product design, we have kept our model intentionally simple to deliver clear and important insights. Our framework can be enriched in various ways. For instance, each firm's advertising may be received by some (not all) customers, and this subset of customers may be chosen randomly or may be targeted by the firm. This may reduce the harm from negative advertising and differentiation. Our model can be extended to account for empirical results regarding how other (if any) competitors benefit from negative advertising wars between two competitors (see Anderson et al. (2016), Gandhi et al. (2016) and Galasso et al. (2020)). Finally, we summarize three ideas with the hope that other

[25] In EU, until 2000, comparative advertising was regarded as an improper business practice, and trading on another firm's reputation and goodwill was considered unfair (Romano, 2004).

[26] https://www.ftc.gov/public-statements/1979/08/statement-policy-regarding-comparative-advertising

scholars can build on these ideas. First, our study highlights the idea that negative advertising shrinks overall industry demand. While a very important subject, there has been little empirical investigation on this issue by marketers, and future research can contribute to this area. Second, we find that a change in a regulation that permits negative advertising might lead to reduced product innovation. Since such a regulation change was experienced in various EU countries, empirical researchers can put this finding to test by collecting data from numerous products over multiple years. Third, and in a similar vein, we find that there can be a consumer welfare loss from permitting negative advertising; empirical studies can also focus on this question.

## References

Alden, D. L., J.-B. E. Steenkamp, and R. Batra (1999). Brand positioning through advertising in Asia, North America, and Europe: The role of global consumer culture. *Journal of Marketing 63*(1), 75–87.

Amaldoss, W. and C. He (2010). Product variety, informative advertising, and price competition. *Journal of Marketing Research 47*(1), 146–156.

Anderson, S. P., F. Ciliberto, J. Liaukonyte, and R. Renault (2016). Push-me pull-you: comparative advertising in the OTC analgesics industry. *The RAND Journal of Economics 47*(4), 1029–1056.

Anderson, S. P. and R. Renault (2009). Comparative advertising: disclosing horizontal match information. *The RAND Journal of Economics 40*(3), 558–581.

Ansolabehere, S., S. Iyengar, A. Simon, and N. Valentino (1994). Does attack advertising demobilize the electorate? *American Political Science Review 88*(04), 829–838.

Arceneaux, K. and D. W. Nickerson (2010). Comparing negative and positive campaign messages. *American Politics Research 38*(1), 54–83.

Austen-Smith, D. (1987). Interest groups, campaign contributions, and probabilistic voting. *Public Choice 54*(2), 123–139.

Barton, J., M. Castillo, and R. Petrie (2016). Negative campaigning, fundraising, and voter turnout: A field experiment. *Journal of Economic Behavior & Organization 121*, 99–113.

Bass, F. M., A. Krishnamoorthy, A. Prasad, and S. P. Sethi (2005). Generic and brand advertising strategies in a dynamic duopoly. *Marketing Science 24*(4), 556–568.

Beard, F. (2010). Comparative advertising wars: An historical analysis of their causes and consequences. *Journal of Macromarketing 30*(3), 270–286.

Beard, F. K. (2013). A history of comparative advertising in the United States. *Journalism & Communication Monographs 15*(3), 114–216.

Carpenter, G. S. (1989). Perceptual position and competitive brand strategy in a two-dimensional, two-brand market. *Management Science 35*(9), 1029–1044.

Chen, Y., Y. V. Joshi, J. S. Raju, and Z. J. Zhang (2009). A theory of combative advertising. *Marketing Science 28*(1), 1–19.

Christou, C. and N. Vettas (2008). On informative advertising and product differentiation. *International Journal of Industrial Organization 26*(1), 92–112.

Coate, S. (2004). Political competition with campaign contributions and informative advertising. *Journal of the European Economic Association 2*(5), 772–804.

Cooper, R. G. and E. J. Kleinschmidt (1987). New products: what separates winners from losers? *Journal of Product Innovation Management 4*(3), 169–184.

Cubbin, J. and S. Domberger (1988). Advertising and post-entry oligopoly behaviour. *The Journal of Industrial Economics*, 123–140.

Diamond, P. A. (1971). A model of price adjustment. *Journal of Economic Theory 3*(2), 156–168.

Dolliver, M. (2009, Oct). An aversion to ads that attack a rival. *Adweek 50*(37).

Ellickson, P. B., S. Misra, and H. S. Nair (2012). Repositioning dynamics and pricing strategy. *Journal of Marketing Research 49*(6), 750–772.

Emons, W. and C. Fluet (2012). Non-comparative versus comparative advertising of quality. *International Journal of Industrial Organization 30*(4), 352–360.

Fruchter, G. E. (1999). The many-player advertising game. *Management Science 45*(11), 1609–1611.

Fuchs, C. and A. Diamantopoulos (2010). Evaluating the effectiveness of brand-positioning strategies from a consumer perspective. *European Journal of Marketing 44*(11/12), 1763–1786.

Galasso, V., T. Nannicini, and S. Nunnari (2020). Positive spillovers from negative campaigning. *CEPR Discussion Paper No. DP14312*.

Gandhi, A., D. Iorio, and C. Urban (2016). Negative advertising and political competition. *The Journal of Law, Economics, and Organization 32*(3), 433–477.

Garcia-Jimeno, C. and P. Yildirim (2017). Matching pennies on the campaign trail: An empirical study of senate elections and media coverage. Technical report, National Bureau of Economic Research.

Gavish, B., D. Horsky, and K. Srikanth (1983). An approach to the optimal positioning of a new product. *Management Science 29*(11), 1277–1297.

Gentzkow, M., J. M. Shapiro, and M. Taddy (2019). Measuring group differences in high-dimensional choices: method and application to congressional speech. *Econometrica 87*(4), 1307–1340.

Grewal, D., S. Kavanoor, E. F. Fern, C. Costley, and J. Barnes (1997). Comparative versus noncomparative advertising: a meta-analysis. *Journal of Marketing 61*(4), 1–15.

Grossman, G. M. and C. Shapiro (1984). Informative advertising with differentiated products. *The Review of Economic Studies 51*(1), 63–81.

Hanley, R. (1927, Dec). An entire industry turns to negative advertising. *Printer's Ink*, 10–12.

Harrington Jr, J. E. and G. D. Hess (1996). A spatial theory of positive and negative campaigning. *Games and Economic Behavior 17*(2), 209–229.

Hauser, J. R. and S. P. Gaskin (1984). Application of the defender consumer model. *Marketing Science 3*(4), 327–351.

Hauser, J. R. and S. M. Shugan (1983). Defensive marketing strategies. *Marketing Science 2*(4), 319–360.

Horsky, D. and P. Nelson (1992). New brand positioning and pricing in an oligopolistic market. *Marketing Science 11*(2), 133–153.

Hotelling, H. (1929). Stability in competition. *The Economic Journal 39*(153), 41–57.

Hume, S. (1986, Apr). Mac chief explodes burger wars': Competition to get fiercer. *Advertising Age*, 3.

Jaben, J. (1992, Aug). Mud wrestling. Microsoft's ads highlight new prominence of negative marketing in business. *Business Marketing*, 2832.

Jewell, R. D. and C. Saenger (2014). Associative and dissociative comparative advertising strategies in broadening brand positioning. *Journal of Business Research 67*(7), 1559–1566.

Johnson, J. P. and D. P. Myatt (2006, June). On the simple economics of advertising, marketing, and product design. *American Economic Review 96*(3), 756–784.

Knight, J. (1978, Feb). Tylenol gives Anacin Excedrin headache no. 2.

Kuksov, D. (2004). Buyer search costs and endogenous product design. *Marketing Science 23*(4), 490–499.

Kumar, K. R. and D. Sudharshan (1988). Defensive marketing strategies: An equilibrium analysis based on decoupled response function models. *Management Science 34*(7), 805–815.

Lau, R. R., L. Sigelman, and I. B. Rovner (2007). The effects of negative political campaigns: A meta-analytic reassessment. *The Journal of Politics 69*(4), 1176–1209.

LeBlanc, G. (1998). Informative advertising competition. *The journal of Industrial Economics 46*(1), 63–77.

Lowry, F. (1929, Dec). Sugar's reply to Lucky Strikes. *Sales Management*, 790803.

Maarit Jalkala, A. and J. Keränen (2014). Brand positioning strategies for industrial firms providing customer solutions. *Journal of Business & Industrial Marketing 29*(3), 253–264.

Marketing News (1980). Sales, company and brand image can suffer if ad research ignores like/dislike dimension. *Marketing News May*, 13. Accessed through University of Pennsylvania archives.

Meenaghan, T. (1995). The role of advertising in brand image development. *Journal of Product & Brand Management 4*(4), 23–34.

Meurer, M. and D. O. Stahl II (1994). Informative advertising and product match. *International Journal of Industrial Organization 12*(1), 1–19.

Montoya-Weiss, M. M. and R. Calantone (1994). Determinants of new product performance: A review and meta-analysis. *Journal of Product Innovation Management 11*(5), 397–417.

Moorthy, K. S. (1988). Product and price competition in a duopoly. *Marketing Science 7*(2), 141–168.

Neff, J. (1999, Nov). Household brands counterpunch: Direct comparison is favored ad ploy in crowded category slugfest. *AdAge*.

Niven, D. (2006). A field experiment on the effects of negative campaign mail on voter turnout in a municipal election. *Political Research Quarterly 59*(2), 203–210.

Petrova, M., A. Sen, and P. Yildirim (2021). Social media and political contributions: the impact of new technology on political competition. *Management Science 67*(5), 2997–3021.

Polborn, M. K. and T. Y. David (2004). A rational choice model of informative positive and negative campaigning. *Quarterly Journal of Political Science 1*(4), 351–372.

Roberts, J. (2005). Defensive marketing. *Harvard Business Review 83*(11), 150–157.

Robinson, W. T. (1988). Marketing mix reactions to entry. *Marketing Science 7*(4), 368–385.

Romano, C. J. (2004). Comparative advertising in the united states and in france. *Nw. J. Int'l L. and Bus. 25*, 371.

Schultz, C. (2007). Strategic campaigns and redistributive politics. *The Economic Journal 117*(522), 936–963.

Seamans, R. and F. Zhu (2017). Repositioning and cost-cutting: The impact of competition on platform strategies. *Strategy Science 2*(2), 83–99.

Singh, S. and G. Iyer (2020). Persuasion contest: Disclosing own and rival information. Johns Hopkins University Working paper.

Skaperdas, S. and B. Grofman (1995). Modeling negative campaigning. *American Political Science Review*, 49–61.

Stegeman, M. (1991). Advertising in competitive markets. *The American Economic Review*, 210–223.

Stevenson, R. W. (1988, Feb). Advertising: Visa aims at american express. *New York Times*, D23.

Thomadsen, R. (2007). Product positioning and competition: The role of location in the fast food industry. *Marketing Science 26*(6), 792–804.

Thomas, L. A. (1999). Incumbent firms' response to entry: Price, advertising, and new product introduction. *International Journal of Industrial Organization 17*(4), 527–555.

Wilke, M. (1997, Feb). Claritin's comparative ads take on Seldane. *Advertising Age*, 57.

Winters, P. (1987, Mar). Credit-card war looms; MasterCard, Visa face Optima threat. *Advertising Age*, 1.

# ONLINE APPENDIX

## A.1 Discussion

### A.1.1 Simultaneous Entry of Competing Firms

In the main model, we demonstrated that the desire to avoid a negative advertising war can cause an entrant to choose a product positioning that is similar to the existing offering in the market. Next, we consider the case where two competing firms enter a new market and choose their positions simultaneously.

**Definition** A Perfect Bayesian Equilibrium (PBE) in a simultaneous entry game consists of positioning policies $x^1, x^2 \in \{L, R\}$, advertising policies $a^2(x_1, x_2, \theta) \in \{P_2, N_1, \varnothing\}$, $a^1(x_1, x_2, \theta) \in \{P_1, N_2, \varnothing\}$ of firms, beliefs of consumers over firms types $\mathcal{F} : \theta_1 \times \theta_2 \to [0, 1]$, and purchase policies of consumers $\{g^j(x_1, x_2, a_1, a_2)\}_j \in \{i, k, o\}$ such that

1. Consumers' choices are sequentially rational, i.e., $\{g^j(.)\}_j$ maximizes $E[U_{ij}|\mathcal{F}]$.
2. $a^2(.)$ and $a^1(.)$ constitute a Nash equilibrium of the advertising sub-game, given $\{g^j(.)\}_j$.
3. The location choices $x^1(.), x^2(.) \in \{L, R\}$ constitute a Nash equilibrium given $a^2(.)$, $a^1(.)$, and $\{g^j(.)\}_j$.
4. The consumers' beliefs $\mathcal{F}$ are updated based on $a^2(.)$ and $a^1(.)$ according to the Bayes' Rule.

When two firms simultaneously enter the market, the second stage of the game where firms determine their advertising strategy is identical to the one in the benchmark model. Therefore, the lemmas in Section 3.2.1 still apply in the backward induction solution. What is different is the first-stage decisions about positioning. Yet, it turns out that the equilibria of the simultaneous entry game are identical to the equilibria of the entrant-incumbent game, with the slight modification that firms can choose to locate both on the right and on the left end of consumer heterogeneity line:

**Proposition A.1. (Positioning under Simultaneous Entry)**
*(i) For each Perfect Bayesian Equilibrium of the entrant-incumbent game under co-location, there are two equivalent Perfect Bayesian Equilibria in the simultaneous entry game, where both firms choose to locate at either $R$ or $L$.*
*(ii) For each Perfect Bayesian Equilibrium of the entrant-incumbent game with location differentiation, there are two equivalent Perfect Bayesian Equilibria in the simultaneous entry game, where one firm locates at $R$ and one firm locates at $L$.*

Proposition A.1 demonstrates that the key insights of the benchmark model are robust to simultaneous entry of firms in a market. The equilibrium outcomes under a simultaneous game map to those under a sequential game.

### A.1.2 Discussion of Multiplicity of Equilibria

In our main analysis, instead of characterizing all possible equilibria, we restricted our attention to the equilibria where firms prioritize both positive and negative advertising over no advertising. Hence, if a firm does no advertising, consumers infer $P_i = 0, N_{-i} = 0$. This is an intuitive restriction that simplifies our analysis. In this section, we will discuss possible other equilibria.

