Controlling for Retailer Synergies when Evaluating Coalition Loyalty Programs

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Abstract

Spatial models in retailing allow for correlations in purchase decisions from consumers within predefined geographic areas. The purpose of these models is to control for unobserved demand side effects at the regional-level (e.g., a neighborhood), but typically ignores synergies among individual retailers within a region. To capture the synergies on both the supply side (e.g., store availability) and the demand side (e.g., population density) for each region, we augment a traditional spatial model with a Bayesian Additive Regression Tree (BART). This allows us to account for unobserved regional differences and observed but potentially complex interactions among individual customer and retailers. We apply this model to a credit card coalition loyalty program. Most credit cards offer rewards that vary by retailer. In a typical co-branded credit card, cardmembers earn higher rewards faster for purchases at the retailer sponsoring the card. However, in a coalition loyalty program (CLP), rewards are higher at specific and potentially unrelated retailers within the coalition retailer network. In our empirical setting, we are interested in analyzing the impact of the loyalty program earnings structure on monthly credit card spend. We do this while controlling for the evolving coalition network, which contains hundreds of geographically dispersed partner retailers. Our data has two key features that permit this. First, the retail partner network evolves over time; this variation in retailer participation status allows us to estimate the value of each retailer in the network. Second, the data contains a natural experiment where the loyalty program changed its earning structure, which allows us to estimate the impact of the rewards rate of the loyalty program on customer spend. Our findings show that failure to control for supply side activity results in severely biased estimates of CLP rewards effectiveness. We discuss the implications of BART in our empirical setting and highlight its potential in other marketing situations that contain numerous, interacting control variables.
1 Introduction

About 75% of consumers have a rewards program attached to their credit card, and three of the top four factors cited as most important as choosing a credit card are related to rewards. The structure of a rewards program is flexible, but a recurring feature is that cardmembers earn higher rewards at selected retailers. At one extreme, rewards are earned only at a single retailer (e.g., a co-branded credit card). At the other extreme are coalition loyalty programs, where rewards are earned from a variety of retailers spanning multiple product and service categories. Estimating the effectiveness of a coalition loyalty program (CLP) is challenging when the network of participating retailers is large and changes over time. In this paper we control for the complex and nonlinear interactions among coalition retail partners with a Bayesian additive regression tree (BART) (Chipman et al., 2010). The primary goal of this paper is to highlight how BART can solve a previously intractable, yet important consideration in the analysis of coalition loyalty problems. Specifically, we use BART to account for the complex and evolving network of partners to evaluate the impact of a change in the rewards schedule of a large CLP. A secondary goal is that this paper serves as a motivating example on the versatility of BART and its potential to augment a variety of marketing models in similar situations facing a large number of interrelated control variables.

Our modeling approach starts with a traditional econometric spatial model of monthly cardmember spend. Spatial models recognize that units within each predefined region are similar, and outcomes might be similar between neighboring regions. In our empirical application, this allows us to estimate the influence of a change in the CLP rewards on spend while controlling for unobserved demand side effects such as differences in population density, socioeconomics, and level of competition associated with each region. However, a coalition loyalty program introduces many supply side issues that spatial models are unable to handle. For instance, in a coalition loyalty program synergies may exist between retailers across regions. In addition, the sheer number of retailers may be too cumbersome to estimate with a traditional spatial model.

To account for supply side synergies among coalition retailers, we need to control for the structure of the coalition network. This introduces non-trivial challenges to the model; in our empirical application hundreds of retailers participate in the coalition loyalty program and the network evolves over time. In addition to handling this high-dimensional data, synergies may exist among multiple partners, either within or across the predefined spatial regions. To address these modeling challenges, we augment our base spatial model with a Bayesian additive regression tree, which offers a flexible, nonparametric approach and accounts for the unobserved interactions among the retailers in the CLP. This allows us to control for both the unobserved demand side effects of each region and the observed supply side effects of partner retail locations. The augmented model reduces the potential for omitted variable bias, where supply side synergies would otherwise
be absorbed into demand side regional-level spatial effects. A secondary benefit of BART is its ability to quantify the marginal influence of each of the many retailers in the coalition network, thereby allowing the firm to evaluate the importance of each retailer conditional on the participation status of others.

We estimate the proposed model using detailed, individual-level credit card data from a large European coalition loyalty program with hundreds of retailer partners spanning a broad geographic area. Of primary interest is the effect of a change in the rewards structure on monthly card usage. The spatial component captures the demand side effects from 26 regions of interest while the BART component controls for supply side synergies among many coalition retailers. The supply side synergies we intend to capture can transcend the predefined spatial regions and include location advantages (e.g., a participating gas station may be more attractive if it is near to a participating grocery store), perceptual advantages (e.g., the credit card may be more attractive if luxurious brands participate in the earnings network, even those not nearby), or economic advantages (e.g., complementary retailers). We use BART to capture these granular supply side synergies and the spatial component to control for demand side effects.

Two features of this data are key to this analysis. First, the coalition network evolves over time, allowing us to observe card spending patterns when individual retailers are both in and out of the coalition network. Second, the coalition changed the loyalty program rewards rate partway through the data. This shock allows us to separate out the impact of the loyalty program earnings structure on customer spending from the influence due to the dynamic network of the participating retailers.

Our empirical results show that failure to control for the evolving coalition network biases the estimated loyalty program effects, some of which become insignificant once the evolving network is accounted for. The analysis presents BART as one solution to account for CLP supply side synergies, which are infeasible to control for using a standard spatial model. Even though our empirical application focuses on augmenting a spatial model with BART, we hope marketers become inspired to use BART to enhance other marketing models in similar situations with numerous, potentially complex control variables.

The remainder of the paper is organized as follows: Section 2 provides background to loyalty programs and spatial models prior to introducing BART. Section 3 discusses the empirical setting. Section 4 presents the augmented spatial model with results in Section 5. Section 6 concludes.

2 Background and Literature Review

This paper draws primarily from two research streams. First, we review relevant work on loyalty programs, of which a coalition loyalty program is one type. Second, we discuss prior research on spatial models in marketing. Our aim is to motivate our empirical application and highlight the modeling challenges that
prior work has not yet resolved, namely related to the inability of spatial models to capture granular supply side effects. We then introduce Bayesian additive regression trees as a solution to these challenges, both as it relates to our empirical application and its applicability in other marketing contexts.

2.1 Loyalty Programs

Loyalty programs have served as an important component for customer relationship management for a variety of firms. The marketing literature has seen a long list of studies dedicated to understand their impacts across many industries, including airline, financial services, hotel, gaming and retail (Ferguson and Hlavinka, 2007). The majority of these studies focused on understanding the impact of LP on changing the consumers’ purchase behaviors before and after joining such a program (Lal and Bell (2003); Verhoef (2003); Taylor and Neslin (2005); Liu (2007); Zhang and Breugelmans (2012), to name a few). With a few exceptions, most of the existing studies examine loyalty programs offered by a single firm (see reviews provided by Breugelmans et al. (2015) and Liu and Yang (2009)). In contrast, a coalition loyalty program (CLP) allows customers to buy and redeem rewards across multiple partner retailers, which have received only limited attention (Dorotic et al., 2011; Stourn et al., 2017). Meyer-Waarden (2008) pointed out that with CLPs, it is even harder to change consumers’ purchase behavior than a LP offered by a single firm. In fact, the literature has documented somewhat conflicting results of the impact of CLPs. Moore and Sekhon (2005) confirmed the lack of evidence that CLPs can improve market share. However, Lemon and Wangenheim (2009) find a reinforcing mechanism in cross-buying across loyalty program partners, using data from a European airline and its partners.

As summarized by Liu and Yang (2009), a LP (and CLP) consist of many moving parts that likely influence its ultimate impact on consumers’ purchase behaviors. These factors include, for example, the point structure, such as the point thresholds (O’Brien and Jones, 1995); and consumer characteristics, such as consumers’ usage levels, where Lal and Bell (2003) and Liu (2007) find that it is the light users who experienced the biggest increase in spending after joining the LP. In a coalition loyalty program, there is an added complexity due to the evolution of the participating retailers over time, which can influence the overall attractiveness of the CLP. Our paper contributes to the study of CLPs, where we develop a model to evaluate the change in purchase behavior due to adjustments in the points structure, while controlling for the potentially nonlinear and complex interactions from the evolving coalition retail network.

For the firm managing the coalition loyalty program, their primary strategic decisions revolve around which retailers to invite into the partnership. Like traditional single-retailer loyalty programs, the program points structure often remains fixed and is difficult to change. In addition, unlike a traditional single-retailer
loyalty program, marketing activity is typically decided by the retail partners and not the coalition itself. Thus, it is the coalition partner network that is easiest to modify to influence customer spend. Ideally, a firm should be able to drop partners that decrease overall engagement with the loyalty program and add those that increase engagement, especially synergistic retail partner combinations. However, assessing the contribution of these supply side effects can be challenging from a modeling perspective since the attractiveness of a given partner in the network may depend on the location and participation of other stores in the network at that time.