When both positive and negative advertising are allowed, there are four relevant firm types:

$$\theta \in \{\{P_i = \Pi, N_i = 0\}, \{P_i = \Pi, N_i = -\beta\}, \{P_i = 0, N_i = 0\}, \{P_i = 0, N_i = -\beta\}\},$$

which implies $4 \times 4 = 16$ possible realizations for the tuple $\{\theta_1, \theta_2\}$. On the other hand, there are only $3 \times 3 = 9$ possible advertising outcomes, which rule out a fully separating equilibrium as in Proposition 1. The truthfulness assumption requires beliefs to put 0 probability on types that contradict the advertised attribute in any Perfect Bayesian Equilibrium. Any other restriction has to come from equilibrium strategies of each firm type. This allows consumers' beliefs to be self-fulfilling and generates the possibility of multiple equilibria.

Suppose first that the consumers believe the equilibrium strategy is to run positive advertising when both are feasible ($P_i = \Pi, N_{-i} = -\beta$). If the firm does not run positive advertising, consumers infer that the positive attribute is missing. Then, a firm may need to prioritize positive advertising to avoid being perceived as a bad type ($P_i = 0$). Now suppose that consumers believe negative advertising is prioritized when both are feasible. If the firm does not run negative advertising, consumers infer the negative attribute is missing from the competitor. Then, a firm may prioritize negative advertising to prevent the competitor from being perceived as a good type ($N_{-i} = 0$). As a result, different advertising equilibria can be observed in otherwise identical markets.

In Proposition 2, we discuss the existence of an equilibrium under a given set of conditions. Proposition A.2 discusses its uniqueness.

**Proposition A.2. (Positioning and Advertising - Uniqueness)** *There is no set of parameters for which the Perfect Bayesian Equilibrium in the full game where the entrant co-locates, and both firms prioritize positive advertising is unique.*

## A.2 Proofs

**Proof of Proposition 1.** The PBE is defined by

1. $x^2 = R$
2. $a^i(x_i, \theta) = \begin{cases} P_i, \text{if } P_i = \Pi \\ \varnothing, \text{if } P_i = 0 \end{cases}$
3. $\mathcal{F}$ puts probability 1 on $P_i = \Pi$ if $a_i = P_i$, 0 otherwise.

4. Let $B_{ij} = A_i - |\chi_j - x_i|$

   - if $\gamma_j < \max_i E[B_{ij}]$ then $g^j(x_2, a_1, a_2) = o$
   - else if $\arg\max_i E[B_{ij}]$ is unique, then $g^j(x_2, a_1, a_2) = \arg\max_i E[B_{ij}]$
   - else $g^j(x_2, a_1, a_2) = \begin{cases} 1, \text{w.p. } 0.5 \\ 2, \text{w.p. } 0.5 \end{cases}$

We next prove that $\{x^2, a^1, a^2, \mathcal{F}, g^j\}$ is the unique PBE.

First, $g^j$ is the unique maximizer for $E[U_{ij}]$ given $\mathcal{F}$.

Second, see that the $\mathcal{F}$ is consistent with the separating advertising strategies of the types. For a pooling equilibrium to exist, both types would have to announce $\varnothing$, and consumers would have to put a probability less than 1 for $P_i = \Pi$ when $P_i$ announced, contradicting the truthfulness assumption. Hence, $\mathcal{F}$ describes the unique beliefs in any possible PBE. Note that, according to $\mathcal{F}$, $E[A_i|a_i = P_i, a_{-i}, x_2] = (1 - \sigma_\Pi)\Pi$ and $E[A_i|a_i = \varnothing, a_{-i}, x_2] = -\sigma_\Pi \Pi$.

Third, in the advertising sub-game, given $\mathcal{F}$, announcing $a^i = P_i$ if $P_i = \Pi$ is strictly dominant:

$$E[A_i|a_i = P_i, a_{-i}, x_2] = (1 - \sigma_\Pi)\Pi > -\sigma_\Pi \Pi = E[A_i|a_i = \varnothing, a_{-i}, x_2], \quad \forall a_{-i}, x_2$$

Hence, $a^1, a^2$ is the unique Nash Equilibrium of the advertising sub-game given $\mathcal{F}$ and $g^j$.

| $\theta$ | Prob. | $a_1$ | $a_2$ | $E[A_1]$ | $E[A_2]$ | $D_2$ |
|---|---|---|---|---|---|---|
| $P_1 = P_2 = \Pi$ | $\sigma_\Pi^2 + \rho\sigma_\Pi(1 - \sigma_\Pi)$ | $P_1$ | $P_2$ | $(1 - \sigma_\Pi)\Pi$ | $(1 - \sigma_\Pi)\Pi$ | $0.5\big[2 - \Gamma(-(1 - \sigma_\Pi)\Pi) - \Gamma(\delta - (1 - \sigma_\Pi)\Pi)\big]$ |
| $P_1 = \Pi, P_2 = 0$ | $\sigma_\Pi(1 - \sigma_\Pi)(1 - \rho)$ | $P_1$ | $\varnothing$ | $(1 - \sigma_\Pi)\Pi$ | $-\sigma_\Pi \Pi$ | 0 |
| $P_1 = 0, P_2 = \Pi$ | $\sigma_\Pi(1 - \sigma_\Pi)(1 - \rho)$ | $\varnothing$ | $P_2$ | $-\sigma_\Pi \Pi$ | $(1 - \sigma_\Pi)\Pi$ | $2 - \Gamma(-(1 - \sigma_\Pi)\Pi) - \Gamma(\delta - (1 - \sigma_\Pi)\Pi)$ |
| $P_1 = P_2 = 0$ | $(1 - \sigma_\Pi)(1 - \sigma_\Pi + \rho\sigma_\Pi)$ | $\varnothing$ | $\varnothing$ | $-\sigma_\Pi \Pi$ | $-\sigma_\Pi \Pi$ | $0.5\big[2 - \Gamma(\sigma_\Pi \Pi) - \Gamma(\delta + \sigma_\Pi \Pi)\big]$ |

Table A1: Co-Location

| $\theta$ | Prob. | $a_1$ | $a_2$ | $E[A_1]$ | $E[A_2]$ | $D_2$ |
|---|---|---|---|---|---|---|
| $P_1 = P_2 = \Pi$ | $\sigma_\Pi^2$ | $P_1$ | $P_2$ | $(1 - \sigma_\Pi)\Pi$ | $(1 - \sigma_\Pi)\Pi$ | $1 - \Gamma(-(1 - \sigma_\Pi)\Pi)$ |
| $P_1 = \Pi, P_2 = 0$ | $\sigma_\Pi(1 - \sigma_\Pi)$ | $P_1$ | $\varnothing$ | $(1 - \sigma_\Pi)\Pi$ | $-\sigma_\Pi \Pi$ | 0 |
| $P_1 = 0, P_2 = \Pi$ | $\sigma_\Pi(1 - \sigma_\Pi)$ | $\varnothing$ | $P_2$ | $-\sigma_\Pi \Pi$ | $(1 - \sigma_\Pi)\Pi$ | $2 - \Gamma(-(1 - \sigma_\Pi)\Pi) - \Gamma(\delta - (1 - \sigma_\Pi)\Pi)$ |
| $P_1 = P_2 = 0$ | $(1 - \sigma_\Pi)^2$ | $\varnothing$ | $\varnothing$ | $-\sigma_\Pi \Pi$ | $-\sigma_\Pi \Pi$ | $1 - \Gamma(\sigma_\Pi \Pi)$ |

Table A2: Locating Apart

Table A3: Demand for the Entrant with Negative Advertising Forbidden

Last, to prove the optimality of $x^2$, consider the potential outcomes following each location choice, summarized in Table A3.[27] The table restricts attention to the realizations of $P_i$. The realizations of $N_i$ are irrelevant for firms' payoff because they cannot be advertised.

We can show the expected payoff under locating apart exceeds the expected payoff under co-location in two steps. Assume the probabilities of realizations were identical across location choices.

[27] Table A3 assumes $\Pi > \delta$. The proof would be identical for $\Pi < \delta$.

The payoff is identical for the asymmetric realizations of $P_i$. For the symmetric realizations, the payoff would have been larger under locating apart since

$$2 - 2\Gamma(-(1-\sigma_\Pi)\Pi) > 2 - \Gamma(-(1-\sigma_\Pi)\Pi) - \Gamma(\delta - (1-\sigma_\Pi)\Pi)$$

and

$$2 - 2\Gamma(\sigma_\Pi\Pi) > 2 - \Gamma(\sigma_\Pi\Pi) - \Gamma(\delta + \sigma_\Pi\Pi).$$

The extra term co-location has due to the difference in probabilities is

$$\frac{\rho\sigma_\Pi(1-\sigma_\Pi)}{2}\Big[\Gamma(-(1-\sigma_\Pi)\Pi) + \Gamma(\delta - (1-\sigma_\Pi)\Pi) - \Gamma(\sigma_\Pi\Pi) - \Gamma(\delta + \sigma_\Pi\Pi)\Big]$$

which is always negative. Hence, the expected payoff is necessarily higher under locating apart.

□

**Proofs of Proposition 2, Lemmas 1, 2, and 4.** The PBE is defined by

1. $x^2 = L$
2. $a^i(x_i,\theta)=\begin{cases} P_i, \text{if } N_i{=}0, P_{-i}{=}0, P_i{=}\Pi \\ N_{-i}, \text{else if } N_{-i}{=}-\beta \\ P_i, \text{else if } P_i{=}\Pi \\ \varnothing, \text{otherwise} \end{cases}$ if $x_i{=}R$ & $\begin{cases} P_i, \text{if } P_i{=}\Pi \\ N_{-i}, \text{else if } N_{-i}{=}-\beta \\ \varnothing, \text{otherwise} \end{cases}$ if $x_i{=}L$
3. $\mathcal{F}$ is as described in Table 1.
4. Let $B_{ij}{=}A_i{-}|\chi_j{-}x_i|$
   - if $\gamma_j{<}\max_i E[B_{ij}]$ then $g^j(x_2,a_1,a_2){=}o$
   - else if $\arg\max_i E[B_{ij}]$ is unique, then $g^j(x_2,a_1,a_2){=}\arg\max_i E[B_{ij}]$
   - else $g^j(x_2,a_1,a_2){=}\begin{cases} 1, \text{w.p. } 0.5 \\ 2, \text{w.p. } 0.5 \end{cases}$

We prove that $\{x^2, a^1, a^2, \mathcal{F}, g^j\}$ is a PBE when

$$\textbf{(i)}\ \beta > (1-\sigma_\Pi)\Pi + \delta \tag{A1a}$$

$$\textbf{(ii)}\ \bar{\gamma} + \sigma_\beta\beta > (1-\sigma_\Pi)\Pi + \delta \tag{A1b}$$

$$\textbf{(iii)}\ \Pi > (1-\rho)(1-\sigma_\beta)\beta \tag{A1c}$$

$$\textbf{(iv)}\ \sigma_\beta(1-\sigma_\beta)(1-\sigma_\Pi)(1-\rho)(1-\rho+\rho\sigma_\Pi)\beta + (1-\sigma_\Pi)\sigma_\Pi(1-\rho)\Pi - \frac{\delta}{2} > \sigma_\beta(1-\sigma_\beta)\beta$$

$$+ (1-\sigma_\Pi)\sigma_\Pi(1-\sigma_\beta)(1-3\sigma_\beta)\Pi - (1-\sigma_\beta)(\sigma_\beta + (1-\sigma_\beta)\sigma_\Pi(1-\sigma_\Pi))\delta \tag{A1d}$$

First, $g^j$ trivially maximizes $E[U_{ij}]$ given $\mathcal{F}$.

Second, see that the $\mathcal{F}$ is consistent with the advertising strategies of the firms. Importantly, when $P_i = \Pi$ and $N_{-i} = -\beta$, consumers expect firm $i$ to use negative advertising against strong opponents under locating apart.

Third, we discuss the optimality of $a^i(.)$, and its uniqueness given $\mathcal{F}$. Please refer to Table 1 for the posterior beliefs following each advertising outcome.

**Locating Apart:** If $N_i = 0$ and $P_{-i} = 0$, i.e., the opponent is weak, then $E[A_i] > E[A_{-i}]$ regardless of whether firm $i$ does positive or negative advertising. Then positive advertising would always lead to more demand except for the scenario where only negative advertising allows stealing consumers, that is,

$$E[A_i - A_{-i}|a_i = N_i, a_{-i} = \varnothing] > \delta \text{ and } E[A_i - A_{-i}|a_i = P_i, a_{-i} = \varnothing] < \delta,$$

which is ruled out by the other parameter inequalities and Assumption 1. Hence, prioritizing positive advertising against 'weak opponents' is strictly dominant under locating apart (Lemma 1). If either $N_i = -\beta$ or $P_{-i} = \Pi$, i.e., the opponent is strong, then, for negative advertising to be prioritized in the advertising equilibrium, negative advertising should be effective enough to steal consumers if the opponent runs positive advertising:

$$\begin{aligned} E[A_i - A_{-i}|a_i = N_i, a_{-i} = P_i] &> \delta \\ \Leftrightarrow \beta - (1-\sigma_\Pi)\Pi &> \delta, \end{aligned} \tag{A2}$$

which is equivalent to condition (A1a) above. Second, for negative advertising to be prioritized against strong opponents in the unique advertising equilibrium, the number of stolen consumers should be sufficient to make up for lost demand in own location:

$$\begin{aligned} D_i(a_i = N_i, a_{-i} = P_i) &> D_i(a_i = P_i, a_{-i} = P_i) \\ \Leftrightarrow 2 - \Gamma(-\sigma_\beta\beta) - \Gamma(\delta - \sigma_\beta\beta) &> 1 - \Gamma(-\sigma_\beta\beta - (1-\sigma_\Pi)\Pi) \\ \Leftrightarrow \bar{\gamma} + \sigma_\beta\beta &> (1-\sigma_\Pi)\Pi + \delta, \end{aligned} \tag{A3}$$

which is equivalent to condition (A1b) above. Since the game is symmetric, conditions (A1a) and (A1b) together imply prioritizing negative advertising against strong opponents is strictly dominant (Lemma 2).