As a simple example, it may not be advantageous for a customer to engage with the loyalty program credit card if only one specific gas station participates in the coalition. However, if there are many locations that participate, and if the gas stations are located near other stores that the customer otherwise frequents, the retailer network may influence store choice and perhaps motivate the customer to shift towards the coalition credit card. We posit that these dynamics depend on both who is in the network and where they are positioned with respect to each other and each cardmember. Because of this, controlling for each retailer partner is of critical importance in understanding the effectiveness of the coalition loyalty program. To our knowledge, this is the first paper to address this issue head on and fills a much needed gap in the limited work on coalition loyalty programs.

### 2.2 Spatial Models

The literature in marketing has an established history of incorporating spatial components into marketing models. The scope of the applications has been quite broad, including models of product adoption, satisfaction ratings, promotional strategies, and advertising allocation (Aravindakshan et al., 2012), among others (see Bradlow et al. (2005) for a more comprehensive review). Spatial models recognize that individuals (or, more generally, any unit of analysis) are located in space and that the decisions of individuals in the vicinity may be correlated. In spatial econometrics the spaces are typically represented on a map that is geographical in nature, but spatial closeness can be also defined using demographics (Yang and Allenby, 2003), latent constructs (Moon and Russell, 2004), or any other construct that determines the relative similarities of the individual units. A common assumption is that the behavior of one individual is conditionally independent of the behavior of another individual. In our empirical application, we assume that geographic proximity implies high correlation between individuals. That is, customers located near each other share similar unobserved traits.

The spatial model presented in this paper is designed to capture spatially correlated errors, the idea being that unobserved variables that drive purchase behavior can be inferred from proximity on a map (see Russell
and Petersen (2000), Yang and Allenby (2003), and Bronnenberg and Sismeiro (2002) as examples). The spatial component of the model is formulated using a typical conditional auto-regressive (CAR) structure: given the random effects from all other areas, the distribution of the spatial effect only depends on its neighbors (Cressie, 1993). In short, each geographic area is similar to its neighbors.

Spatial modeling usually involves specifying a covariance matrix between regions, from which a correlation coefficient is estimated to measure influence between neighboring regions. Because of this matrix, empirical applications in prior work typically specify only a relatively small number of areas. However, in our empirical application, we observe hundreds of retailers. While we want to control for demand side effects at the regional level, it would be intractable to try and make the regions granular enough to capture supply side effects at the retailer level. Our solution is to retain a spatial modeling structure to capture demand side effects at the regional level, but offload the supply side effects from individual coalition retailers to BART.

2.3 Bayesian Additive Regression Trees (BART)

Marketing modeling has encountered many situations where the complexity of relationships among variables can be non-trivial. For example, optimizing store shelf layout (Van Nierop et al., 2008), identifying influential users in a network (Trusov et al., 2010), and managing the marketing mix in CRM (Rust and Verhoef, 2005), to name a few. In this paper we focus on the inherent complexity in controlling for the supply side effects of a large network of retailers whose locations could create synergies with each other (e.g., complementary stores located near each other) or could exert a highly non-linear relationships to cardmember spend.

From modeling perspective, conditioning on these cross-effects within a large coalition network is quite difficult. To account for the potentially high order interactions among multiple retailers and handle the high-dimensionality of the data we extend a spatial model with a Bayesian additive regression tree as proposed by Chipman et al. (2010). At its core, BART provides a Bayesian inference on a model of \( y = f(X) + \varepsilon \) with \( \varepsilon \sim N(0, \sigma^2) \) where \( f \) is a sum of regression trees, and therefore a non-parametric function of the input \( X \) variables.

The regression tree serves as the basic building block to a BART model. Figure 1 illustrates a simple regression tree based on our empirical application. In this example, only two variables \((x_1 \text{ and } x_2)\) are considered, which represent the distances between a customer and two partner retailers. The regression tree partitions the \( X \) space into subgroups. Observations are assigned to each subgroup by working down the tree’s decision rules. If the distance to the first retailer, \( x_1 \), is less than 9km then the distance of the second retailer, \( x_2 \), is considered and the predicted monthly spend on the credit card is either \( \mu_2 \) when \( x_2 \) is less than 4km or \( \mu_3 \) otherwise. The values at these terminal nodes (\( \mu \)) are typically the average of the observations.
in each subgroup (e.g., the average monthly spend for observations that meet all the decision rules on the branch).

On the left is a regression tree with two decision rules and three terminal nodes. The decision rules are processed from top to bottom until a terminal node is reached, where the terminal node contains the predicted outcome value. The predicted value is typically the average of the outcome variable $y$ associated with all $X$ that satisfy the decision rules in the data used to fit the model. On the right is a visual representation of the total $X$ space partitioned, according to the regression tree presented on the left.

**Figure 1: Example Regression Tree Reproduced from Hill et al. (2020)**

BART extends this single-tree concept by combining multiple such trees, where a regularization prior controls the tree growing process. Models with sums of regression trees, such as BART, have greater ability and flexibility than single trees to capture interactions and non-linearities among the input variables (Hill, 2011). Relative to a traditional tree model, BART offers two additional key advantages in practice, as outlined in Allenby et al. (2014). First, the default priors have been shown to consistently provide reasonable estimates, so there is less parameter tuning using cross-validation. Second, the depth of the tree (i.e., its complexity) is inferred naturally through the MCMC process so that the level of interactions is inferred somewhat automatically. The non-parametric nature and its easy setup of BART lend itself the unique advantages of capturing sophisticated interactions among a large number of input predictors, in addition to their possible nonlinear effects. Furthermore, the Bayes posterior distributions obtained through MCMC provides the usual representation of uncertainty in the parameter estimates. Although there are many modeling incidences that are well suited to a flexible approach such as BART, most of the applications have been limited to evaluating heterogeneous treatment effects (for example, Hill (2011), Green and Kern (2012), Carnegie et al. (2019), Heckman et al. (2014), Hahn et al. (2020), and Athey and Imbens (2017)).

A significant feature of BART, especially in marketing, is that it can easily be incorporated into a variety
of models as its modularity allows it to be simply inserted into the MCMC sampling process. This presents BART as an attractive option to control for potentially complex, auxiliary variables while still adhering to a traditional functional form on the key econometric variables of interest. As demonstrated in Chipman et al. (2010), the BART model can be applied in a probit model, with only minor modifications of the methods. Kindo et al. (2016) further expanded it to a multinomial probit model. In addition, BART has also been applied in the context of survival analysis. Bonato et al. (2011) incorporated BART into parametric survival models, including Cox Proportional Hazard model, Weibull and Accelerated Failure time models; and Sparapani et al. (2016) applied BART in nonparametric survival analysis using a nonproportional hazard approach. In this application we augment a spatial model with BART, similar to Zhang et al. (2007) but who use the model for merging datasets.

The implementation of BART in the marketing literature is surprisingly limited. In our empirical application, we use BART to control for complex supply side effects among many coalition retailer partners in order to address our managerial question of interest: what is the impact of changing the coalition loyalty program on customer spending patterns? We find that in doing so, we reduce the bias of the coefficients of interest related to the effects of the loyalty program. To our knowledge this paper is one of the first to examine the spatially driven impacts of a coalition loyalty program and one of the early attempts to incorporate the BART model into research in marketing.

3 Empirical Setting: Coalition Loyalty Program

In this section we present the empirical context and describe the data used for model estimation. We highlight the unique aspects of the data, namely the availability of a natural experiment that allows us to estimate the impact of changes in the LP rewards structure on credit card spend while controlling for the evolving network of participating retailers.

The dataset comes from a large European coalition loyalty program. Cardmembers sign up for the loyalty program combined with a credit card. The credit card is used for transactions as any other typical credit card, including purchases and cash withdrawals. Purchases can be made (and points accumulated) anywhere credit cards are accepted, however points are earned at a faster rate with purchases at in-network retailers (i.e., “partner” stores) relative to out-of-network stores. Earned points are redeemed for cash vouchers, which can be used to purchase additional goods and services at selected partner retailers. In addition to managing the points tracking and voucher generation and redemption, the firm sends outbound marketing campaigns to encourage sign-up, purchases, and voucher usage.

The network of partners is considerably large and diverse, both in terms of partner type (e.g., gas
stations, clothing retailer, etc.) and regional footprint – in an average month there are 1,956 unique partners. Typically, partners are independent retailers looking to benefit from the network effects of the coalition, but there is a substantial number of chain retailers included as well. Over the observed time frame of four years the network size ranges from 1,891 partners to 2,017 partners. While the overall network size remains relatively stable over time, each month there are an average of nine partners joining the network and nine partners leaving the network.

We observe monthly spend from 619 customers. About 71% months have zero in-network spend and 44% of the months have zero out-of-network spend. Figure 2 shows the distribution of non-zero monthly spend. The average monthly in-network monthly spend is $258 and the out-of-network monthly spend about $594 (for months with nonzero spend), but there is a long tail to this distribution.

The geographic footprint of the retail partners in the coalition network is rather large – the average distance between each cardmember and all retail partners is about 100km. However, more than half of all transactions occur within 8km. Figure 3 shows the distribution of distance traveled for all in-network transactions, where the majority occur within 20km.
The dashed line represents the median of the distance traveled by each cardmember for in-network transactions, about 8km.

Figure 3: In-Network Distance Traveled

This suggests the much of the coalition retailer network is irrelevant to many cardmembers, since they will be unlikely to travel very far to retail partners. Figure 4 compares the distribution of distance traveled to retailers “available” to each cardmember, depending on their location. We see the vast majority of cardmembers stick with a local network of partners, and do not engage with the coalition network further away. The implication of this is that a partner entry or exit from the coalition is likely to be only relevant to the cardmembers located nearby.
The distribution of coalition partner retailers available to each cardmember is relatively uniform in terms of distance. However, cardmembers tend to frequent retailers located close by.