**Co-Locating:** If the opponent is weak, then $E[A_i] > E[A_{-i}]$ regardless of whether firm $i$ does positive or negative advertising. Since this automatically allows stealing customers, prioritizing

positive advertising against 'weak opponents' is strictly dominating under co-locating (Lemma 3). If the opponent is strong, then, for positive advertising to be prioritized in the unique advertising equilibrium, it must be effective enough to steal consumers if the opponent runs negative advertising:

$$E[A_i - A_{-i}|a_i = P_i, a_{-i} = N_i] > 0 \\ \Leftrightarrow \Pi > (1-\rho)(1-\sigma_\beta)\beta, \tag{A4}$$

which is equivalent to condition $(ii)$ above. Since the game is symmetric, this condition by itself implies that prioritizing positive advertising against strong opponents is strictly dominant under co-location (Lemma 4).

Lastly, to prove the optimality of $x^2$, consider the potential outcomes following each location choice, summarized in Table A6. Let $\xi_1 = \sigma_\Pi(1-\sigma_\Pi)(1-\rho)$ and $\xi_2 = (1-\sigma_\Pi)(1-\sigma_\Pi+\rho\sigma_\Pi)$ for convenience.

| $\theta$ | Probability | $a_1$ | $a_2$ | $D_2$ |
|---|---|---|---|---|
| $\{\Pi, \Pi, , \}$ | $\sigma_\Pi^2 + \rho\sigma_\Pi(1-\sigma_\Pi)$ | $P_1$ | $P_2$ | $0.5\big[2 - \Gamma(-(1-\sigma_\Pi)\Pi) - \Gamma(\delta - (1-\sigma_\Pi)\Pi)\big]$ |
| $\{\Pi, 0, -\beta, \}$ | $\xi_1\sigma_\beta$ | $P_1$ | $N_1$ | $0$ |
| $\{0, \Pi, , -\beta\}$ | $\xi_1\sigma_\beta$ | $N_2$ | $P_2$ | $2 - \Gamma((1-\sigma_\beta)\beta - (1-\sigma_\Pi)\Pi) - \Gamma(\delta + (1-\sigma_\beta)\beta - (1-\sigma_\Pi)\Pi)$ |
| $\{\Pi, 0, 0, \}$ | $\xi_1(1-\sigma_\beta)$ | $P_1$ | $\varnothing$ | $0$ |
| $\{0, \Pi, , 0\}$ | $\xi_1(1-\sigma_\beta)$ | $\varnothing$ | $P_2$ | $2 - \Gamma(-\sigma_\beta\beta - (1-\sigma_\Pi)\Pi) - \Gamma(\delta - \sigma_\beta\beta - (1-\sigma_\Pi)\Pi)$ |
| $\{0, 0, -\beta, -\beta\}$ | $\xi_2\big(\sigma_\beta^2 + \rho\sigma_\beta(1-\sigma_\beta)\big)$ | $N_2$ | $N_1$ | $0.5\big[2 - \Gamma(\sigma_\Pi\Pi + (1-\sigma_\beta)\beta) - \Gamma(\delta + \sigma_\Pi\Pi + (1-\sigma_\beta)\beta)\big]$ |
| $\{0, 0, 0, -\beta\}$ | $\xi_2\big(\sigma_\beta(1-\sigma_\beta)(1-\rho)\big)$ | $N_2$ | $\varnothing$ | $0$ |
| $\{0, 0, -\beta, 0\}$ | $\xi_2\big(\sigma_\beta(1-\sigma_\beta)(1-\rho)\big)$ | $\varnothing$ | $N_1$ | $2 - \Gamma(\sigma_\Pi\Pi - \sigma_\beta\beta) - \Gamma(\delta + \sigma_\Pi\Pi - \sigma_\beta\beta)$ |
| $\{0, 0, 0, 0\}$ | $\xi_2(1-\sigma_\beta)(1-\sigma_\beta+\rho\sigma_\beta)$ | $\varnothing$ | $\varnothing$ | $0.5\big[2 - \Gamma(\sigma_\Pi\Pi - \sigma_\beta\beta) - \Gamma(\delta + \sigma_\Pi\Pi - \sigma_\beta\beta)\big]$ |

Table A4: Co-Location

| $\theta$ | Probability | $a_1$ | $a_2$ | $D_2$ |
|---|---|---|---|---|
| $\{, , -\beta, -\beta\}$ | $\sigma_\beta^2$ | $N_2$ | $N_1$ | $1 - \Gamma(1-\sigma_\beta)\beta)$ |
| $\{, \Pi, 0, -\beta\}$ | $\sigma_\beta(1-\sigma_\beta)\sigma_\Pi$ | $N_2$ | $P_2$ | $0$ |
| $\{\Pi, , -\beta, 0\}$ | $\sigma_\beta(1-\sigma_\beta)\sigma_\Pi$ | $P_1$ | $N_1$ | $2 - \Gamma(-\sigma_\beta\beta) - \Gamma(\delta - \sigma_\beta\beta)$ |
| $\{, 0, 0, -\beta\}$ | $\sigma_\beta(1-\sigma_\beta)(1-\sigma_\Pi)$ | $N_2$ | $\varnothing$ | $0$ |
| $\{0, , -\beta, 0\}$ | $\sigma_\beta(1-\sigma_\beta)(1-\sigma_\Pi)$ | $\varnothing$ | $N_1$ | $2 - \Gamma(\sigma_\Pi\Pi - \sigma_\beta\beta) - \Gamma(\delta + \sigma_\Pi\Pi - \sigma_\beta\beta)$ |
| $\{\Pi, \Pi, 0, 0\}$ | $(1-\sigma_\beta)^2\sigma_\Pi^2$ | $P_1$ | $P_2$ | $1 - \Gamma(-(1-\sigma_\Pi)\Pi - \sigma_\beta\beta)$ |
| $\{\Pi, 0, 0, 0\}$ | $(1-\sigma_\beta)^2\sigma_\Pi(1-\sigma_\Pi)$ | $P_1$ | $\varnothing$ | $0$ |
| $\{0, \Pi, 0, 0\}$ | $(1-\sigma_\beta)^2\sigma_\Pi(1-\sigma_\Pi)$ | $\varnothing$ | $P_2$ | $2 - \Gamma(-(1-\sigma_\Pi)\Pi - \sigma_\beta\beta) - \Gamma(\delta - (1-\sigma_\Pi)\Pi - \sigma_\beta\beta)$ |
| $\{0, 0, 0, 0\}$ | $(1-\sigma_\beta)^2(1-\sigma_\Pi)^2$ | $\varnothing$ | $\varnothing$ | $1 - \Gamma(\sigma_\Pi\Pi - \sigma_\beta\beta)$ |

Table A5: Locating Apart

Table A6: Demand for the Entrant with Negative Advertising Allowed Note. $D_2$ denotes the demand for the entrant. See Table 1 for $E[A_1]$ and $E[A_2]$ associated with each outcome. $\theta = \{P_1, P_2, N_1, N_2\}$

With some tedious algebra, we can simplify to

$$E[D_2|x_2 = L] = \frac{1}{\bar{\gamma} - \underline{\gamma}}\Big[\bar{\gamma} + \sigma_\beta(1-\sigma_\beta)(1-\sigma_\Pi)(1-\rho)(1-\rho+\rho\sigma_\Pi)\beta \\ + (1-\sigma_\Pi)\sigma_\Pi(1-\rho)\Pi - \frac{\delta}{2}\Big], \text{ and} \tag{A5}$$

$$E[D_2|x_2 = R] = \frac{1}{\bar{\gamma} - \underline{\gamma}}\Big[\bar{\gamma} + \sigma_\beta(1-\sigma_\beta)\beta + (1-\sigma_\Pi)\sigma_\Pi(1-\sigma_\beta)(1-3\sigma_\beta)\Pi \\ - (1-\sigma_\beta)(\sigma_\beta + (1-\sigma_\beta)\sigma_\Pi(1-\sigma_\Pi))\delta\Big]. \tag{A6}$$

Hence, $E[D_2|x_2 = L] > E[D_2|x_2 = R]$ becomes equivalent to condition (A1d) above.

To sum up, once conditions (A1a)-(A1d) are satisfied, there exists a PBE as defined in $1-4$, which is unique given the beliefs specified by $\mathcal{F}$. □

**Proof of Corollary 1.** To prove the corollary, first, notice that conditions (A1a) and (A1b), which are necessary for the entrant to co-locate given $\mathcal{F}$ as proven in Proposition 2, are only satisfied when $\delta$ is small enough. Let $\delta_1$ and $\delta_2$ be the values of $\delta$ which make the lefthand sides equal to the righthand sides of (A1a) and (A1b), respectively. Second, the condition in (A1d) is necessary for the entrant to co-locate given $\mathcal{F}$. The term multiplying $\delta$ on the lefthand side is 0.5 while it is $(1-\sigma_\beta)(\sigma_\beta + (1-\sigma_\beta)\sigma_\Pi(1-\sigma_\Pi))$ on the righthand side. Since the maximum $x(1-x)$ is 0.25 for $x < 1$, the latter term is bounded above by 0.5. Hence, as $\delta$ grows, the term with $\delta$ on the lefthand side dominates the other terms and (A1d) fails. Let $\delta_3$ be the value of $\delta$ which makes the lefthand side equal to the righthand side. Then, for $\bar{\delta} = \min\{\delta_1, \delta_2, \delta_3\}$, the corollary follows. □

**Proof of Corollary 2.** First, see that (A1c) is only satisfied when $\rho$ is large enough. Let $\underline{\rho}$ be the value of $\rho$ where the lefthand side equals righthand side. Second, in (A1d), see that the terms with $\beta$ and $\Pi$ on the lefthand side disappear as $\rho$ approaches 1. Because the coefficient of $\delta$ on the righthand side is always smaller in magnitude relative to the coefficient of $\delta$ on the lefthand side (See the Proof of Corollary 1), (A1d) fails as $\rho$ approaches 1. Let $\bar{\rho}$ be the largest value of $\rho$ where the lefthand side equals righthand side.[28] The corollary follows. □

**Proof of Corollary 3.** Let the value of $\sigma_\beta$ where the lefthand side equals righthand side in (A1c) be $\underline{\sigma_\beta}$. Similarly, let the values of $\sigma_\Pi$ where the lefthand side equals righthand side in (A1a) and (A1b) be $\sigma_{\Pi,1}$ and $\sigma_{\Pi,2}$ respectively, conditional on $\sigma_\beta = \underline{\sigma_\beta}$. Then, let $\underline{\sigma_\Pi} = \max\{\sigma_{\Pi,1}, \sigma_{\Pi,2}\}$. The corollary follows. □

**Proof of Lemma 6.** We start by deriving a simple expression for welfare comparisons. Since the outside option provides 0 utility, the total consumer surplus can be written as

$$CS = \sum_\theta \left[\int_{\gamma_L^*(\theta)}^{\bar{\gamma}} (\gamma - \gamma_L^{true}(\theta))d\Gamma(\gamma) + \int_{\gamma_R^*(\theta)}^{\bar{\gamma}} (\gamma - \gamma_R^{true}(\theta))d\Gamma(\gamma)\right], \tag{A7}$$

[28] See that neither $\underline{\rho}$ nor $\bar{\rho}$ are necessarily between 0 and 1.

where $\theta$ refers to the vector of values for product attributes, and $\gamma^*(\theta)$ and $\gamma^{true}(\theta)$ denote the reservation values for the consumers who are indifferent between buying the superior product or the outside option ex-ante and ex-post, respectively. The two values can differ because advertising does not always reveal all attributes of the product. If $\gamma^* > \gamma^{true}$, there are some consumers who don't buy a product, but would have enjoyed a positive utility and when $\gamma^* < \gamma^{true}$, there are some consumers who buy a product, but would have been better off with the outside option. When $\Gamma$ is the uniform *cdf*, the expression becomes

$$\begin{aligned} CS &= \sum_{\theta} \left[ \int_{\gamma_L^*(\theta)}^{\bar{\gamma}} \frac{\gamma - \gamma_L^{true}(\theta)}{\bar{\gamma} - \underline{\gamma}} d\gamma \int_{\gamma_R^*(\theta)}^{\bar{\gamma}} \frac{\gamma - \gamma_R^{true}(\theta)}{\bar{\gamma} - \underline{\gamma}} d\gamma \right] \\ &= \sum_{\theta} \left[ \frac{(\bar{\gamma} - \gamma_L^*(\theta))(\bar{\gamma} + \gamma_L^*(\theta) - 2\gamma_L^{true}(\theta))}{2(\bar{\gamma} - \underline{\gamma})} + \frac{(\bar{\gamma} - \gamma_R^*(\theta))(\bar{\gamma} + \gamma_R^*(\theta) - 2\gamma_R^{true}(\theta))}{2(\bar{\gamma} - \underline{\gamma})} \right] \\ &= \sum_{\theta} \Big[ \Phi + \gamma_L^*(\theta)(2\gamma_L^{true}(\theta) - \gamma_L^*(\theta)) - 2\bar{\gamma}\gamma_L^{true}(\theta) \\ &\qquad + \gamma_R^*(\theta)(2\gamma_R^{true}(\theta) - \gamma_R^*(\theta)) - 2\bar{\gamma}\gamma_R^{true}(\theta) \Big], \end{aligned} \tag{A8}$$

where $\Phi$ is only a function of $\bar{\gamma}$ and $\underline{\gamma}$, hence, it is invariant to advertising policy. Using the equilibrium strategies of firms and consumers in Propositions 1 and 2, we can characterize the values of $\gamma^*$ and $\gamma^{true}$ for each realization of product attributes (See Table A9).

| $\theta$ | a$^1$ | a$^2$ | W | $\gamma_L^*$ | $\gamma_R^*$ | $\gamma_L^{true}$ | $\gamma_R^{true}$ |
|---|---|---|---|---|---|---|---|
| $\{\Pi,\Pi,\beta,\beta\}$ | P | P | - | $-(1-\sigma_\Pi)\Pi$ | $-(1-\sigma_\Pi)\Pi$ | $(1-\sigma_\beta)\beta-(1-\sigma_\Pi)\Pi$ | $(1-\sigma_\beta)\beta-(1-\sigma_\Pi)\Pi$ |
| $\{\Pi,\Pi,\beta,0\}$ | P | P | - | $-(1-\sigma_\Pi)\Pi$ | $-(1-\sigma_\Pi)\Pi$ | $(1-\sigma_\beta)\beta-(1-\sigma_\Pi)\Pi$ | $-\sigma_\beta\beta-(1-\sigma_\Pi)\Pi$ |
| $\{\Pi,\Pi,0,\beta\}$ | P | P | - | $-(1-\sigma_\Pi)\Pi$ | $-(1-\sigma_\Pi)\Pi$ | $-\sigma_\beta\beta-(1-\sigma_\Pi)\Pi$ | $(1-\sigma_\beta)\beta-(1-\sigma_\Pi)\Pi$ |
| $\{\Pi,\Pi,0,0\}$ | P | P | - | $-(1-\sigma_\Pi)\Pi$ | $-(1-\sigma_\Pi)\Pi$ | $-\sigma_\beta\beta-(1-\sigma_\Pi)\Pi$ | $-\sigma_\beta\beta-(1-\sigma_\Pi)\Pi$ |
| $\{\Pi,0,\beta,\beta\}$ | P | 0 | 1 | $-(1-\sigma_\Pi)\Pi$ | $\delta-(1-\sigma_\Pi)\Pi$ | $(1-\sigma_\beta)\beta-(1-\sigma_\Pi)\Pi$ | $\delta+(1-\sigma_\beta)\beta-(1-\sigma_\Pi)\Pi$ |
| $\{\Pi,0,\beta,0\}$ | P | 0 | 1 | $-(1-\sigma_\Pi)\Pi$ | $\delta-(1-\sigma_\Pi)\Pi$ | $(1-\sigma_\beta)\beta-(1-\sigma_\Pi)\Pi$ | $\delta+(1-\sigma_\beta)\beta-(1-\sigma_\Pi)\Pi$ |
| $\{\Pi,0,0,\beta\}$ | P | 0 | 1 | $-(1-\sigma_\Pi)\Pi$ | $\delta-(1-\sigma_\Pi)\Pi$ | $-\sigma_\beta\beta-(1-\sigma_\Pi)\Pi$ | $\delta-\sigma_\beta\beta-(1-\sigma_\Pi)\Pi$ |
| $\{\Pi,0,0,0\}$ | P | 0 | 1 | $-(1-\sigma_\Pi)\Pi$ | $\delta-(1-\sigma_\Pi)\Pi$ | $-\sigma_\beta\beta-(1-\sigma_\Pi)\Pi$ | $\delta-\sigma_\beta\beta-(1-\sigma_\Pi)\Pi$ |
| $\{0,\Pi,\beta,\beta\}$ | 0 | P | 2 | $\delta-(1-\sigma_\Pi)\Pi$ | $-(1-\sigma_\Pi)\Pi$ | $\delta+(1-\sigma_\beta)\beta-(1-\sigma_\Pi)\Pi$ | $(1-\sigma_\beta)\beta-(1-\sigma_\Pi)\Pi$ |
| $\{0,\Pi,\beta,0\}$ | 0 | P | 2 | $\delta-(1-\sigma_\Pi)\Pi$ | $-(1-\sigma_\Pi)\Pi$ | $\delta-\sigma_\beta\beta-(1-\sigma_\Pi)\Pi$ | $-\sigma_\beta\beta-(1-\sigma_\Pi)\Pi$ |
| $\{0,\Pi,0,\beta\}$ | 0 | P | 2 | $\delta-(1-\sigma_\Pi)\Pi$ | $-(1-\sigma_\Pi)\Pi$ | $\delta+(1-\sigma_\beta)\beta-(1-\sigma_\Pi)\Pi$ | $(1-\sigma_\beta)\beta-(1-\sigma_\Pi)\Pi$ |
| $\{0,\Pi,0,0\}$ | 0 | P | 2 | $\delta-(1-\sigma_\Pi)\Pi$ | $-(1-\sigma_\Pi)\Pi$ | $\delta-\sigma_\beta\beta-(1-\sigma_\Pi)\Pi$ | $-\sigma_\beta\beta-(1-\sigma_\Pi)\Pi$ |
| $\{0,0,\beta,\beta\}$ | 0 | 0 | - | $\sigma_\Pi\Pi$ | $\sigma_\Pi\Pi$ | $(1-\sigma_\beta)\beta+\sigma_\Pi\Pi$ | $(1-\sigma_\beta)\beta+\sigma_\Pi\Pi$ |
| $\{0,0,\beta,0\}$ | 0 | 0 | - | $\sigma_\Pi\Pi$ | $\sigma_\Pi\Pi$ | $(1-\sigma_\beta)\beta+\sigma_\Pi\Pi$ | $\sigma_\Pi\Pi-\sigma_\beta\beta$ |
| $\{0,0,0,\beta\}$ | 0 | 0 | - | $\sigma_\Pi\Pi$ | $\sigma_\Pi\Pi$ | $\sigma_\Pi\Pi-\sigma_\beta\beta$ | $(1-\sigma_\beta)\beta+\sigma_\Pi\Pi$ |
| $\{0,0,0,0\}$ | 0 | 0 | - | $\sigma_\Pi\Pi$ | $\sigma_\Pi\Pi$ | $\sigma_\Pi\Pi-\sigma_\beta\beta$ | $\sigma_\Pi\Pi-\sigma_\beta\beta$ |

Table A7: Negative Advertising Banned, Firms are Located Apart

| $\theta$ | a$^1$ | a$^2$ | W | $\gamma_L^*$ | $\gamma_R^*$ | $\gamma_L^{true}$ | $\gamma_R^{true}$ |
|---|---|---|---|---|---|---|---|
| $\{\Pi,\Pi,\beta,\beta\}$ | P | P | - | $-(1-\sigma_\Pi)\Pi$ | $\delta-(1-\sigma_\Pi)\Pi$ | $(1-\sigma_\beta)\beta-(1-\sigma_\Pi)\Pi$ | $\delta+(1-\sigma_\beta)\beta-(1-\sigma_\Pi)\Pi$ |
| $\{\Pi,\Pi,\beta,0\}$ | P | P | - | $-(1-\sigma_\Pi)\Pi$ | $\delta-(1-\sigma_\Pi)\Pi$ | $(0.5-\sigma_\beta)\beta-(1-\sigma_\Pi)\Pi$ | $\delta+(0.5-\sigma_\beta)\beta-(1-\sigma_\Pi)\Pi$ |
| $\{\Pi,\Pi,0,\beta\}$ | P | P | - | $-(1-\sigma_\Pi)\Pi$ | $\delta-(1-\sigma_\Pi)\Pi$ | $(0.5-\sigma_\beta)\beta-(1-\sigma_\Pi)\Pi$ | $\delta+(0.5-\sigma_\beta)\beta-(1-\sigma_\Pi)\Pi$ |
| $\{\Pi,\Pi,0,0\}$ | P | P | - | $-(1-\sigma_\Pi)\Pi$ | $\delta-(1-\sigma_\Pi)\Pi$ | $-\sigma_\beta\beta-(1-\sigma_\Pi)\Pi$ | $\delta-\sigma_\beta\beta-(1-\sigma_\Pi)\Pi$ |
| $\{\Pi,0,\beta,\beta\}$ | P | N | 1 | $(1-\sigma_\beta)\beta-(1-\sigma_\Pi)\Pi$ | $\delta+(1-\sigma_\beta)\beta-(1-\sigma_\Pi)\Pi$ | $(1-\sigma_\beta)\beta-(1-\sigma_\Pi)\Pi$ | $\delta+(1-\sigma_\beta)\beta-(1-\sigma_\Pi)\Pi$ |
| $\{\Pi,0,\beta,0\}$ | P | N | 1 | $(1-\sigma_\beta)\beta-(1-\sigma_\Pi)\Pi$ | $\delta+(1-\sigma_\beta)\beta-(1-\sigma_\Pi)\Pi$ | $(1-\sigma_\beta)\beta-(1-\sigma_\Pi)\Pi$ | $\delta+(1-\sigma_\beta)\beta-(1-\sigma_\Pi)\Pi$ |
| $\{\Pi,0,0,\beta\}$ | P | 0 | 1 | $-\sigma_\beta\beta-(1-\sigma_\Pi)\Pi$ | $\delta-\sigma_\beta\beta-(1-\sigma_\Pi)\Pi$ | $-\sigma_\beta\beta-(1-\sigma_\Pi)\Pi$ | $\delta-\sigma_\beta\beta-(1-\sigma_\Pi)\Pi$ |
| $\{\Pi,0,0,0\}$ | P | 0 | 1 | $-\sigma_\beta\beta-(1-\sigma_\Pi)\Pi$ | $\delta-\sigma_\beta\beta-(1-\sigma_\Pi)\Pi$ | $-\sigma_\beta\beta-(1-\sigma_\Pi)\Pi$ | $\delta-\sigma_\beta\beta-(1-\sigma_\Pi)\Pi$ |
| $\{0,\Pi,\beta,\beta\}$ | N | P | 2 | $(1-\sigma_\beta)\beta-(1-\sigma_\Pi)\Pi$ | $\delta+(1-\sigma_\beta)\beta-(1-\sigma_\Pi)\Pi$ | $(1-\sigma_\beta)\beta-(1-\sigma_\Pi)\Pi$ | $\delta+(1-\sigma_\beta)\beta-(1-\sigma_\Pi)\Pi$ |
| $\{0,\Pi,\beta,0\}$ | 0 | P | 2 | $-\sigma_\beta\beta-(1-\sigma_\Pi)\Pi$ | $\delta-\sigma_\beta\beta-(1-\sigma_\Pi)\Pi$ | $-\sigma_\beta\beta-(1-\sigma_\Pi)\Pi$ | $\delta-\sigma_\beta\beta-(1-\sigma_\Pi)\Pi$ |
| $\{0,\Pi,0,\beta\}$ | N | P | 2 | $(1-\sigma_\beta)\beta-(1-\sigma_\Pi)\Pi$ | $\delta+(1-\sigma_\beta)\beta-(1-\sigma_\Pi)\Pi$ | $(1-\sigma_\beta)\beta-(1-\sigma_\Pi)\Pi$ | $\delta+(1-\sigma_\beta)\beta-(1-\sigma_\Pi)\Pi$ |
| $\{0,\Pi,0,0\}$ | 0 | P | 2 | $-\sigma_\beta\beta-(1-\sigma_\Pi)\Pi$ | $\delta-\sigma_\beta\beta-(1-\sigma_\Pi)\Pi$ | $-\sigma_\beta\beta-(1-\sigma_\Pi)\Pi$ | $\delta-\sigma_\beta\beta-(1-\sigma_\Pi)\Pi$ |
| $\{0,0,\beta,\beta\}$ | N | N | - | $\sigma_\Pi\Pi+(1-\sigma_\beta)\beta$ | $\delta+\sigma_\Pi\Pi+(1-\sigma_\beta)\beta$ | $\sigma_\Pi\Pi+(1-\sigma_\beta)\beta$ | $\delta+\sigma_\Pi\Pi+(1-\sigma_\beta)\beta$ |
| $\{0,0,\beta,0\}$ | 0 | N | 2 | $\sigma_\Pi\Pi-\sigma_\beta\beta$ | $\delta+\sigma_\Pi\Pi-\sigma_\beta\beta$ | $\sigma_\Pi\Pi-\sigma_\beta\beta$ | $\delta+\sigma_\Pi\Pi-\sigma_\beta\beta$ |
| $\{0,0,0,\beta\}$ | N | 0 | 1 | $\sigma_\Pi\Pi-\sigma_\beta\beta$ | $\delta+\sigma_\Pi\Pi-\sigma_\beta\beta$ | $\sigma_\Pi\Pi-\sigma_\beta\beta$ | $\delta+\sigma_\Pi\Pi-\sigma_\beta\beta$ |
| $\{0,0,0,0\}$ | 0 | 0 | - | $\sigma_\Pi\Pi-\sigma_\beta\beta$ | $\delta+\sigma_\Pi\Pi-\sigma_\beta\beta$ | $\sigma_\Pi\Pi-\sigma_\beta\beta$ | $\delta+\sigma_\Pi\Pi-\sigma_\beta\beta$ |

Table A8: Negative Advertising Allowed, Firms are Co-located

Table A9: The $\gamma$ values for which Ex-ante and Ex-post Utilities are 0, See Table 1 for $E[A_1]$ and $E[A_2]$ associated with each outcome. $\theta = \{P_1, P_2, N_1, N_2\}$. $a^1$ and $a^2$ are equilibrium advertising strategies associated with equilibria described in Propositions 1 and 2. W refers to which firm's product (if any) serves the whole market.