Figure 4: Coalition Usage

There is also substantial heterogeneity in the size of the coalition network around each cardmember. Figure 5 shows the distribution of the number of stores within 10km of each cardmember.\textsuperscript{1} The average cardmember has access to 88 retail partners within 10km, but we see this distribution is bimodal: some cardmembers are in high areas of saturation (e.g., a city center with many partners) while others are located in areas with only a dozen or so partners nearby (e.g., an area of low interest to potential coalition partners). Considering both the limited travel distances and the variation in nearby network composition, it would be naive to assume that cardmembers value all partners equally. This motivates the need to account for the unique distances between each cardmember and all partner retail locations.

\textsuperscript{1}Since the coalition evolves, this distribution represents the average number of stores across time.
Within a 10km travel range (slightly above the median travel distance of 8km), there is substantial variation in the number of coalition partners available.

Figure 5: Stores within 10km of Cardmembers

3.1 Change in the Loyalty Program Structure

A key feature of this data is that it contains a natural experiment where the firm decided to change the LP earnings structure. The change in LP design was meant to make the program more competitive, and is therefore exogenous with respect to individual cardmember locations or spend behavior. The four year observation period spans the two years on either side of this change, which serves as the focal point of our analysis: to what extent did the change in the LP structure influence cardmember behavior? Since we do not have access to a control group (that is, individuals who did not experience a change in the coalition loyalty program) we instead compare the impact across two customer segments: high spenders and low spenders, defined by a median split in monthly spend prior to the change. This allows us to determine if the change was equally attractive to each segment, or if one segment was more sensitive to the change.

The change introduced by the firm involved two primary components: 1) The relative attractiveness of using the card out-of-network and 2) the exchange rate (i.e., the monetary voucher value of each point earned). Table 1 outlines the structure of the LP both before and after the change occurred. The in-network earning rate remained constant at .5 or 1 point per dollar spent, depending on the partner. However the out-of-network earnings rate doubled from .1 to .2 points per dollar, thereby making out-of-network spend relatively more attractive. In addition, the points were devalued by a factor of three in that after the

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While there are countless ways to segment customers, we chose a median split since it maximizes the number of cardmembers in the fewest segments.
change it requires three times as many points to earn an equivalent voucher value. This change played out in a predictable pattern in the data: overall spending declined (perhaps from point devaluation) and the proportion of spend in-network declined (perhaps due to the relative gain in attractiveness of out-of-network card usage).

<table>
<thead>
<tr>
<th></th>
<th>Pre LP Change</th>
<th>Post LP Change</th>
</tr>
</thead>
<tbody>
<tr>
<td>Points Earned In-Network</td>
<td>.5 or 1*</td>
<td>.5 or 1*</td>
</tr>
<tr>
<td>Points Earned Out-Of-Network</td>
<td>.1</td>
<td>.2</td>
</tr>
<tr>
<td>Exchange Rate</td>
<td>33 1/3</td>
<td>100</td>
</tr>
<tr>
<td>Avg. In-Network Spend</td>
<td>$90</td>
<td>$61</td>
</tr>
<tr>
<td>Avg. Out-Of-Network Spend</td>
<td>$330</td>
<td>$334</td>
</tr>
<tr>
<td>Avg. Total Spend</td>
<td>$420</td>
<td>$395</td>
</tr>
</tbody>
</table>

*Depending on the retail partner.

Table 1: Coalition Program Schedule

Interestingly, we find that the change in the behavior varies drastically between the “High” and “Low” spend groups. Figure 6 shows the monthly spend by segment before and after the LP change. After the LP changed, high spenders reduced their spend and low spenders reduced in-network spend but increased out-of-network spend. These patterns are reproduced with a regression discontinuity shown in Table 2. The challenge with this simple regression is that we have not conditioned on the evolving coalition network. This makes it difficult to disentangle the effect of the change in the LP structure from the influences of the partner locations. Furthermore, any changes in the coalition network may be correlated with each spend group. For example, the decrease in in-network spend from the low segment may reflect coalition partners leaving the network who are attractive to this group, and not have any relation to the change in the loyalty program.
The results from this regression discontinuity suggest that the change in the loyalty program negatively impacted all spend except for out-of-network spending in the low segment.

Table 2: Regression Discontinuity

To isolate the influence of the change in the LP structure, we need to control for the state of the coalition.
network. This is a non-trivial task for a variety of reasons. The first is that the network is very large, so including individual partner level controls is simply not feasible using a traditional functional form model. The second more substantial concern is that it is not unreasonable to expect synergies to exist between locations in such a large network. In other words, it is unlikely that a single partner will make a material difference with its entry or exit but rather it is the holistic network composition that plays a role in the attractiveness of the network. For instance, it may be the case that a handful of complementary stores, as a group, makes the card attractive and only if all these stores exit card usage may drop significantly. Capturing these higher order interactions is especially challenging, and in the next section we present a model to handle these issues.

4 Model Specification

In this section we introduce the spatial model, which is then augmented with BART. The primary purpose of this model is to estimate the effect of the change in the loyalty program, conditional on unobserved regional level effects and the evolving network structure. We begin with the base model specification with spatially correlated errors and discuss the modifications to account for the correlations between our two dependent variables and their censored distribution. Then, we augment this model with the BART component to control for the complex coalition network structure.

4.1 A Conditional Auto-Regressive Spatial Model

In this section we outline our base model for each dependent variable, which takes the form of a traditional spatial model. We observe in-network \( y_{it}^I \) and out-of-network \( y_{it}^O \) credit card spend for cardmembers \( i = 1 \ldots N \) in month \( t \). Since the independent variables are identical in both models, we use superscripts to differentiate between in-network (\( I \)) and out-of-network (\( O \)) coefficients and for clarity we drop the superscripts when feasible. The base models are specified as follows:

\[
\begin{align*}
y_{it}^I &= \alpha_I^I + \Gamma_I S_{[i,t]} + \kappa^I G_{it} + \zeta^I Z_{it} + \omega^I G_{it} Z_{it} + \theta_a^I + \varepsilon_{it}^I \\
y_{it}^O &= \alpha_O^O + \Gamma_O S_{[i,t]} + \kappa^O G_{it} + \zeta^O Z_{it} + \omega^O G_{it} Z_{it} + \theta_a^O + \varepsilon_{it}^O
\end{align*}
\]

Both dependent variables are log transformed to accommodate their skewed distributions. In months with zero spend we treat the outcome as a censored variable, where the realized \( y = \hat{y} \) if the latent variable \( \hat{y} > 0 \) and 0 otherwise, as in a standard Tobit Type II specification.
For the coefficients, $\alpha$ is an individual-level fixed effect and $\Gamma$ controls for 11 monthly fixed effects with December as a baseline corresponding with the $(1 \times 11)$ seasonality matrix $S$ associated with cardmember $i$ at time $t$. Of primary interest are the coefficients associated with the spend group of the customer, the time period after the LP change, and their interaction: $\kappa$, $\zeta$, and $\omega$. $G$ is an indicator which equals one for customers in the high spend group, determined by a median split of spend prior to the LP change. $Z$ is an indicator that equals one for all observations occurring after the LP change. Lastly, the coefficient $\omega$ captures the interaction effect between the LP change and the customer segment.

We control for spatially correlated errors and estimate random effects $\theta$ for areas $a = 1, \ldots, A$ using a conditional autoregressive (CAR) prior. The key idea of the CAR component is that each area is similar its neighboring areas. The density of each regional effect $\theta_a$ is defined by the set of conditional distributions:

$$p(\theta_a | \theta_{-a}) \sim N\left(\frac{\rho}{h_a} \sum_{k \neq a} C_{ak} \theta_k, \frac{\delta^2}{h_a}\right), \ a = 1 \ldots A \tag{3}$$

The distribution of each area effect $\theta_a$ follows a normal distribution where the mean is a function of the following: 1) an adjacency matrix $C$, where $C_{ak} = 1$ if areas $a$ and $k$ are neighbors and $C_{ak} = 0$ otherwise (including $C_{aa} = 0$), 2) the number of neighboring areas to $a$, $h_a$, where $h_a = \sum_{k=1}^{A} C_{ak}$, and 3) the estimated spillover coefficient, $\rho$. Positive (negative) values of $\rho$ indicate positive (negative) values of correlation between regions. If $\rho$ is not significantly different from zero, we conclude that there are no spillover effects across regions. When $\rho$ approaches 1 the conditional mean of $\theta_a$ equals the average of its neighbors. The variance of this distribution is a function of the estimated cross-area variance $\delta^2$, which is scaled by the number of neighboring regions $h_a$. In our empirical application each cardmember is assigned to one of 26 areas.

A primary difference between the CAR model and an alternative simultaneous autoregressive (SAR) model (for example, as used by Yang and Allenby (2003)) is that the SAR model allows for a non-symmetric spatial matrix $\theta$. However, as Cressie (1993) points out, a space-time model is probably more appropriate if the modeler desires non-symmetric spatial dependence matrix. Since the parameters are more naturally interpreted from a conditional-expectation structure, it is argued that the CAR model be used from the outset.