Given the values for $\gamma^*$ and $\gamma^{true}$, we can characterize the consumer surplus associated with each realization of product attributes. In Table A10, we tabulate the associated Consumer Surplus separately for consumers located in $L$ and $R$. We omit the $\Phi$ term in (A8), since it does not vary across different scenarios. Let $\xi_3 = (1 - \sigma_\Pi)\Pi$ for convenience.

| | Negative Advertising Banned | Negative Advertising Allowed | |
|---|---|---|---|
| $\theta$ | Consumer Surplus (L) | Consumer Surplus (L) * | Consumer Surplus (R) |
| $\{\Pi,\Pi,\beta,\beta\}$ | $\xi_3(\xi_3-2(1-\sigma_\beta)\beta+2\bar{\gamma})-2\bar{\gamma}(1-\sigma_\beta)\beta$ | $\xi_3(\xi_3-2(1-\sigma_\beta)\beta+2\bar{\gamma})-2\bar{\gamma}(1-\sigma_\beta)\beta$ | $*-\delta(2\xi_3-2(1-\sigma_\beta)\beta+2\bar{\gamma}-\delta)$ |
| $\{\Pi,\Pi,\beta,0\}$ | $\xi_3(\xi_3-2(1-\sigma_\beta)\beta+2\bar{\gamma})-2\bar{\gamma}(1-\sigma_\beta)\beta$ | $\xi_3(\xi_3-2(0.5-\sigma_\beta)\beta+2\bar{\gamma})-2\bar{\gamma}(0.5-\sigma_\beta)\beta$ | $*-\delta(2\xi_3-2(0.5-\sigma_\beta)\beta+2\bar{\gamma}-\delta)$ |
| $\{\Pi,\Pi,0,\beta\}$ | $\xi_3(\xi_3+2\sigma_\beta\beta+2\bar{\gamma})+2\bar{\gamma}\sigma_\beta\beta$ | $\xi_3(\xi_3-2(0.5-\sigma_\beta)\beta+2\bar{\gamma})-2\bar{\gamma}(0.5-\sigma_\beta)\beta$ | $*-\delta(2\xi_3-2(0.5-\sigma_\beta)\beta+2\bar{\gamma}-\delta)$ |
| $\{\Pi,\Pi,0,0\}$ | $\xi_3(\xi_3+2\sigma_\beta\beta+2\bar{\gamma})+2\bar{\gamma}\sigma_\beta\beta$ | $\xi_3(\xi_3+2\sigma_\beta\beta+2\bar{\gamma})+2\bar{\gamma}\sigma_\beta\beta$ | $*-\delta(2\xi_3+2\sigma_\beta\beta+2\bar{\gamma}-\delta)$ |
| $\{\Pi,0,\beta,\beta\}$ | $\xi_3(\xi_3-2(1-\sigma_\beta)\beta+2\bar{\gamma})-2\bar{\gamma}(1-\sigma_\beta)\beta$ | $\xi_3(\xi_3-2(1-\sigma_\beta)\beta+2\bar{\gamma}))+(1-\sigma_\beta)\beta((1-\sigma_\beta)\beta-2\bar{\gamma})$ | $*-\delta(2\xi_3-2(1-\sigma_\beta)\beta+2\bar{\gamma}-\delta)$ |
| $\{\Pi,0,\beta,0\}$ | $\xi_3(\xi_3-2(1-\sigma_\beta)\beta+2\bar{\gamma})-2\bar{\gamma}(1-\sigma_\beta)\beta$ | $\xi_3(\xi_3-2(1-\sigma_\beta)\beta+2\bar{\gamma}))+(1-\sigma_\beta)\beta((1-\sigma_\beta)\beta-2\bar{\gamma})$ | $*-\delta(2\xi_3-2(1-\sigma_\beta)\beta+2\bar{\gamma}-\delta)$ |
| $\{\Pi,0,0,\beta\}$ | $\xi_3(\xi_3+2\sigma_\beta\beta+2\bar{\gamma})+2\bar{\gamma}\sigma_\beta\beta$ | $\xi_3(\xi_3+2\sigma_\beta\beta+2\bar{\gamma}))+\sigma_\beta\beta(-\sigma_\beta\beta+2\bar{\gamma})$ | $*-\delta(2\xi_3-2(1-\sigma_\beta)\beta+2\bar{\gamma}-\delta)$ |
| $\{\Pi,0,0,0\}$ | $\xi_3(\xi_3+2\sigma_\beta\beta+2\bar{\gamma})+2\bar{\gamma}\sigma_\beta\beta$ | $\xi_3(\xi_3+2\sigma_\beta\beta+2\bar{\gamma}))+\sigma_\beta\beta(-\sigma_\beta\beta+2\bar{\gamma})$ | $*-\delta(2\xi_3-2(1-\sigma_\beta)\beta+2\bar{\gamma}-\delta)$ |
| $\{0,\Pi,\beta,\beta\}$ | $\xi_3(\xi_3-2(1-\sigma_\beta)\beta+2\bar{\gamma}-2\delta)+2(1-\sigma_\beta)\beta(\delta-\bar{\sigma})-2\bar{\gamma}\delta+\delta^2$ | $\xi_3(\xi_3-2(1-\sigma_\beta)\beta+2\bar{\gamma}))+(1-\sigma_\beta)\beta((1-\sigma_\beta)\beta-2\bar{\gamma})$ | $*-\delta(2\xi_3-2(1-\sigma_\beta)\beta+2\bar{\gamma}-\delta)$ |
| $\{0,\Pi,\beta,0\}$ | $\xi_3(\xi_3+2\sigma_\beta\beta+2\bar{\gamma}-2\delta)+2\sigma_\beta\beta(\bar{\sigma}-\delta)-2\bar{\gamma}\delta+\delta^2$ | $\xi_3(\xi_3+2\sigma_\beta\beta+2\bar{\gamma}))+\sigma_\beta\beta(-\sigma_\beta\beta+2\bar{\gamma})$ | $*-\delta(2\xi_3-2(1-\sigma_\beta)\beta+2\bar{\gamma}-\delta)$ |
| $\{0,\Pi,0,\beta\}$ | $\xi_3(\xi_3-2(1-\sigma_\beta)\beta+2\bar{\gamma}-2\delta)+2(1-\sigma_\beta)\beta(\delta-\bar{\sigma})-2\bar{\gamma}\delta+\delta^2$ | $\xi_3(\xi_3-2(1-\sigma_\beta)\beta+2\bar{\gamma}))+(1-\sigma_\beta)\beta((1-\sigma_\beta)\beta-2\bar{\gamma})$ | $*-\delta(2\xi_3-2(1-\sigma_\beta)\beta+2\bar{\gamma}-\delta)$ |
| $\{0,\Pi,0,0\}$ | $\xi_3(\xi_3+2\sigma_\beta\beta+2\bar{\gamma}-2\delta)+2\sigma_\beta\beta(\bar{\sigma}-\delta)-2\bar{\gamma}\delta+\delta^2$ | $\xi_3(\xi_3+2\sigma_\beta\beta+2\bar{\gamma}))+\sigma_\beta\beta(-\sigma_\beta\beta+2\bar{\gamma})$ | $*-\delta(2\xi_3-2(1-\sigma_\beta)\beta+2\bar{\gamma}-\delta)$ |
| $\{0,0,\beta,\beta\}$ | $\sigma_\Pi\Pi(\sigma_\Pi\Pi+2(1-\sigma_\beta)\beta-2\bar{\gamma})-2\bar{\gamma}(1-\sigma_\beta)\beta$ | $\sigma_\Pi\Pi(\sigma_\Pi\Pi+2(1-\sigma_\beta)\beta-2\bar{\gamma})+(1-\sigma_\beta)\beta((1-\sigma_\beta)\beta-2\bar{\gamma})$ | $*-\delta(-2\sigma_\Pi\Pi-2(1-\sigma_\beta)\beta+2\bar{\gamma}-\delta)$ |
| $\{0,0,\beta,0\}$ | $\sigma_\Pi\Pi(\sigma_\Pi\Pi+2(1-\sigma_\beta)\beta-2\bar{\gamma})-2\bar{\gamma}(1-\sigma_\beta)\beta$ | $\sigma_\Pi\Pi(\sigma_\Pi\Pi-2\sigma_\beta\beta-2\bar{\gamma})+\sigma_\beta\beta(\sigma_\beta\beta+2\bar{\gamma})$ | $*-\delta(-2\sigma_\Pi\Pi+2\sigma_\beta\beta+2\bar{\gamma}-\delta)$ |
| $\{0,0,0,\beta\}$ | $\sigma_\Pi\Pi(\sigma_\Pi\Pi-2\sigma_\beta\beta-2\bar{\gamma})+2\bar{\gamma}\sigma_\beta\beta$ | $\sigma_\Pi\Pi(\sigma_\Pi\Pi-2\sigma_\beta\beta-2\bar{\gamma})+\sigma_\beta\beta(\sigma_\beta\beta+2\bar{\gamma})$ | $*-\delta(-2\sigma_\Pi\Pi+2\sigma_\beta\beta+2\bar{\gamma}-\delta)$ |
| $\{0,0,0,0\}$ | $\sigma_\Pi\Pi(\sigma_\Pi\Pi-2\sigma_\beta\beta-2\bar{\gamma})+2\bar{\gamma}\sigma_\beta\beta$ | $\sigma_\Pi\Pi(\sigma_\Pi\Pi-2\sigma_\beta\beta-2\bar{\gamma})+\sigma_\beta\beta(\sigma_\beta\beta+2\bar{\gamma})$ | $*-\delta(-2\sigma_\Pi\Pi+2\sigma_\beta\beta+2\bar{\gamma}-\delta)$ |

Table A10: The consumer surplus following the equilibrium described in Proposition 2 **Notes:** (L) and (R) refer to the consumer surplus associated with consumers in locations $L$ and $R$, respectively. The values are suppressed for consumers in $R$ for when negative advertising is not permitted, as they are symmetrical to those for consumers in $L$. $*$ refers to the Consumer Surplus value for the consumers in location $L$ for each respective realization of $\theta$.

When both products have the positive attribute, both firms utilize positive advertising in equilibrium regardless of whether negative advertising is allowed and regardless of whether the products have the negative attribute or not. Hence, consumers can conclude the positive attribute is present in both products with probability 1, yet gain no additional information about the presence of the negative attribute. This is reflected in the first four rows of Table A10 which show that when $P_1 = P_2 = \Pi$, the consumer welfare becomes identical between the two scenarios as $\delta \longrightarrow 0$.

If one or more products lacks the positive attribute, the firm(s) that lacks the positive attribute (say firm $i$) will not be able to run positive advertising. Because positive advertising is the prioritized strategy, consumers can conclude that the positive attribute is missing in firm $i$'s product with probability 1, regardless of whether negative advertising is allowed. If negative advertising is forbidden, consumers do not learn anything else, because firm $i$ has to run no advertising regardless of the presence of negative attributes. If negative advertising is allowed, however, consumers also learn whether firm $-i$ has the negative attribute. If firm $i$ runs negative advertising, consumers can conclude the negative attribute is present in firm $-i$'s product with probability 1. If firm $i$ runs no advertising, consumers can conclude that the negative attribute is present in firm $-i$'s product with probability 0. This is reflected in the additional $\beta$ terms in rows 5-16 when negative advertising is permitted in Table A10.

To tease out the change in surplus through changes in the available information, we take two steps. First, we compute the change in expected surplus when negative advertising is permitted as if the probability distribution of $\theta$ is the same for both and equal to the one under locating apart (i.e. $\rho = 0$). Second, we take $\delta \longrightarrow 0$ to suppress the change in consumer surplus due to reduced product diversity. The resulting term can be simplified as

$$4\sigma_\beta(1-\sigma_\beta)(1-\sigma_\Pi)\beta\big(2\sigma_\Pi\beta + (1-\sigma_\Pi)\bar{\gamma}\big), \tag{A9}$$

which is always positive. Hence, we conclude that the consumers have welfare gains due to additional information when negative advertising is permitted.

Lastly, the variance of $\beta$ equals $\sigma_\beta(1-\sigma_\beta)$ as it has a Bernoulli distribution. See that both this variance and (A9) approach 0 as $\sigma_\beta$ approaches 0 or 1. Hence, the welfare gains due to additional information disappear as the variance of $\beta$ goes to 0. □

**Proof of Lemma 7.** The welfare loss due to differentiation is reflected in the additional $\delta$ terms when negative advertising is permitted in Table A10. When one firm serves the whole market, there are no additional $\delta$ terms, because consumers in one of the locations end up purchasing from a firm in the other location, regardless of whether negative advertising is permitted or not (rows 5-12). In the other outcomes (rows 1-4 and 13-16), however, there are additional terms with $\delta$ when negative advertising is permitted. The presence of such terms indicate that some consumers who could buy a product that exactly matches their preference when negative advertising is not

allowed can only buy from a firm that doesn't match their preference when negative advertising is permitted. This is because when negative advertising is permitted, the firms co-locate at $L$, hence, consumers located at $R$ will necessarily buy from the other location.

To tease out the change in surplus through changes in the product differentiation, we take two steps. First, we compute the change in expected surplus when negative advertising is permitted as if the probability distribution of $\theta$ is the same for both and equal to the one under locating apart (i.e. $\rho = 0$). Second, we isolate the terms with $\delta$.[29] The resulting term can be simplified as:

$$\delta\Big[2\sigma_\Pi(1-\sigma_\Pi)(1-2\sigma_\Pi)\Pi - 2\sigma_\beta(1-\sigma_\Pi)^2(1-\sigma_\beta)\beta - (\sigma_\Pi^2 + (1-\sigma_\Pi)^2)(2\bar{\gamma}-\delta)\Big]. \tag{A10}$$

Next, we prove the term in (A10) is always negative, i.e., there is some welfare *loss* associated with the change in product differentiation. Using the normalization made earlier, plugging in $\sigma_\Pi\Pi = \sigma_\beta\beta$ yields:

$$\delta\Big[2\sigma_\Pi(1-\sigma_\Pi)(\sigma_\beta - \sigma_\Pi - \sigma_\beta\sigma_\Pi)\Pi - (\sigma_\Pi^2 + (1-\sigma_\Pi)^2)(2\bar{\gamma}-\delta)\Big]. \tag{A11}$$

Replacing $\bar{\gamma}$ in (A11) with $\delta + \sigma_\Pi\Pi + \sigma_\beta\beta$:

$$\delta\Big[2\sigma_\Pi(1-\sigma_\Pi)(\sigma_\beta - \sigma_\Pi - \sigma_\beta\sigma_\Pi)\Pi - (\sigma_\Pi^2 + (1-\sigma_\Pi)^2)(2\sigma_\Pi\Pi + 2\sigma_\beta\beta + \delta)\Big]. \tag{A12}$$

From Assumption 1, $\bar{\gamma} > \delta + \sigma_\Pi\Pi + \sigma_\beta\beta$. Hence, proving (A12) is negative is sufficient to prove (A11) is negative. Next, notice that the term on the left, which is the only positive term in the expression, is maximized when $\sigma_\beta = 1$. Setting $\sigma_\beta = 1$ and collecting the terms with $\Pi$ gives

$$\delta\Big[-2\sigma_\Pi^2\Pi - (\sigma_\Pi^2 + (1-\sigma_\Pi)^2)(2\sigma_\beta\beta + \delta)\Big], \tag{A13}$$

which is necessarily negative. Then (A10) has to be negative. Hence, consumers have welfare losses due to reduced product differentiation when negative advertising is permitted.