The joint distribution of $p(\theta)$ implied from equation 3 is

$$p(\theta | \rho, \delta^2) = N \left(0, \delta^2 \left(\mathbb{I} - \rho C\right)^{-1}\right) \tag{4}$$

where $\mathbb{I}$ is an $A \times A$ diagonal matrix with $h_a$ as the diagonal elements. We employ a uniform prior on
\( \rho \) over a specified range. The parameter \( \rho \) must lie in the interval \( [\lambda_{\text{min}}^{-1}, \lambda_{\text{max}}^{-1}] \) where \( \lambda_{\text{min}} \) and \( \lambda_{\text{max}} \) denote the minimum and maximum eigenvalues of \( C \) for which the matrix \( (H - \rho C) \) can be inverted (Sun et al., 1999). Figure 7 provides a simple illustration of the CAR structure in a hypothetical region with five areas.

\[
C = \begin{bmatrix}
0 & 1 & 1 & 1 & 0 \\
1 & 0 & 0 & 1 & 0 \\
1 & 0 & 0 & 1 & 1 \\
1 & 1 & 1 & 0 & 1 \\
0 & 0 & 1 & 1 & 0 \\
\end{bmatrix}
\]

\[
H = \begin{bmatrix}
3 & 0 & 0 & 0 & 0 \\
0 & 2 & 0 & 0 & 0 \\
0 & 0 & 3 & 0 & 0 \\
0 & 0 & 0 & 4 & 0 \\
0 & 0 & 0 & 0 & 2 \\
\end{bmatrix}
\]

A CAR structure in a hypothetical five area region. \( \rho \) controls spillover among areas, and \( \delta^2 \) captures the variance in the area fixed effects – both of these parameters are estimated. The corresponding \( C \) and \( H \) matrices are fixed and set by the analyst depending on where the area boundaries lie.

Figure 7: CAR Structure

Lastly, in our model \( \varepsilon_{it} \sim N(0, \sigma^2) \) and reflects unobservable factors modeled as error that is independent over time. We allow for correlated errors between monthly in-network spend and out-of-network spend by framing the two equations as a seemingly unrelated regression (SUR) (Zellner, 1962). Estimation of our censored SUR follows Huang (2001), where the only modification from a typical SUR estimation is that we substitute augmented data \( \tilde{y} \) in the estimation rather than than the censored data \( y \). During estimation this is generated from a truncated normal distribution.

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3 More details on the estimation procedure are available in the Appendix.
4.2 Augmenting the Spatial Model with BART

In section 3.1, we discussed some of the challenges in trying to measure the impact of the change in the loyalty program by segment with a simple regression discontinuity. Even in the presence of a natural experiment, the naive regression discontinuity fails to control for potentially important demand side and supply side effects and may bias the results and in turn managerial interpretations. The unobserved demand side effects are accounted for with the spatial CAR specification discussed in the prior section. That leaves the observed supply side effects of individual partner locations and entry/exit behavior. We augment our specified spatial model with BART to control for these supply side effects. The addition of BART is intended to control for the state of the network in each month, so that we have an unbiased estimate of our effects of interest: the impact of the change in the LP. Our augmented model is specified as:

\[ y_{it} = \alpha_i \left( \text{Individual FE} \right) + \Gamma \left( \text{Seasonality} \right) + \kappa G_{it} + \zeta Z_{it} + \omega G_{it} Z_{it} + \theta_a \left( \text{Area RE} \right) + f \left( X_{[it,i]} \right) + \varepsilon_{it} \]  

(5)

where \( X_{[it,i]} \) is a \((1 \times 2,329)\) matrix which represents the state of the coalition network with respect to cardmember \( i \) at time \( t \) for all 2,329 coalition partners. Each element in this matrix represents the geographic influence measured by \( 1/d_{is}^* \), where \( d_{is}^* \) is the distance in kilometers between cardmember \( i \) and store \( s \).\(^4\) For example, \( X_{[12,5,760]} \) denotes the geographic influence of store 760 to cardmember 12 in month 5 of the data.\(^5\)

The building block of the BART model is a regression tree. Let \( T \) denote a single binary tree consisting of a set of decision rules and terminal nodes. The collection of terminal nodes associated with this tree is denoted by \( M = \{ \mu_1, \mu_2, \ldots, \mu_N \} \) for the \( N \) terminal nodes associated with this tree. In the example provided in section 2.3, we had \( M = \{1.2, 0.8, 2.3\} \) to represent the three expected values of \( y \) given a particular partition of \( X \) as determined by the decision tree. We let the function \( g(X; T, M) \) represent a non-parametric function that assigns the terminal node \( \mu_n \in M \) to \( X \), based on the decision rules followed for each record in the data. With this notation, the sum-of-trees model is defined as follows across \( m \) trees:

\[ f(X) = \sum_{j=1}^{m} g(X; T_j, M_j) \]  

(6)

\(^4\)Given the flexibility of the BART estimation process, the model is somewhat agnostic to the exact specification of the geographic influence and its form is mostly dictated to account for properly denoting a retailer’s out-of-network status with a zero.

\(^5\)In some CLPs, it is possible that the decision of a retailer to join the coalition is endogenous with respect to cardmember spending patterns. Controlling for this endogeneity would introduce much sophistication to the model, especially considering a CLP with a large number of stores. However, in the web appendix we provide evidence suggesting that in our empirical application the decision of an individual retailer is exogenous with respect to overall coalition spend and therefore do not consider endogeneity a concern in our model.
Through combining such regression trees, BART allows any potential interactions and nonlinearities among the input $X$ variables. To estimate this sum-of-trees model, BART uses a variant of Bayesian backfitting as outlined by Hastie and Tibshirani (2000) that iteratively constructs and fits residuals across MCMC draws, an approach similar to the gradient boosting approach of (Friedman, 2001). To avoid overfitting, each tree is intended to be small, by limiting its depth and by each parameter $\mu_k$ being shrunk towards 0, which is achieved through the regularization prior. The sum-of-trees, backfitting MCMC algorithm, and regularization prior form the essential features of BART. We review the prior specifications in the Appendix, and additional technical details on the estimation procedure can be found in Chipman et al. (2010). Figure 8 presents the basic features of a Bayesian additive regression tree.
A new tree is proposed using one of four moves:
1) Growing a terminal node
2) Pruning a pair of terminal nodes
3) Changing a nonterminal rule
4) Swapping a parent and child rule

The priors keep trees small so each explains only a portion of $f(X)$

During draws the trees grow or shrink in incremental changes becoming only as complex as necessary

Here we present the basic building blocks of a two tree BART model. The first tree, $T_1$, in the left column ends up getting “trimmed” during the MCMC sampling process moving from draw $r$ to draw $r + 1$ while the second tree, $T_2$, “grows” a terminal node. Each individual tree explains a relatively small portion of the data, but by adding the trees together the predictions can accomodate a highly nonlinear mapping from $X$ to $y$.

Figure 8: Fundamental BART Components

We now have a fully specified model to that controls for both 1) demand side effects through the regional fixed effects and spatially correlated error structure, and 2) supply side retailer level effects through BART. As discussed in Chipman et al. (2010), the BART component of this model can simply be added in to the MCMC sampling process, allowing us to easily incorporate it into our spatial model. The primary advantage of this specification over our based model is that individual store effects will no longer be absorbed into the regional effects and in turn potentially bias our coefficients of interest related to the change in the loyalty
program. In the data, there is variation both over time and across cardmembers that will assist the BART component with estimation. The variation over time comes from partners entering and leaving the network each month. The variation across cardmembers stems from the geographic distribution of the network with respect to each cardmember. As previously discussed, due to the limited distance traveled for in-network transactions, it is as if each cardmember essentially has their own local network with which they interact.

Figure 9 shows the primary differences between the CAR and BART components and highlights why both are advantageous in our model specification. The CAR prior allows us to control for unobserved, regional level differences whereas BART allows us to control for observed, but complex, retailer locations. Excluding either of these components could bias our main effects of interest related to the change in the loyalty program. Furthermore, including CAR but excluding BART could confound estimated spatial effects since all coalition partner entry and exit behavior would be absorbed (incorrectly) into regional effects. Our augmented model contrasts with most prior spatial work in marketing, which simply groups the firm's locations into regional areas thus potentially confounds location effects specific to the firm with regional fixed effects.
Cardmembers within each area are similar and may be correlated with cardmembers in neighboring areas.

Cardmembers located near the edge of an area will be grouped with cardmembers on other side of an area rather than cardmembers nearby.

Area borders are pre-defined by the analyst to capture unobserved differences such as level of competition.

In both the top and bottom maps 8 cardmembers (the black dots) are located in the same spots. In the CAR prior (on top), no consideration is given to individual locations within an area: only differences across areas matter. This means that two cardmembers located near each other will separated (and have different regional effects) if a border crosses between them. The analyst needs to be careful that the specified borders reflect natural groupings that would reasonably capture unobserved differences specific to an area. In the BART model (on bottom), no consideration is given to unobserved area level differences. In our specification, only the distance between each cardmember and each retailer (the white squares) matters. In essence, each cardmember experiences whichever “local” network surrounds them, which may evolve over time. The local network for one of the cardmembers is highlighted by the solid black lines. This variation across cardmembers allows BART to estimate the value of each partner that participates in the coalition.

Figure 9: CAR and BART Comparison with 8 Cardmembers and 3 Coalition Retailers
5 Results

In this section we discuss the estimation results. We briefly review the SUR, spatial, and monthly results before moving onto the primary coefficients of interest related to the change in the LP structure.

5.1 SUR, Spatial, and Monthly Estimates

Table 3 shows the correlation estimate between the in-network and out-of-network spend from the SUR specification. A positive correlation is not surprising, since a high spending in-network cardmember is likely to be a high out-of-network spender, but the low magnitude of .09 suggests that in-network spend is only loosely indicative of out-of-network spend once the coalition network has been accounted for. This may be a reflection of the large heterogeneity in coalition partner access near each cardmember.

| $E \left| \varepsilon^I \varepsilon^O \right|$ | 0.088 | 0.073 | 0.104 |
|----------------------------------------|--------|--------|--------|

Table 3: Estimated Correlation Between Spend and Redemption

Figure 10 presents the coefficients from the eleven month dummy variables (with December as a baseline). For each of the two dependent variables (in-network and out-of-network monthly spend) we estimate the coefficients both excluding and including BART to control for the coalition network structure. The purpose of doing this is to determine to what extent the presence of BART changes the other coefficients in the model. We see that most of the monthly coefficients remain relatively stable after adding BART – this is somewhat expected since the structure of the network is unlikely to have a large influence on monthly seasonality effects.
Table 4 presents the estimated model variance and spatial modeling parameters. We see that by including BART, the model variance decreases slightly, suggesting that it is fitting more unexplained variation. More interestingly, adding BART leads to a dramatic decrease in the cross-regional effects (captured through $\delta^2$) for both dependent variables. Further, the regional spillover coefficient, $\rho$, becomes insignificant in the model with BART. By controlling for retailer-specific information using BART, information that otherwise would be attributed to unobserved demand side regional effects has been properly allocated into observed supply side effects.

<table>
<thead>
<tr>
<th>Coef.</th>
<th>Description</th>
<th>$y^I$: In-Network Spend</th>
<th>$y^O$: Out-Of-Network Spend</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\sigma$</td>
<td>Model SD</td>
<td>6.00 (5.89, 6.11)</td>
<td>6.00 (5.89, 6.11)</td>
</tr>
<tr>
<td>$\delta$</td>
<td>Regional SD</td>
<td>11.76 (8.79, 15.65)</td>
<td>11.76 (8.79, 15.65)</td>
</tr>
<tr>
<td>$\rho$</td>
<td>Reg. spillover</td>
<td>0.16 (0.05, 0.19)</td>
<td>0.16 (0.04, 0.19)</td>
</tr>
</tbody>
</table>

Table 4: Model Variance and Spatial Results. Posterior Median and 95% HPD

To further illustrate the effect of augmenting a traditional spatial model with BART, Figure 11 plots the posterior median of each of the 26 region effects, along with the 95% highest posterior density. Once again, we include the estimates both with and without the BART component. The regional effects virtually disappear once we control for retailer specific information via BART. Without BART, the analyst would
mistakenly believe that the variation in the data is a result of the unobserved demand side effects unique to each region, when in fact the variation appears related to observed retailer locations in the coalition.

Recall that regional effects, and similarly regional spillovers, are intended to capture unobserved effects outside of firms’ individual location influences. Our spatial BART specification allows us to separate the influence of the firm’s contribution at each location against regional level effects and more accurately portray the uncertainty associated with regional level estimates. For firms with many locations, either their own or through a coalition network, this distinction is critical in correctly distinguishing unobserved spatial effects from effects within the firm’s control.

5.2 Impact of Change in CLP Structure

In this section we focus on our coefficients of interest, the influence of the LP change and how it varies by customer segment. Our premise is that to properly measure the loyalty program rewards effect, even in the presence a natural experiment, we must control for the evolving coalition network. Otherwise, any changes in the coalition network over time may be confounded with the estimated impact of the change in the LP. As previously discussed, we decided to employ BART to control for these complex relationships among retail partners. While BART will not explain the mechanism by which the store relationships influence the LP estimates, there is still value in controlling for these auxiliary variables in an attempt to reduce omitted variable bias in our coefficients of interest.
First, we consider a model similar to the regression discontinuity presented in Table 2 from section 3.1. We include this table because after transforming the dependent variables, the regression discontinuity coefficients from earlier are not directly comparable to any coefficients presented in this section. Specifically, we take the full model specification but “turn off” both the CAR prior and BART component, but still retain individual fixed effects, seasonality, and our LP and segment coefficients of interest. With these components are turned off, this model fails to control for any demand side effects at the region level through the CAR prior, nor does it control for supply side effects from the coalition retail partners through BART. The results are presented in Table 5 which, as before, show that the in-network spend from the low segment exhibits a significant decline after the LP changed its rewards structure.

<table>
<thead>
<tr>
<th>Coef.</th>
<th>Description</th>
<th>$y^I$: In-Network Spend</th>
<th>$y^O$: Out-Of-Network Spend</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\kappa$</td>
<td>High Spend</td>
<td>1.44 (-0.18, 2.98)</td>
<td>3.72 (2.18, 5.29)</td>
</tr>
<tr>
<td>$\zeta$</td>
<td>Post LP</td>
<td>-0.28 (-0.46, -0.10)</td>
<td>0.38 (0.17, 0.58)</td>
</tr>
<tr>
<td>$\omega$</td>
<td>HS x Post LP</td>
<td>-0.53 (-0.77, -0.27)</td>
<td>-0.94 (-1.23, -0.66)</td>
</tr>
</tbody>
</table>

These are results are from the model specified but with the CAR prior and BART component removed. The coefficients have a similar interpretation to the regression discontinuity shown earlier in Table 5.

Table 5: Regression Discontinuity LP x Segment Results. Posterior Median and 95% HPD

We next move on to the two primary models we’ve discussed: the CAR model without BART, and then the full model which includes the BART component. The coefficients for the high spend segment and post LP change indicators, along with their interaction, are shown in Table 6. The results with the CAR prior (but without BART) are similar to the results without the CAR prior. That is, we find that change in the LP appears to have a significant negative effect on the low segment in-network spend. However, after controlling for supply side effects with BART, the coefficient is no longer significantly different from zero. This suggests that the change in the LP rewards structure is biased downward if the coalition structure is not accounted for. In other words, it is likely that key coalition partners exited after the coalition changed the LP rewards structure. The firm managing the coalition would fail to capture this insight unless they conditioned on coalition network structure.

<table>
<thead>
<tr>
<th>Coef.</th>
<th>Description</th>
<th>$y^I$: In-Network Spend</th>
<th>$y^O$: Out-Of-Network Spend</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\kappa$</td>
<td>High Spend</td>
<td>1.90 (0.75, 3.07)</td>
<td>2.79 (1.46, 4.23)</td>
</tr>
<tr>
<td>$\zeta$</td>
<td>Post LP</td>
<td>-0.32 (-0.56, -0.10)</td>
<td>-0.07 (-0.37, 0.27)</td>
</tr>
<tr>
<td>$\omega$</td>
<td>HS x Post LP</td>
<td>-0.64 (-0.96, -0.33)</td>
<td>-0.68 (-0.99, -0.35)</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Coef.</th>
<th>Description</th>
<th>Without BART</th>
<th>With BART</th>
<th>Without BART</th>
<th>With BART</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\zeta$</td>
<td>Post LP</td>
<td>-0.32 (-0.56, -0.10)</td>
<td>-0.07 (-0.37, 0.27)</td>
<td>0.32 (0.14, 0.49)</td>
<td>0.60 (0.36, 0.86)</td>
</tr>
<tr>
<td>$\omega$</td>
<td>HS x Post LP</td>
<td>-0.64 (-0.96, -0.33)</td>
<td>-0.68 (-0.99, -0.35)</td>
<td>-0.83 (-1.06, -0.58)</td>
<td>-0.91 (-1.12, -0.71)</td>
</tr>
</tbody>
</table>

Table 6: LP x Segment Results. Posterior Median and 95% HPD

These biases are illustrated in Figure 12. Here we present the posterior distributions of post LP effect by spend segment for each of the two dependent variables. By controlling for the evolving coalition network
structure, we find that the change in the loyalty program shifts all spend predictions up, so much so that for the low spend group the impact of the LP change on in-network spend becomes insignificant. In short, changes in the network structure that lead to decreases in spend were being absorbed into the LP coefficients. After these the evolving network was controlled for using BART, the downward biases were reduced.

![Posterior Distribution of LP x Segment Effects](graph.png)

**Figure 12: LP Effects**

The findings from this section highlight the importance of implementing (or at the very least considering) methods like BART in marketing models. Even in the presence of a natural experiment, there may be numerous, complex ancillary variables that need to be conditioned on and accounted for. In our empirical setting, the coalition loyalty program would fail to see that the change in the LP had no negative impact on the in-network spending from the low segment.

### 5.3 Relative Impact of Network to LP Change

A natural extension to the prior analysis is to determine which is more impactful: a change in the LP design or a change in the network structure? In other words, what type of coalition network reconfiguration would have the same impact as changing the loyalty program? For a coalition loyalty program, there is much more strategic flexibility in being able to select individual partners to add or remove from the coalition (which can be tailored to individual cardmember locations) rather than change the LP design, which affects all cardmembers equally. By using BART, we are able to estimate spend patterns under various network structures using coefficients that would have been intractable to obtain using traditional
econometric approaches alone.