Lastly, to prove that welfare losses increase as $\delta$ increases, we take the derivative of (A10) with respect to $\delta$, which yields

$$(2\delta - 2\bar{\gamma})(\sigma_\Pi^2 + (1-\sigma_\Pi)^2) + 2\sigma_\Pi(1-\sigma_\Pi)(\sigma_\beta(1-\sigma_\Pi) - \sigma_\Pi)\Pi. \tag{A14}$$

Notice that $\sigma_\Pi^2 + (1-\sigma_\Pi)^2 > (1-\sigma_\Pi)(\sigma_\beta(1-\sigma_\Pi) - \sigma_\Pi)$ for any value of $\sigma_\Pi$ and $\sigma_\beta$. Since $\bar{\gamma} > \delta + \sigma_\Pi\Pi + \sigma_\beta\beta$ by Assumption 1, the derivative term is necessarily negative. Therefore, the welfare loss due to reduced product differentiation increases with higher $\delta$. □

[29] In other words, we remove the terms that are associated with additional information, given in the Proof of Lemma 6.

**Proof of Proposition 3.** Using Table A10, we can write down the change in expected consumer surplus when negative advertising is permitted:

$$\begin{aligned}\Delta CS = &\underbrace{4\sigma_\beta(1-\sigma_\beta)(1-\sigma_\Pi)\beta\big(2\sigma_\Pi\beta+(1-\sigma_\Pi)\bar{\gamma}\big)}_{\text{Information Gains}} \\ &+\underbrace{\delta\Big[2\sigma_\Pi(1-\sigma_\Pi)(1-2\sigma_\Pi)\Pi-2\sigma_\beta(1-\sigma_\Pi)^2(1-\sigma_\beta)\beta-(\sigma_\Pi^2+(1-\sigma_\Pi)^2)(2\bar{\gamma}-\delta)\Big]}_{\text{Product Differentiation Losses}} \\ &+2\rho(1-\sigma_\Pi)\Big[\sigma_\Pi(2\sigma_\Pi-1)\Pi(\Pi+\delta)-2\sigma_\Pi\Pi\bar{\gamma}-\sigma_\Pi\sigma_\beta(1-\sigma_\beta)\beta^2 \\ &\qquad+\sigma_\beta(1-\sigma_\beta)(2\sigma_\Pi-\rho\sigma_\Pi-1)\beta(2\bar{\gamma}-\beta-\delta)\Big].\end{aligned} \tag{A15}$$

The first and second terms denote the welfare change due to information and reduced product differentiation that were derived in Lemmas 6 and 7, respectively. These terms were derived keeping the distribution of $\theta$ fixed when negative advertising was permitted. The third term is the 'correction' term, which reflects the change in welfare due to the change in the distribution of $\theta$ and disappears as $\rho \longrightarrow 0$.

Collecting the terms, and using the normalization $\sigma_\Pi\Pi = \sigma_\beta\beta$, we can re-write (A15) as the summation of two terms:

$$\begin{aligned}\Delta CS &= 2\sigma_\Pi(1-\sigma_\Pi)\Pi\Big[4\sigma_\Pi(1-\sigma_\beta)\beta+2(1-\sigma_\Pi)(1-\sigma_\beta)\bar{\gamma}+(\sigma_\beta-\sigma_\Pi-\rho\sigma_\Pi)\delta \\ &\qquad+\rho(2\sigma_\Pi-1)\Pi+\rho(2\sigma_\Pi-1)\delta-2\rho\bar{\gamma}-\rho\sigma_\Pi(1-\sigma_\beta)\beta \\ &\qquad+\rho(1-\sigma_\beta)(2\sigma_\Pi-1-\rho\sigma_\Pi)(2\bar{\gamma}-\beta-\delta)\Big] \\ &\qquad-\delta(2\bar{\gamma}-\delta)\Big[\sigma_\Pi^2+(1-\sigma_\Pi)^2\Big] \\ &= 2\sigma_\Pi(1-\sigma_\Pi)\Pi\Big[(1-\sigma_\beta)\big(\sigma_\Pi(4-3\rho+\rho^2)+\rho\big)\beta \\ &\qquad+2\big((1-\sigma_\beta)(1-\sigma_\Pi+\rho(2-\rho)\sigma_\Pi-\rho)-\rho\big)\bar{\gamma} \\ &\qquad+\big(\sigma_\beta(1-\sigma_\Pi)-\sigma_\Pi(1-\rho^2)+\sigma_\beta\sigma_\Pi(2\rho-rho^2-1)\big)\delta \\ &\qquad+\rho(2\sigma_\Pi-1)\Pi\Big] \\ &\qquad-\delta(2\bar{\gamma}-\delta)\Big[\sigma_\Pi^2+(1-\sigma_\Pi)^2\Big].\end{aligned} \tag{A16}$$

Notice that $\Delta CS$ is continuous in all its arguments. The second term is necessarily negative due to Assumption 1. The first term in (A16), whose sign cannot be established with certainty

- disappears if $\sigma_\Pi$ approaches 0 or 1, or $\sigma_\beta$ approaches 0, and
- becomes negative as $\sigma_\beta$ approaches 1.

It is straightforward how the first term in (A16) disappears as $\sigma_\Pi$ approaches 0 or 1. Notice that, $\sigma_\beta$ approaching 0 is equivalent to $\sigma_\Pi$ approaching 0, given the normalization $\sigma_\beta\beta = \sigma_\Pi\Pi$.[30] Lastly, to show that the term becomes negative as $\sigma_\beta$ approaches 1, we plug $\sigma_\beta = 1$ in (A16), which yields

$$\begin{aligned}\Delta CS = 2\sigma_\Pi(1-\sigma_\Pi)\Pi\Big[0\beta - 2\rho\bar{\gamma} + (1-3\sigma_\Pi + 2\rho\sigma_\Pi)\delta + \rho(2\sigma_\Pi - 1)\Pi\Big] \\ -\delta(2\bar{\gamma}-\delta)\Big[\sigma_\Pi^2 + (1-\sigma_\Pi)^2\Big].\end{aligned} \tag{A17}$$

The term in the brackets is strictly negative because (1) $1 - 3\sigma_\Pi + 2\rho\sigma_\Pi$ is bounded above by 1, (2) $\rho(2\sigma_\Pi - 1)$ is bounded above by $\sigma_\Pi$ and (3) $\bar{\gamma} > \delta + \sigma_\Pi\Pi + \sigma_\beta\beta$ by Assumption 1. Moreover, $\Delta CS$ will be negative for any $\sigma_\beta > \underline{\sigma_\beta}$ for some $\underline{\sigma_\beta} < 1$ because $\Delta CS$ is continuous in $\sigma_\beta\beta$.

Lastly, we prove that the change in welfare decreases with $\delta$. Notice that the first term in (A15) does not change with $\delta$ while the second term decreases as shown in Lemma 7. Taking the derivative of (A15) with respect to $\delta$ and using $\sigma_\Pi\Pi = \sigma_\beta\beta$ yields

$$2(\delta-\bar{\gamma})(\sigma_\Pi^2 + (1-\sigma_\Pi)^2) + 2\sigma_\Pi(1-\sigma_\Pi)\Pi\Big[(\sigma_\beta(1-\sigma_\Pi) - \sigma_\Pi) + \rho\Big((2\sigma_\Pi - 1) - (1-\sigma_\beta)(2\sigma_\Pi - \rho\sigma_\Pi - 1)\Big)\Big]. \tag{A18}$$

By Assumption 1, $\bar{\gamma} > \delta + \sigma_\Pi\Pi + \sigma_\beta\beta$, so a sufficient condition for the term to be negative is:

$$\begin{aligned}(1-\sigma_\Pi)\Big[\sigma_\beta(1-\sigma_\Pi) - \sigma_\Pi + \rho\Big((2\sigma_\Pi - 1) - (1-\sigma_\beta)(2\sigma_\Pi - \rho\sigma_\Pi - 1)\Big)\Big] \\ < (\sigma_\Pi^2 + (1-\sigma_\Pi)^2).\end{aligned} \tag{A19}$$

See that the term in brackets is bounded above by $1 - \sigma_\Pi$:

$$\begin{aligned}&\sigma_\beta(1-\sigma_\Pi) - \sigma_\Pi + \rho\Big((2\sigma_\Pi - 1) - (1-\sigma_\beta)(2\sigma_\Pi - \rho\sigma_\Pi - 1)\Big) \\ &= \sigma_\beta(1-\sigma_\Pi) - \sigma_\Pi + \rho\Big((1-\sigma_\beta)\rho\sigma_\Pi + \sigma_\beta\sigma_\Pi - \sigma_\beta(1-\sigma_\Pi))\Big) \\ &= \sigma_\beta(1-\rho)(1-\sigma_\Pi) - \sigma_\Pi + \rho\Big((1-\sigma_\beta)\rho\sigma_\Pi + \sigma_\beta\sigma_\Pi)\Big) \\ &< \sigma_\beta(1-\rho)(1-\sigma_\Pi) - \sigma_\Pi + \sigma_\Pi \\ &< 1 - \sigma_\Pi.\end{aligned}$$

Thus, the condition is satisfied for $\sigma_\Pi > 0$ and the total change in consumer surplus given in (A15) decreases with $\delta$. □

**Proof of Lemma 8.** We postulate equilibrium strategies for the pricing search sub-games after

[30] A similar argument cannot be made for $\Pi$ and $\beta$ because $\delta$ is bounded above by the two in the equilibrium parameter set, hence, the second term disappears together with the first term.

each location and advertising choices, and prove that the postulated strategies indeed constitute a Nash equilibrium.

**If $x_2 = R$ and $A_1 = A_2$:** We postulate that, in this sub-game, consumers do not search and both firms charge $p^i = \frac{E[A_i]+\overline{\gamma}}{2}$. First, since both firms charge the same price, and the consumers receive the first quote from the firm that matches their taste, consumers have no incentive to search for a second quote. In other words, there is no profitable deviation for the consumers. Second, because the consumers do not search for a second quote, neither firm can steal consumers from the other through reduced prices. Then, the pricing problem of firm $i$ boils down to:

$$\max_{p_i>0} p_i\left(1+\frac{A_i-p_i+\underline{\gamma}}{\overline{\gamma}-\underline{\gamma}}\right) \tag{A20}$$

Hence, changing prices does not increase profits, since $p^i = \frac{E[A_i]+\overline{\gamma}}{2}$ is already the price that equates marginal revenue to marginal cost for firm $i$. Thus, the postulated strategies constitute a Nash Equilibrium of the pricing-search sub-game.

**If $x_2 = L$ and $A_1 = A_2$:** We postulate that, in this sub-game, consumers do not search and both firms charge $p^i = \frac{E[A_i]+\overline{\gamma}}{2} - \frac{\delta}{4}$ for $\delta$ small enough and $p^i = \frac{E[A_i]+\overline{\gamma}}{2}$ otherwise. First, since both firms charge the same price, and $A_1 = A_2$, consumers have no incentive to search for a second quote. Second, because the consumers do not search for a second quote, neither firm can steal consumers from the other through reduced prices. Then, the pricing problem of a monopolist boils down to:

$$\max_{p_i>0} \frac{1}{2}p_i\left(\max\{1+\frac{A_i-p_i+\underline{\gamma}}{\overline{\gamma}-\underline{\gamma}},0\}+\max\{\frac{A_i-p_i+\underline{\gamma}-\delta}{\overline{\gamma}-\underline{\gamma}},0\}\right) \tag{A21}$$

The pricing problem looks different from (A20) because now, if the price is sufficiently small, firm $i$ can serve consumers whose tastes do not exactly match firm $i$'s product, i.e. consumers located in $R$.[31]

The problem may be non-convex around the solution, due to the presence of two separate markets: it may be optimal for the firm to set a price where the demand from location $R$ equals 0. This becomes more likely as $\delta$ grows. First, assume that the firm serves both markets in the optimal solution. Then, the price that solves

$$\max_{p_i>0} \frac{1}{2}p_i\left(1+\frac{A_i-p_i+\underline{\gamma}}{\overline{\gamma}-\underline{\gamma}}+\frac{A_i-p_i+\underline{\gamma}-\delta}{\overline{\gamma}-\underline{\gamma}}\right), \tag{A22}$$

or, $p_i^* = \frac{E[A_i]+\overline{\gamma}}{2} - \frac{\delta}{4}$, also solves (A21). If, on the other hand, $A_i - p_i^* + \underline{\gamma} - \delta < 0$, then the firm will charge $p_i^* = \frac{E[A_i]+\overline{\gamma}}{2}$ and only serve consumer located at $L$. Hence, one of these prices equates

[31] The $\frac{1}{2}$ term in the beginning signifies the fact that only half of the consumers in either location receive their quote from firm $i$.

marginal revenue to marginal cost for firm $i$. Thus, the postulated strategies constitute a Nash Equilibrium of the pricing-search sub-game.

**If $A_1 \neq A_2$:** We postulate that, in this sub-game, consumers do not search, and both firms charge $p^i = \frac{E[A_i]+\overline{\gamma}}{2} - \frac{\delta}{4}$ for $\delta$ small enough and $p^i = \frac{E[A_i]+\overline{\gamma}}{2}$ otherwise. The strategies here are identical to the previous sub-game, and proving that they constitute a Nash Equilibrium follows the same steps. The only difference in this case is that consumer $j$ buys from the firm that has a larger $E[A_i] - |x_i - \chi_j|$.[32] □

**Proof of Proposition 4.** We restrict attention to the case where monopolists serve both markets, i.e., where $\delta$ is sufficiently small. This is also the interesting case where co-location leads to reduced prices for consumers.