We present the two simplest comparisons: 1) change in expected monthly spend with no partners in the coalition versus all partners in the coalition (assuming the LP already changed) and 2) change in expected monthly spend pre LP and post LP (assuming the average network composition). The estimates are shown in Figure 13. We find that going from an empty to a full network increases estimated monthly spend by about $24,930 (about $40/cardmember per month), but the change in the LP decreases estimated monthly spend by about $33,460 (about $52/cardmember per month). In short, the change in the LP structure was more detrimental than the entire benefits of the coalition network. Note that this is total spend across segments, so even if there was no impact to the in-network spend for low segment cardmembers, the negative effects from the high segment prevailed.

![Figure 13: Impact of Network Structure Relative to LP Change](image)

While this simplistic example gives a sense of the relative impact of the change in the LP structure relative to potential changes in the network structure, it gets us no further in understanding which of the partners may be contributing most to the coalition network. While BART does not provide individual coefficients on each of the partner stores, we can easily characterize the marginal effects by using a partial dependent function approach. Specifically, we evaluated the difference in estimated spend between when a coalition partner is in-network (at its current distance from each cardmember) from when the partner is out-of-network (and thus the geographic influence is 0). We do this for all 2,329 partners in the data, and find individual marginal effects are overwhelmingly negligible. In fact, about over 80% of the locations

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6Since our model is non-linear, the impact of the change in the LP depends on the coalition network structure, and vice versa.
have no effect on monthly spend predictions and drop out of the BART predictions entirely. This may seem unusual given that we previously showed a substantial change in spend when all stores are in the network versus removed, but it reflects a key advantage of the BART model and its ability to manage higher order interactions between individual retail locations. In other words, the effects among stores is not necessarily additive but instead can be synergistic among select partners. Once estimation is complete, identifying these key partners is non-trivial and outside the scope of this paper, as it involves an extensive search routine.

5.4 Marginal Effect of Distance

In this section we further investigate the relationship between partner distance and spend. We first consider how distance to each partner influences spend at an aggregate level. To do this, we set the average customer-store distance as a baseline and then deviate from the average distance to each store at fixed intervals. Figure 14 presents the results when we grow or contract the average distance by increments of 10%. For example, 1 is the current network at the average distance, whereas \(1.1^5\) is the distance after five 10% increases, or a 61% increase relative to the baseline \((1.1^5 = 1.61)\). Since the model contains other components (such as individual fixed effects), in this graph we we simply plot the changes in the BART component \(f(X)\). We see that as distance increases, the log of in-network spend decreases, everything else held constant. On the other hand, out-of-network spend remains relatively constant and somewhat impervious to changes in the network structure.

![Sensitivity to Network Contraction and Expansion](image)

Figure 14: Aggregate Distance Sensitivity

To investigate this phenomenon at the individual level, we select three cardmembers from the dataset,
each with slightly different sensitivities to distance. Each of these cardmembers will have their own fixed
effect, which will account for different baseline spending patterns, and their own regional effect, which will
control for unobserved demand side effects. The predicted spend patterns and their local network structure
(the coalition network within 15km, since most cardmembers do not travel much further), are presented in
Figure 15.

We see that for Cardmember 1, expected monthly spend is around $150 but slowly tapers off as the stores
move further away. This first cardmember has relatively strong spend considering the network is relatively
sparse. This could be a reflection of limited outside spend opportunities. For Cardmember 2, expected
revenue is slightly higher at $200 for the current network, and then increases as the stores contract slightly
but then declines if the stores contract too much. This could reflect a synergy between store locations which
is only maximized when a “sweet spot” is reached in terms of relative distances to each other. Finally,
Cardmember 3 has very low projected spend, even though their network is most dense. Cardmember 3
may be in a region that has a high level of competition, or the composition of stores may not be attractive.
Alternatively, this may reflect what we discussed in section 5.3: that most coalition partners simply do not
have any influence on monthly spending patterns.

These predictions demonstrate the flexibility of incorporating BART into the spatial model. Not only can
we capture the granular spatial relationship between each retail partner and cardmember, we are also able
to estimate very flexible (i.e., non-linear) relationships between the distance to a particular retailer and its
influence on card spend. Recall that this type of analysis is infeasible without a method, such as BART, to
deal with a large number of potentially interacting parameters. While our primary interest is in the effect of
the change in the LP, the coalition can use supplementary analyses like this one to strategically identify
retail partners that are most influential on cardmember spend.
The top graph shows the changes in the expected median total spend as the network around three cardmember from the data expands or contracts. The bottom graph shows the actual network of stores the cardmember has access to, where the graph areas is limited to a 15km radius and the cardmember placed in the center. The latitude and longitude has been normalized to ensure anonymity for the firm. The cardmembers each have very different spend patterns, which do not necessarily correlate strictly with the number of stores nearby. BART identifies which retail partner locations add value to the coalition.

Figure 15: Network Sensitivity for Three Cardmembers

5.5 Identifying Synergies and Non-linearities with BART

In this section we briefly emphasize the advantages of BART in handling higher order interactions and non-linear relationships. As this is slightly tangential to our primary goal of estimating a LP treatment effect we keep this discussion short. One of the advantages of BART is its ability to control for complex relationships.
without the need to commit to a functional form. These complex interactions are either of primary interest to the analyst or instead act as nuisance parameters that need to be accounted for, the latter case analogous to our empirical application where BART handled variables that were ancillary to our main analysis. The modularity of BART allows the analyst to keep a functional specification of interest while offloading the complexities to BART. It is easy to demonstrate the power of BART in its ability to account for high level interactions and nonlinearities, building a tree in such a way to only be as complex as required. Because of this, BART in a way acts as an insurance against potentially complex relationships if it is unclear a priori whether they exist.

We attempted to quantify the presence of partner level interactions within the network. If there are in fact interactions among stores, the marginal effect of adding a store to an empty network would be different than the marginal effect of adding a store to a full network (i.e., all of the potential partners are participating in the coalition). For simplicity, we set the geographic influence to one, thereby replicating a network where a customer has easy access to all stores in the network. In doing so, we identify seven stores whose marginal effects are different in the full network versus the empty network, based on the difference in the posterior medians. Of the 2,329 partners, only 711 unique stores have any influence on monthly spend, and of these we find just 7 that exhibit significant interaction effects. Of course, this is just a rough approximation and illustrates only one approach to identifying the importance of interactions.

Another important feature of BART is its ability to include other non-linear relationships, not simply interactions between variables. For illustrative purposes, in Table 7 below we compare the root mean squared error (RMSE) between two simple specifications: when the geographic influence enters linearly, and when it enters through BART. If the relationship was truly linear, we would expect BART to capture this and thus show no improvement in RMSE, but we find this is not the case. For both dependent variables RMSE improves substantially when the variables are entered via BART versus being constrained to a linear relationship.

<table>
<thead>
<tr>
<th>Linear Model</th>
<th>BART</th>
</tr>
</thead>
<tbody>
<tr>
<td>In-Network Spend</td>
<td>2.49</td>
</tr>
<tr>
<td>Out-of-Network Spend</td>
<td>3.19</td>
</tr>
</tbody>
</table>

Table 7: RMSE Comparison

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7 See section 5.2 from Chipman et al. (2010)
8 See Damien et al. (2013) for an approach that measures interactions based on the shared splitting rules within each tree.
6 Discussion and Conclusion

In this paper we measure the impact of a change in a coalition loyalty program rewards structure on card-member spend. To properly measure this effect, even in the presence of a natural experiment, we needed to control for both unobserved demand side effects at the regional level and observed supply side effects specific to the coalition network structure. We controlled for demand side effects using a CAR prior, much like traditional spatial models. To control for the supply side effects we augmented the spatial model with a BART component. Controlling for both the demand side and supply side effects should lead to less bias in our estimates of interest related to the change in the loyalty program.

Our empirical results show that a simple regression discontinuity and the model with only the CAR prior to account for demand side effects lead to the conclusion that the change in the LP had a negative effect on spend in the low spend segment. However, when we control for the evolving coalition network this effect is reduced to zero, suggesting that the decrease in spend after the loyalty program change was likely influenced by an undesirable change in the coalition network. Our empirical findings are simply one of many potential examples where a marketing analyst is faced with a large number of potentially complex and interacting variables that are too cumbersome to specify using a traditional functional form. By augmenting our model with BART, we were able to deal with these control variables while still retaining a specified functional form on our coefficients of interest. In other words, we did not need to send the whole analysis through a machine learning “black box” – BART was used only for variables that we suspected would influence the outcome variable but which were of secondary interest.

While our primary goal was to estimate the impact of the change in the LP on spend, we also hoped to illustrate the potential of augmenting models with BART, or similar methods, so that marketers can accommodate evermore complex modeling situations.
References


Appendix 1: Markov Chain Monte Carlo Estimation

We carried out estimation sequentially generating draws from the following distributions. With the exception of the SUR components, the drawing process between in-network spend ($y^I$) and out-of-network spend ($y^O$) spend is nearly identical. The outline below refers to the in-network spend draws.

1. Update BART
   
   (a) Create $y^*_{it} = y_{it} - \theta - W_i \Delta - \frac{\sigma_{OI}}{\sigma_{OO}} e^O_i$ where $e^O$ is the residuals between the predicted out-of-network spend and augmented data $y^O$ and $W_i$ is the matrix specific to cardmember $i$ containing all the fixed effects, seasonality variables $S$, spend group $G$, post-LP indicator $Z$, and the interaction $GZ$.
   
   (b) Estimate function BART $f()$ in $y^* = f(X)$

2. Update missing data $z$

   (a) Create $y^*_{it} = f(X_{it}) + \theta + W_i \Delta - \frac{\sigma_{OI}}{\sigma_{OO}} e^O_i$

   (b) For observations with missing data in $y_{it}$ (e.g., zero in-network spend), draw $f(z|\cdot) \sim$ Truncated Normal ($y^*, -\infty, 0$)

   (c) Replace the missing data in $y_{it}$ with the draws from $z$

3. Update $\Sigma$

   (a) Update $e^O_{it} = \hat{y}^O_{it} - y^O_{it}$ and $e^I_{it} = \hat{y}^I_{it} - y^I_{it}$ and join the columns into $E$

   (b) Draw $p(\Sigma|\cdot) \sim$ $IW\left(\nu_\Sigma + n, \left(E' E + V\right)^{-1}\right)$

   (c) $\nu_\Sigma = 3$ and $V$ is a $2 \times 2$ identity matrix multiplied by $\nu_\Sigma$

4. Update regional effects $\theta$

   (a) Create $y^*_{it} = y_{it} - f(X_{it}) - W_i \Delta$

   (b) Set $\theta_\Sigma = \left[\frac{U' U}{\sigma_{xx}} + \frac{(H - \rho C)}{\sigma^2}\right]^{-1}$ where $U$ is an indicator design matrix of size $I \times A$ (number of cardmembers by number of regions)

   (c) Set $\theta_\mu = \theta_\sigma \left(\frac{U' y^*}{\sigma_{xx}}\right)$

   (d) Draw $p(\theta|\cdot) \sim$ MVN ($\theta_\mu, \theta_\Sigma$)

5. Update regional variance $\delta^2$

   (a) $p(\delta^2|\cdot) \sim$ IG ($a, b$)

   (b) $a = a_2 + A/2$ with $a_2 = .001$ and $A = 26$ regions

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(c) \( b = b_2 + \left( \theta' B' \theta B \right) \) where \( B = (H - \rho C) \) and \( b_2 = .001 \)

6. Update regional correlation \( \rho \) using Metropolis-Hastings algorithm with a random walk, similar to Yang and Allenby (2003)

(a) Let the proposed draw be given by \( \rho^{(n)} = \rho^{(p)} + \epsilon \) where \( \rho^{(p)} \) is the previous draw and \( \epsilon \) is a draw from the normal density with mean 0 and variance .0025

(b) Accept the proposed draw with the following probability \( \alpha = \min \left\{ \frac{|B(\rho^{(n)})|^2 \exp \left[ -0.5 (1/\sigma^2) \theta' B(\rho^{(n)})' \theta \right]}{|B(\rho^{(p)})|^2 \exp \left[ -0.5 (1/\sigma^2) \theta' B(\rho^{(p)})' \theta \right]}, 1 \right\} \)

where \( B(\rho) = H - \rho C \) and the probability of acceptance is zero if \( \rho^{(n)} \) is outside the range of \( [\lambda_{\text{min}}, \lambda_{\text{max}}] \)

7. Update all other coefficients \( \Delta \)

(a) Create \( y_{it}^* = y_{it} - f(X_{it}) - \theta_a \)

(b) Estimate \( \Delta \) from model of \( y_{it}^* = W_i \Delta + \epsilon \) where \( \epsilon \sim N(0, \sigma^2) \) where \( \sigma^2 \) is already drawn in \( \Sigma \)

(c) Draw \( p(\Delta|\cdot) \sim N(\bar{\Delta}, V_\Delta) \) where \( V_\Delta \) equals the inverse of an identity matrix times .01, with number of columns equal to that of \( W \)
Appendix 2: Prior Specification

We estimate the conditional spatial BART model using Markov chain Monte Carlo methods. The method of estimation requires specification of prior distributions for the model parameters and derivation of the full conditional distributions. We set the priors to be diffuse and use conjugate priors when possible. First, we complete the Bayesian hierarchical model with priors \(p(f), p(\theta), p(\Delta)\) for \(f, \theta,\) and \(\Delta\), respectively. We save our discussion of \(p(\sigma^2)\) for later when we introduce the SUR structure. We assume a priori independence.

The BART model is indexed by \(\{(T_j, M_j), j = 1, \ldots m\}\). We then have:

\[
p(f) = \prod_{j=1}^{m} p(T_j, M_j) = \prod_{j=1}^{m} \{p(T_j) \cdot p(M_j|T_j)\}
\]

(7)

As in Chipman et al. (2010), we define \(p(T_j)\) by three factors, corresponding to a node being non-terminal, the selection of the splitting variable for the a non-terminal node, and the choice of the splitting value conditional on a chosen splitting variable. The probability that a node at depth \(d\) is non-terminal is assumed to be

\[
\alpha (1 + d)^{-\gamma}
\]

(8)

where \(\alpha \in (0, 1)\) and \(\gamma \in [0, \infty)\) are two hyperparameters reflecting the prior belief about the tree. If the depth of the tree is believed to be small, a large value is assigned for \(\gamma\), so that the probability decays fast with \(d\). We use the proposed \(\alpha = 0.95\) and \(\gamma = 2\) as default values, which implies that with prior probability 0.05, 0.55, 0.38, 0.09 and 0.03 the tree as 1, 2, 3, 4, and 5 or more terminal nodes, respectively.

A natural choice for the selection of the splitting variable, conditional on a node being non-terminal, is a uniform prior over all available variances. A default choice for the distribution of the splitting value is a uniform choice of available splitting values.

We also define a prior for \(M_j\). Let \(M_{jk}\) be the \(k\)th element of \(M_j\). Conditional on \(T_j\), we assume i.i.d. normal priors for \(M_{jk}\). The mean and variance of the normal prior are specified in such a way that each tree is a weak learner, and each individual tree plays a small role in the overall fit. See section 3.2 of Chipman et al. (2010) for more details.

For all other coefficients \(\Delta\) we use a multivariate normal prior:

\[
\Delta \sim MVN(\bar{\Delta}, V_\Delta)
\]

(9)

We complete the prior model with probability models for the hyperparameters \(\rho, \delta^2, \bar{\Delta},\) and \(V_\Delta\). We
assume $p(\delta^2)$ to be an inverse Gamma distribution, denoted by $\text{IG}(a, b)$, with density function

$$p(\delta^2) \propto \frac{1}{(\delta^2)^{a+1}} \exp\left(-\frac{b}{\delta^2}\right)$$

(10)

where $a$ and $b$ are fixed hyperparameters. Finally, the hyperparameters on the demographic effects $\tilde{\Delta}$ and $V_\Delta$ are fixed.

The Markov chain proceeds by generating draws from the set of conditional posterior distributions of the parameters (Gelfand and Smith, 1990). After subtracting out the spatial random effects $\theta_\alpha$ and demographic effects $\Delta$, from $y_{it}$ the estimation of $f$ is a standard BART model. The conditional distributions of the remaining model parameters, given $f$, are of standard form. A detailed description of the full conditional distributions is provided in the Appendix.
## Appendix 3: Full Model Results