The PBE is defined by:

1. $x^2 = L$
2. $a^i(.) = \begin{cases} P_i, \text{if } N_i = 0, P_{-i} = 0, P_i = \Pi \\ N_{-i}, \text{else if } N_{-i} = -\beta \\ P_i, \text{else if } P_i = \Pi \\ \varnothing, \text{otherwise} \end{cases}$ if $x_i = R$ & $\begin{cases} P_i, \text{if } P_i = \Pi \\ N_{-i}, \text{else if } N_{-i} = -\beta \\ \varnothing, \text{otherwise} \end{cases}$ if $x_i = L$
3. $\mathcal{F}$ is as described in Table 1.
4. $s^j(.) =$not, $\forall \theta, a_1, a_2, x_2$
5. $p^i(.) = \begin{cases} \frac{E[A_i]+\overline{\gamma}}{2}, \text{if } A_1 = A_2 \\ \frac{E[A_i]+\overline{\gamma}}{2} - \frac{\delta}{4}, \text{otherwise} \end{cases}$ if $x_i = R$ & $\left\{ \frac{E[A_i]+\overline{\gamma}}{2} - \frac{\delta}{4} \right.$ if $x_i = L$
6. Let $B_{ij} = A_i - |\chi_j - x_i| - p_i$
   - if $\gamma_j < \max_{i \in \mathcal{S}_j} E[B_{ij}]$ then $g^j(x_2, a_1, a_2) = o$
   - else if $\arg\max_{i \in \mathcal{S}_j} E[B_{ij}]$ is unique, then $g^j(x_2, a_1, a_2) = \arg\max_i E[B_{ij}]$
   - else $g^j(x_2, a_1, a_2) = \begin{cases} 1, \text{w.p. } 0.5 \\ 2, \text{w.p. } 0.5 \end{cases}$

   where $\mathcal{S}_j = 1, 2$ if $s^j$=search and $\mathcal{S}_j = i$ otherwise, where $i$ denotes the product whose free quote is received by consumer $j$.

[32] The assumption that consumers receive the first quote from the firm with larger $E[A_i] - |x_i - \chi_j|$ greatly simplifies characterizing the equilibrium. Otherwise, some consumers would be better off searching for a second quote even under identical prices, which would create incentives for firms to undercut each other's prices.

We prove that $\{x^2, a^1, a^2, \mathcal{F}, g^j, s^j, p^1, p^2\}$ is a PBE when

$$\textbf{(i)}\ \beta > (1-\sigma_\Pi)\Pi + \delta \tag{A23a}$$

$$\textbf{(ii)}\ (\bar{\gamma} + \sigma_\beta\beta)(\bar{\gamma} + \sigma_\beta\beta - 2(1-\sigma_\Pi)\Pi - 2\delta) > ((1-\sigma_\Pi)\Pi - 0.5\delta)((1-\sigma_\Pi)\Pi + \delta) \tag{A23b}$$

$$\textbf{(iii)}\ \Pi > (1-\rho)(1-\sigma_\beta)\beta \tag{A23c}$$

$$\begin{aligned}
&\textbf{(iv)}\ \big(\sigma_\Pi^2 + \rho\sigma_\Pi(1-\sigma_\Pi)\big)(\bar{\gamma} + (1-\sigma_\Pi)\Pi)^2 \\
&+ 2\rho(1-\sigma_\Pi)(1-\rho)\sigma_\beta(\bar{\gamma} + (1-\sigma_\Pi)\Pi - (1-\sigma_\beta)\beta)^2 \\
&+ 2\sigma_\Pi(1-\sigma_\Pi)(1-\rho)(1-\sigma_\beta)(\bar{\gamma} + (1-\sigma_\Pi)\Pi + \sigma_\beta\beta)^2 \\
&+ (1-\sigma_\Pi)(1-\sigma_\Pi+\rho\sigma_\Pi)(\sigma_\beta^2 + \rho\sigma_\beta(1-\sigma_\beta))(\bar{\gamma} - \sigma_\Pi\Pi - (1-\sigma_\beta)\beta)^2 \\
&+ (1-\sigma_\Pi)(1-\sigma_\beta)(1-\sigma_\Pi+\rho\sigma_\Pi)(1+\sigma_\beta-\rho\sigma_\beta)(\bar{\gamma} - \sigma_\Pi\Pi + \sigma_\beta\beta)^2 - 0.5\delta^2 \\
&> \big(2\sigma_\beta(1-\sigma_\beta)\sigma_\Pi + (1-\sigma_\beta)^2(1-\sigma_\Pi)^2\big)(\bar{\gamma} + \sigma_\beta\beta)^2 \\
&+ (1-\sigma_\beta)^2\sigma_\Pi(2-\sigma_\Pi)(\bar{\gamma} + \sigma_\beta\beta + (1-\sigma_\Pi)\Pi)^2 + \sigma_\beta^2(\bar{\gamma} - (1-\sigma_\beta)\beta)^2 \\
&+ (1-\sigma_\beta)(1-\sigma_\Pi)(1+\sigma_\beta-\sigma_\Pi+\sigma_\Pi\sigma_\beta)(\bar{\gamma} + \sigma_\beta\beta - \sigma_\Pi\Pi)^2 \\
&- 0.5(1-\sigma_\beta)(\sigma_\beta + (1-\sigma_\beta)\sigma_\Pi(1-\sigma_\Pi))\delta^2
\end{aligned} \tag{A23d}$$

First, $g^j$ trivially maximizes $E[U_{ij}]$ given $\mathcal{F}$ and $s_j$. Second, $\mathcal{F}$ is consistent with the advertising strategies of the types. Third, we discuss the optimality of $a^i(.)$ given $\mathcal{F}$. Notice that positive advertising would be prioritized against 'weak opponents' after both location choices for the same reasons as described in the Proof of Proposition 2.

**Locating Apart:** If the opponent is strong, then, for negative advertising to be prioritized in the advertising equilibrium, negative advertising should be effective enough to steal consumers if the opponent runs positive advertising. Since consumers don't search in equilibrium and prices are symmetric, firm $i$ would steal consumers by guaranteeing $E[A_i - A_{-i}] > \delta$:

$$\begin{aligned}
E[A_i - A_{-i} | a_i = N_i, a_{-i} = P_i] &> \delta \\
\Leftrightarrow \beta - (1-\sigma_\Pi)\Pi &> \delta,
\end{aligned} \tag{A24}$$

which is equivalent to condition (A23a) above. Second, for negative advertising to be prioritized against strong opponents in the unique advertising equilibrium, the revenues[33] from stolen consumers should be sufficient to make up for the lost revenues in own location ($R_i(a_i = N_i, a_{-i} =$

[33]The revenues equal profits since there is no cost to producing the product for the firms.

$P_i) > R_i(a_i = P_i, a_{-i} = P_i))$:

$$\left(\frac{\sigma_\beta\beta + \overline{\gamma}}{2} - \frac{\delta}{4}\right)\left(2 - \Gamma\big(\frac{\sigma_\beta\beta + \overline{\gamma}}{2} - \frac{\delta}{4} - \sigma_\beta\beta\big) - \Gamma\big(\frac{\sigma_\beta\beta + \overline{\gamma}}{2} - \frac{\delta}{4} + \delta - \sigma_\beta\beta\big)\right) >$$
$$\left(\frac{(1-\sigma_\Pi)\Pi + \sigma_\beta\beta + \overline{\gamma}}{2}\right)\left(1 - \Gamma\big(\frac{(1-\sigma_\Pi)\Pi + \sigma_\beta\beta + \overline{\gamma}}{2} - \sigma_\beta\beta - (1-\sigma_\Pi)\Pi\big)\right),$$

which can be simplified as

$$(\bar{\gamma} + \sigma_\beta\beta)(\bar{\gamma} + \sigma_\beta\beta - 2(1-\sigma_\Pi)\Pi - 2\delta) > ((1-\sigma_\Pi)\Pi - 0.5\delta)((1-\sigma_\Pi)\Pi + \delta),$$

which is equivalent to condition (A23b) above. Since the game is symmetric, conditions (A23a) and (A23b) together imply prioritizing negative advertising against strong opponents is strictly dominant (Lemma 2). See that (A23b) is harder to satisfy than (A1b) because

$$(\bar{\gamma} + \sigma_\beta\beta - 2(1-\sigma_\Pi)\Pi - 2\delta) > ((1-\sigma_\Pi)\Pi - 0.5\delta)$$
$$\Leftrightarrow \bar{\gamma} + \sigma_\beta\beta > 3(1-\sigma_\Pi)\sigma_\Pi + \frac{3\delta}{2},$$

is harder to satisfy than

$$\bar{\gamma} + \sigma_\beta\beta > (1-\sigma_\Pi)\Pi + \delta.$$

Hence, negative advertising under locating apart is less likely when pricing is introduced to the model. Because this is a necessary condition for the entrant to co-locate, co-location is also less likely when pricing is introduced.[34]

**Co-Locating:** If the opponent is strong, for positive advertising to be prioritized in the unique advertising equilibrium, positive advertising should be effective enough to steal consumers if the opponent runs negative advertising. Since consumers don't search in equilibrium and prices are symmetric, firm $i$ would steal consumers by guaranteeing $E[A_i - A_{-i}] > 0$:

$$E[A_i - A_{-i}|a_i = P_i, a_{-i} = N_i] > 0$$
$$\Pi > (1-\rho)(1-\sigma_\beta)\beta, \tag{A25}$$

which is equivalent to condition (A23c) above.

Fourth, to prove the optimality of $x^2$, consider the potential outcomes following each location choice, summarized in Table A13.

[34] Condition (A23d) also changes once pricing is introduced. However, condition (A23b) has precedence over condition (A23d) because (A23d) assumes (A23b) is satisfied.

| $\theta$ | $a_1$ | $a_2$ | $p_2$ $(\frac{1}{2}*)$ | $R_2$ $(\frac{1}{4(\bar{\gamma}-\underline{\gamma})}*)$ |
|---|---|---|---|---|
| $\{\Pi, \Pi, , \}$ | $P_1$ | $P_2$ | $\bar{\gamma} + (1-\sigma_\Pi)\Pi - \frac{\delta}{2}$ | $(\bar{\gamma} + (1-\sigma_\Pi)\Pi)^2 - \frac{\delta^2}{4}$ |
| $\{\Pi, 0, -\beta, \}$ | $P_1$ | $N_1$ | $\bar{\gamma} + (1-\sigma_\Pi)\Pi - (1-\sigma_\beta)\beta - \frac{\delta}{2}$ | $0$ |
| $\{0, \Pi, , -\beta\}$ | $N_2$ | $P_2$ | $\bar{\gamma} + (1-\sigma_\Pi)\Pi - (1-\sigma_\beta)\beta - \frac{\delta}{2}$ | $2\big((\bar{\gamma} + (1-\sigma_\Pi)\Pi - (1-\sigma_\beta)\beta)^2 - \frac{\delta^2}{4}\big)$ |
| $\{\Pi, 0, 0, \}$ | $P_1$ | $\varnothing$ | $\bar{\gamma} + (1-\sigma_\Pi)\Pi + \sigma_\beta\beta - \frac{\delta}{2}$ | $0$ |
| $\{0, \Pi, , 0\}$ | $\varnothing$ | $P_2$ | $\bar{\gamma} + (1-\sigma_\Pi)\Pi + \sigma_\beta\beta - \frac{\delta}{2}$ | $2\big((\bar{\gamma} + (1-\sigma_\Pi)\Pi + \sigma_\beta\beta)^2 - \frac{\delta^2}{4}\big)$ |
| $\{0, 0, -\beta, -\beta\}$ | $N_2$ | $N_1$ | $\bar{\gamma} - \sigma_\Pi\Pi - (1-\sigma_\beta)\beta - \frac{\delta}{2}$ | $(\bar{\gamma} - \sigma_\Pi\Pi - (1-\sigma_\beta)\beta)^2 - \frac{\delta^2}{4}$ |
| $\{0, 0, 0, -\beta\}$ | $N_2$ | $\varnothing$ | $\bar{\gamma} - \sigma_\Pi\Pi + \sigma_\beta\beta - \frac{\delta}{2}$ | $0$ |
| $\{0, 0, -\beta, 0\}$ | $\varnothing$ | $N_1$ | $\bar{\gamma} - \sigma_\Pi\Pi + \sigma_\beta\beta - \frac{\delta}{2}$ | $2\big((\bar{\gamma} - \sigma_\Pi\Pi + \sigma_\beta\beta)^2 - \frac{\delta^2}{4}\big)$ |
| $\{0, 0, 0, 0\}$ | $\varnothing$ | $\varnothing$ | $\bar{\gamma} - \sigma_\Pi\Pi + \sigma_\beta\beta - \frac{\delta}{2}$ | $(\bar{\gamma} - \sigma_\Pi\Pi + \sigma_\beta\beta)^2 - \frac{\delta^2}{4}$ |

Table A11: Co-Location

| $\theta$ | $a_1$ | $a_2$ | $p_2$ $(\frac{1}{2}*)$ | $R_2$ $(\frac{1}{4(\bar{\gamma}-\underline{\gamma})}*)$ |
|---|---|---|---|---|
| $\{, , -\beta, -\beta\}$ | $N_2$ | $N_1$ | $\bar{\gamma} - (1-\sigma_\beta)\beta$ | $(\bar{\gamma} - (1-\sigma_\beta\beta)^2$ |
| $\{, \Pi, 0, -\beta\}$ | $N_2$ | $P_2$ | $\bar{\gamma} + \sigma_\beta\beta - \frac{\delta}{2}$ | $0$ |
| $\{\Pi, , -\beta, 0\}$ | $P_1$ | $N_1$ | $\bar{\gamma} + \sigma_\beta\beta - \frac{\delta}{2}$ | $2\big((\bar{\gamma} + \sigma_\beta\beta)^2 - \frac{\delta^2}{4}\big)$ |
| $\{, 0, 0, -\beta\}$ | $N_2$ | $\varnothing$ | $\bar{\gamma} + \sigma_\beta\beta - \sigma_\Pi\Pi - \frac{\delta}{2}$ | $0$ |
| $\{0, , -\beta, 0\}$ | $\varnothing$ | $N_1$ | $\bar{\gamma} + \sigma_\beta\beta - \sigma_\Pi\Pi - \frac{\delta}{2}$ | $2\big(\bar{\gamma} - \sigma_\Pi\Pi + \sigma_\beta\beta)^2 - \frac{\delta^2}{4}\big)$ |
| $\{\Pi, \Pi, 0, 0\}$ | $P_1$ | $P_2$ | $\bar{\gamma} + \sigma_\beta\beta + (1-\sigma_\Pi)\Pi$ | $(\bar{\gamma} + (1-\sigma_\Pi)\Pi + \sigma_\beta\beta)^2$ |
| $\{\Pi, 0, 0, 0\}$ | $P_1$ | $\varnothing$ | $\bar{\gamma} + \sigma_\beta\beta - \sigma_\Pi\Pi - \frac{\delta}{2}$ | $0$ |
| $\{0, \Pi, 0, 0\}$ | $\varnothing$ | $P_2$ | $\bar{\gamma} + \sigma_\beta\beta + (1-\sigma_\Pi)\Pi - \frac{\delta}{2}$ | $2\big((\bar{\gamma} + (1-\sigma_\Pi)\Pi + \sigma_\beta\beta)^2 - \frac{\delta^2}{4}\big)$ |
| $\{0, 0, 0, 0\}$ | $\varnothing$ | $\varnothing$ | $\bar{\gamma} + \sigma_\beta\beta - \sigma_\Pi\Pi - \frac{\delta}{2}$ | $(\bar{\gamma} - \sigma_\Pi\Pi + \sigma_\beta\beta)^2$ |

Table A12: Locating Apart

Table A13: Revenues for the Entrant with Negative Advertising Allowed, $p_2$ and $R_2$ refer to the price and revenues for the entrant for each outcome. See Table 1 for $E[A_1]$ and $E[A_2]$ associated with each outcome. $\theta = \{P_1, P_2, N_1, N_2\}$. Refer to Table A6 for the associated probabilities for each scenario.

With some tedious algebra, we can simplify to

$$\begin{aligned} E[R_2|x_2 = L] = \frac{1}{4(\bar{\gamma}-\underline{\gamma})}\Big[&\big(\sigma_\Pi^2 + \rho\sigma_\Pi(1-\sigma_\Pi)\big)(\bar{\gamma} + (1-\sigma_\Pi)\Pi)^2 \\ &+ 2\rho(1-\sigma_\Pi)(1-\rho)\sigma_\beta(\bar{\gamma} + (1-\sigma_\Pi)\Pi - (1-\sigma_\beta)\beta)^2 \\ &+ 2\sigma_\Pi(1-\sigma_\Pi)(1-\rho)(1-\sigma_\beta)(\bar{\gamma} + (1-\sigma_\Pi)\Pi + \sigma_\beta\beta)^2 \\ &+ (1-\sigma_\Pi)(1-\sigma_\Pi+\rho\sigma_\Pi)(\sigma_\beta^2 + \rho\sigma_\beta(1-\sigma_\beta))(\bar{\gamma} - \sigma_\Pi\Pi - (1-\sigma_\beta)\beta)^2 \\ &+ (1-\sigma_\Pi)(1-\sigma_\beta)(1-\sigma_\Pi+\rho\sigma_\Pi)(1+\sigma_\beta-\rho\sigma_\beta)(\bar{\gamma} - \sigma_\Pi\Pi + \sigma_\beta\beta)^2 - 0.5\delta^2\Big] \end{aligned} \tag{A26}$$

$$\begin{aligned} E[R_2|x_2 = R] = \frac{1}{4(\bar{\gamma}-\underline{\gamma})}\Big[&\big(2\sigma_\beta(1-\sigma_\beta)\sigma_\Pi + (1-\sigma_\beta)^2(1-\sigma_\Pi)^2\big)(\bar{\gamma} + \sigma_\beta\beta)^2 \\ &+ (1-\sigma_\beta)^2\sigma_\Pi(2-\sigma_\Pi)(\bar{\gamma} + \sigma_\beta\beta + (1-\sigma_\Pi)\Pi)^2 + \sigma_\beta^2(\bar{\gamma} - (1-\sigma_\beta)\beta)^2 \\ &+ (1-\sigma_\beta)(1-\sigma_\Pi)(1+\sigma_\beta-\sigma_\Pi+\sigma_\Pi\sigma_\beta)(\bar{\gamma} + \sigma_\beta\beta - \sigma_\Pi\Pi)^2 \\ &- 0.5(1-\sigma_\beta)(\sigma_\beta + (1-\sigma_\beta)\sigma_\Pi(1-\sigma_\Pi))\delta^2\Big]. \end{aligned} \tag{A27}$$

Hence, $E[R_2|x_2 = L] > E[R_2|x_2 = R]$ becomes equivalent to condition *iv* above.

Last, the fact that consumers' search decisions and firms' pricing decisions constitute a Nash equilibrium of the pricing-search subgame has been established in Lemma 8.

To sum up, once conditions (A23a)-(A23d) are satisfied, there exists a PBE as defined in $1-6$. □

**Proof of Proposition 5.** First, consider incentives to run negative advertising against weak opponents where the entrant locates apart. In product competition, the necessary condition for negative advertising to be prioritized was $E[A_i - A_{-i}|a_i = N] > \delta > E[A_i - A_{-i}|a_i = P]$, i.e., only negative advertising allows stealing consumers. In political competition, negative advertising may be utilized even when it doesn't lead to stolen consumers. As long as the opponent loses sufficiently many consumers, negative advertising can be utilized. In other words, the necessary condition is $E[A_i - A_{-i}|a_i = N] > E[A_i - A_{-i}|a_i = P]$, which is weaker than the condition above. In the scenario where the entrant co-locates, there would be no change in the incentives and positive advertising would be prioritized against weak opponents in both political and product competition.

Second, consider incentives to run negative advertising against strong opponents. In product competition, there are two necessary conditions for negative advertising to be prioritized: (1) negative advertising allows stealing consumers when the competitor runs positive advertising and (2) the number of stolen consumers is sufficiently large. In political competition, the first condition is sufficient by itself, because a decline in total number of votes is not problematic as long as the opponent loses more votes. Hence, negative advertising is more likely against strong opponents under both co-location and locating apart.

To sum up, under any parameter set where negative advertising is prioritized in product competition, negative advertising is also prioritized in political competition. Notice that this result is independent of the beliefs of consumers $\mathcal{F}$. □

**Proof of Proposition 6.** **(i)** In the benchmark where $\eta = 0$, the game is symmetric between the firms. Hence, the expected winning probability is 0.5 following both co-location and locating apart. This result is independent of which equilibrium is played in the advertising subgames, because all advertising equilibria are symmetric.

**(ii)** In political competition, the presence of $\eta > 0$ implies that co-location always leads to a negative winning probability. This is because the winning probability is discontinuous in $\eta$.[35] When a mass $\eta > 0$ prefers the incumbent, then any symmetric advertising choice leads to incumbent winning the race with probability 1 (instead of 0.5), reducing the (expected) winning probability of the entrant following co-location significantly below 0.5.

[35] The discontinuity in $\eta$ follows from the fact that the winning probability is discontinuous in the vote difference between the candidates.

If the entrant candidate locates apart, however, there always exists an $\eta > 0$ which does not impact the winning probability of the entrant candidate. First, if the advertising outcomes favor the entrant sufficiently more than the incumbent, so much so that the entrant can steal the regular voters (measure $1-\eta$), then, the entrant would win with probability 1 as long as $\eta < 0.5$. Otherwise, voters in $L$ would vote for the incumbent regardless of the incumbency advantage. Hence, for $\eta < 0.5$, the winning probability of the entrant is still 0.5 following locating apart. Hence, the entrant candidate would always locate apart.

The same reasoning does not work in product competition because profits/demand are continuous in $\eta$. The presence of $\eta$ can only reduce the expected demand for the entrant firm by an amount proportional to $\eta$. Hence, when $\eta$ is small enough, its impact on the expected payoffs is negligible. For any situation where the entrant strictly prefers to co-locate, there exists an $\eta$ small enough so that the entrant still strictly prefers to co-locate □

**Proof of Proposition A.1. Part (i)** The proof follows the steps of the of Proof for Proposition 2 almost line by line. The steps for the optimality of purchase and advertising decisions, and how consumer beliefs $\mathcal{F}$ satisfy the Bayes rule given the advertising decisions is identical to the proof for Proposition 2. The only difference now is that location choice is not made in isolation by the entrant, but decided simultaneously within a game for two firms. Hence, the updated equilibrium condition would state that location decisions $x^1$ and $x^2$ constitute a Nash equilibrium. For the parameter set where the entrant decides to co-locate, the expected payoff following co-location should be larger than the expected payoff following locating apart, conditional on the location of the incumbent. Then, the Nash equilibria of the location decisions game (conditional on $g^j$, $a^1$ and $a^2$) would be $\{x^1, x^2\} = \{L, L\}$ and $\{x^1, x^2\} = \{R, R\}$. If $\{x^1, x^2\} = \{L, R\}$ or $\{x^1, x^2\} = \{R, L\}$, then both firms would have a profitable deviation to the other location. The observable equilibrium outcomes are identical to those of the entrant-incumbent game, up to a symmetric change in where firms are located.

**Part (ii)** Similarly, for the parameter set where the entrant decides to locate apart, the expected payoff following locating apart should be larger than the expected payoff following co-location, conditional on the location of the incumbent. Then, the Nash equilibria of the location decision game (conditional on $g^j$, $a^1$ and $a^2$) would be $\{x^1, x^2\} = \{L, R\}$ and $\{x^1, x^2\} = \{R, L\}$. If $\{x^1, x^2\} = \{L, L\}$ or $\{x^1, x^2\} = \{R, R\}$, then both firms would have a profitable deviation to the other location. The observable equilibrium outcomes are identical to those of the entrant-incumbent game, up to a symmetric change in where firms are located. □

**Proof of Proposition A.2.** Here we will postulate a set of beliefs $\mathcal{F}'$ such that there is no set of parameters where the advertising and positioning decisions given in Proposition 2 constitute

a PBE with $\mathcal{F}'$. To this end, we will demonstrate that it is impossible to completely rule out an equilibrium where firms prioritize positive advertising under locating apart and negative advertising under co-location against strong opponents if $\mathcal{F}'$ is such that consumers expect firms to do exactly that (See Table A14.

| | | Co-Location | | Locating Apart | |
|---|---|---|---|---|---|
| $a_i$ | $a_{-i}$ | $E[A_i]$ | $E[A_{-i}]$ | $E[A_i]$ | $E[A_{-i}]$ |
| $P_i$ | $P_{-i}$ | $(1-\sigma_\Pi)\Pi+\sigma_\beta\beta$ | $(1-\sigma_\Pi)\Pi+\sigma_\beta\beta$ | $(1-\sigma_\Pi)\Pi$ | $(1-\sigma_\Pi)\Pi$ |
| $P_i$ | $N_i$ | $(1-\sigma_\Pi)\Pi-(1-\sigma_\beta)\beta$ | $\sigma_\beta\beta$ | $(1-\sigma_\Pi)\Pi-(1-\sigma_\beta)\beta$ | $-\sigma_\Pi\Pi-\rho(1-\sigma_\beta)\beta$ |
| $N_{-i}$ | $N_i$ | $-(1-\sigma_\beta)\beta$ | $-(1-\sigma_\beta)\beta$ | $-\sigma_\Pi\Pi-(1-\sigma_\beta)\beta$ | $-\sigma_\Pi\Pi-(1-\sigma_\beta)\beta$ |
| $P_i$ | $\varnothing$ | $(1-\sigma_\Pi)\Pi+\sigma_\beta\beta$ | $-\sigma_\Pi\Pi$ | $(1-\sigma_\Pi)\Pi+\sigma_\beta\beta$ | $-\sigma_\Pi\Pi+\rho\sigma_\beta\beta$ |
| $N_{-i}$ | $\varnothing$ | $\sigma_\beta\beta-\sigma_\Pi\Pi$ | $-\sigma_\Pi\Pi-(1-\sigma_\beta)\beta$ | $-\sigma_\Pi\Pi+\sigma_\beta\beta$ | $-\sigma_\Pi\Pi-(1-\sigma_\beta)\beta$ |
| $\varnothing$ | $\varnothing$ | $-\sigma_\Pi\Pi+\sigma_\beta\beta$ | $-\sigma_\Pi\Pi+\sigma_\beta\beta$ | $-\sigma_\Pi\Pi+\sigma_\beta\beta$ | $-\sigma_\Pi\Pi+\sigma_\beta\beta$ |

Table A14: Posterior Beliefs Given Advertising Outcomes Under $\mathcal{F}'$

Given $\mathcal{F}'$, when the entrant locates apart from the incumbent ($x_2$=R), there exists a unique PBE in the sub-game where both firms prioritize negative advertising if and only if

$$(1-\sigma_\beta)\beta > \Pi+\delta, \tag{A28}$$

and

$$\overline{\gamma} > (1+\sigma_\Pi)\Pi+\delta. \tag{A29}$$

The first condition ensures that negative advertising is sufficiently more effective to allow a firm to steal consumers from the opposite location, even when consumers believe negative advertising signals inferior positive attribute. The second condition is the stronger counterpart of A1a, and ensures that aggressiveness through negative advertising is also profitable, even under 'adverse' beliefs.

When the entrant co-locates with the incumbent ($x_2$=L), there exists a unique PBE in the sub-game where both firms prioritize positive advertising if and only if

$$(1-\rho)(1-\sigma_\Pi)\Pi > \beta, \tag{A30}$$

or

$$\delta > 2\overline{\gamma}+2\sigma_\beta\beta+(4\rho-2)(1-\sigma_\Pi)\Pi. \tag{A31}$$

The first condition is a stronger version of (A1c), and ensures that positive advertising has the upper hand when the opponent running negative advertising, even under adverse beliefs. The second condition ensures that even when negative advertising brings the upper hand against an opponent, the loss in aggregate demand is larger than the stolen consumers.

First, see that (A31) is incompatible with (A29). If negative advertising steals sufficiently many consumers when consumers expect positive advertising to be prioritized, then it should steal sufficiently many consumers when the consumers expect negative advertising to be prioritized. Second, see that (A30) is incompatible with (A1b). If negative advertising allows stealing consumers from the distant location, then it should allow stealing consumers from the same location when consumers expect negative advertising to be prioritized.

Hence, there are no set of parameters where advertising and location choices that constitute the PBE in Proposition 2 with $\mathcal{F}$ can also constitute a PBE with $\mathcal{F}'$. □