### Table 8: In-Network Partial Model Results

<table>
<thead>
<tr>
<th>Coefficient</th>
<th>Model 1</th>
<th>Model 2</th>
<th>Model 3</th>
<th>Model 4</th>
<th>Model 5</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\sigma$</td>
<td>6.00 (5.90, 6.09)</td>
<td>6.00 (5.90, 6.10)</td>
<td>6.00 (5.90, 6.10)</td>
<td>6.00 (5.89, 6.11)</td>
<td>5.32 (5.23, 5.42)</td>
<td>model variance</td>
</tr>
<tr>
<td>$\gamma_1$</td>
<td>-0.62 (-0.97, -0.23)</td>
<td>-0.69 (-1.07, -0.31)</td>
<td>-0.67 (-1.05, -0.30)</td>
<td>-0.68 (-1.05, -0.30)</td>
<td>-1.04 (-1.42, -0.67)</td>
<td>January</td>
</tr>
<tr>
<td>$\gamma_2$</td>
<td>-1.30 (-1.77, -1.02)</td>
<td>-1.44 (-1.84, -1.07)</td>
<td>-1.43 (-1.80, -1.06)</td>
<td>-1.44 (-1.81, -1.05)</td>
<td>-2.30 (-2.65, -1.90)</td>
<td>February</td>
</tr>
<tr>
<td>$\gamma_3$</td>
<td>-0.97 (-1.33, -0.60)</td>
<td>-1.02 (-1.39, -0.63)</td>
<td>-1.01 (-1.38, -0.63)</td>
<td>-1.02 (-1.37, -0.64)</td>
<td>-1.39 (-1.96, -1.24)</td>
<td>March</td>
</tr>
<tr>
<td>$\gamma_4$</td>
<td>-0.77 (-1.15, -0.41)</td>
<td>-0.83 (-1.20, -0.43)</td>
<td>-0.81 (-1.18, -0.41)</td>
<td>-0.82 (-1.22, -0.47)</td>
<td>-1.26 (-1.60, -0.89)</td>
<td>April</td>
</tr>
<tr>
<td>$\gamma_5$</td>
<td>-0.84 (-1.22, -0.48)</td>
<td>-0.90 (-1.28, -0.54)</td>
<td>-0.88 (-1.27, -0.51)</td>
<td>-0.90 (-1.27, -0.51)</td>
<td>-1.24 (-1.63, -0.86)</td>
<td>May</td>
</tr>
<tr>
<td>$\gamma_6$</td>
<td>-0.79 (-1.15, -0.42)</td>
<td>-0.85 (-1.23, -0.46)</td>
<td>-0.84 (-1.19, -0.44)</td>
<td>-0.84 (-1.19, -0.43)</td>
<td>-1.12 (-1.49, -0.73)</td>
<td>June</td>
</tr>
<tr>
<td>$\gamma_7$</td>
<td>-0.63 (-1.00, -0.24)</td>
<td>-0.69 (-1.05, -0.30)</td>
<td>-0.67 (-1.05, -0.27)</td>
<td>-0.68 (-1.05, -0.31)</td>
<td>-0.87 (-1.25, -0.49)</td>
<td>July</td>
</tr>
<tr>
<td>$\gamma_8$</td>
<td>-1.17 (-1.57, -0.80)</td>
<td>-1.24 (-1.61, -0.87)</td>
<td>-1.21 (-1.61, -0.85)</td>
<td>-1.22 (-1.59, -0.83)</td>
<td>-1.77 (-2.16, -1.37)</td>
<td>August</td>
</tr>
<tr>
<td>$\gamma_9$</td>
<td>-0.82 (-1.19, -0.46)</td>
<td>-0.88 (-1.26, -0.51)</td>
<td>-0.86 (-1.25, -0.48)</td>
<td>-0.87 (-1.27, -0.52)</td>
<td>-1.16 (-1.55, -0.79)</td>
<td>September</td>
</tr>
<tr>
<td>$\gamma_{10}$</td>
<td>-0.93 (-1.31, -0.56)</td>
<td>-0.99 (-1.39, -0.63)</td>
<td>-0.97 (-1.36, -0.59)</td>
<td>-0.98 (-1.37, -0.62)</td>
<td>-1.20 (-1.57, -0.80)</td>
<td>October</td>
</tr>
<tr>
<td>$\gamma_{11}$</td>
<td>-0.84 (-1.21, -0.50)</td>
<td>-0.90 (-1.26, -0.55)</td>
<td>-0.83 (-1.18, -0.48)</td>
<td>-0.83 (-1.22, -0.50)</td>
<td>-1.12 (-1.47, -0.78)</td>
<td>November</td>
</tr>
<tr>
<td>$\kappa$</td>
<td>1.44 (0.28, 2.55)</td>
<td>1.75 (0.60, 2.92)</td>
<td>1.90 (0.75, 3.07)</td>
<td>2.79 (1.46, 4.23)</td>
<td>Customer Group</td>
<td></td>
</tr>
<tr>
<td>$\zeta$</td>
<td>-0.64 (-0.80, -0.49)</td>
<td>-0.32 (-0.56, -0.10)</td>
<td>-0.07 (-0.37, 0.27)</td>
<td>Group x LP</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$\omega$</td>
<td>-0.64 (-0.96, -0.33)</td>
<td>-0.68 (-0.99, -0.33)</td>
<td>-0.99 (-0.96, -0.33)</td>
<td>-0.99 (-0.96, -0.33)</td>
<td>Group x LP</td>
<td></td>
</tr>
<tr>
<td>$\delta$</td>
<td>11.75 (8.92, 15.74)</td>
<td>11.72 (8.60, 15.44)</td>
<td>11.65 (8.51, 15.29)</td>
<td>11.76 (8.79, 15.65)</td>
<td>2.84 (1.76, 4.11)</td>
<td>regional SD</td>
</tr>
<tr>
<td>$\rho$</td>
<td>0.16 (0.05, 0.19)</td>
<td>0.16 (0.06, 0.19)</td>
<td>0.16 (0.06, 0.19)</td>
<td>0.16 (0.05, 0.19)</td>
<td>0.07 (-0.33, 0.17)</td>
<td>reg. spillover</td>
</tr>
</tbody>
</table>
### Table 9: Out-Of-Network Partial Model Results

<table>
<thead>
<tr>
<th>Coefficient</th>
<th>Model 1</th>
<th>Model 2</th>
<th>Model 3</th>
<th>Model 4</th>
<th>Model 5</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>σ</td>
<td>4.99 (4.93, 5.05)</td>
<td>4.99 (4.92, 5.04)</td>
<td>4.99 (4.93, 5.05)</td>
<td>4.99 (4.93, 5.05)</td>
<td>4.00 (3.96, 4.05)</td>
<td>model variance</td>
</tr>
<tr>
<td>γ&lt;sub&gt;1&lt;/sub&gt;</td>
<td>-0.47 (-0.75, -0.18)</td>
<td>-0.51 (-0.79, -0.21)</td>
<td>-0.51 (-0.81, -0.23)</td>
<td>-0.52 (-0.82, -0.24)</td>
<td>-0.54 (-0.77, -0.28)</td>
<td>January</td>
</tr>
<tr>
<td>γ&lt;sub&gt;2&lt;/sub&gt;</td>
<td>-0.67 (-0.96, -0.38)</td>
<td>-0.71 (-1.00, -0.43)</td>
<td>-0.72 (-1.02, -0.43)</td>
<td>-0.72 (-0.99, -0.41)</td>
<td>-0.67 (-0.94, -0.42)</td>
<td>February</td>
</tr>
<tr>
<td>γ&lt;sub&gt;3&lt;/sub&gt;</td>
<td>-0.30 (-0.61, -0.04)</td>
<td>-0.35 (-0.64, -0.07)</td>
<td>-0.35 (-0.63, -0.05)</td>
<td>-0.36 (-0.64, -0.06)</td>
<td>-0.27 (-0.52, -0.01)</td>
<td>March</td>
</tr>
<tr>
<td>γ&lt;sub&gt;4&lt;/sub&gt;</td>
<td>-0.09 (-0.38, 0.21)</td>
<td>-0.13 (-0.42, 0.16)</td>
<td>-0.14 (-0.41, 0.17)</td>
<td>-0.14 (-0.43, 0.16)</td>
<td>-0.02 (-0.25, 0.24)</td>
<td>April</td>
</tr>
<tr>
<td>γ&lt;sub&gt;5&lt;/sub&gt;</td>
<td>0.02 (-0.26, 0.32)</td>
<td>-0.02 (-0.30, 0.28)</td>
<td>-0.03 (-0.32, 0.27)</td>
<td>-0.03 (-0.34, 0.25)</td>
<td>0.12 (-0.13, 0.38)</td>
<td>May</td>
</tr>
<tr>
<td>γ&lt;sub&gt;6&lt;/sub&gt;</td>
<td>-0.09 (-0.39, 0.20)</td>
<td>-0.14 (-0.44, 0.14)</td>
<td>-0.15 (-0.42, 0.16)</td>
<td>-0.15 (-0.44, 0.14)</td>
<td>-0.02 (-0.27, 0.23)</td>
<td>June</td>
</tr>
<tr>
<td>γ&lt;sub&gt;7&lt;/sub&gt;</td>
<td>0.36 (0.10, 0.67)</td>
<td>0.32 (0.03, 0.61)</td>
<td>0.32 (0.02, 0.61)</td>
<td>0.31 (-0.01, 0.39)</td>
<td>0.51 (0.27, 0.77)</td>
<td>July</td>
</tr>
<tr>
<td>γ&lt;sub&gt;8&lt;/sub&gt;</td>
<td>0.08 (-0.20, 0.37)</td>
<td>0.04 (-0.25, 0.33)</td>
<td>0.03 (-0.23, 0.34)</td>
<td>0.03 (-0.26, 0.31)</td>
<td>0.19 (-0.07, 0.43)</td>
<td>August</td>
</tr>
<tr>
<td>γ&lt;sub&gt;9&lt;/sub&gt;</td>
<td>-0.07 (-0.36, 0.23)</td>
<td>-0.12 (-0.41, 0.18)</td>
<td>-0.12 (-0.42, 0.17)</td>
<td>-0.12 (-0.42, 0.17)</td>
<td>0.00 (-0.24, 0.26)</td>
<td>September</td>
</tr>
<tr>
<td>γ&lt;sub&gt;10&lt;/sub&gt;</td>
<td>0.15 (-0.12, 0.46)</td>
<td>0.11 (-0.19, 0.39)</td>
<td>0.10 (-0.18, 0.41)</td>
<td>0.10 (-0.18, 0.41)</td>
<td>0.27 (0.00, 0.51)</td>
<td>October</td>
</tr>
<tr>
<td>γ&lt;sub&gt;11&lt;/sub&gt;</td>
<td>-0.37 (-0.65, -0.10)</td>
<td>-0.41 (-0.70, -0.15)</td>
<td>-0.41 (-0.66, -0.12)</td>
<td>-0.41 (-0.68, -0.13)</td>
<td>-0.38 (-0.63, -0.15)</td>
<td>November</td>
</tr>
<tr>
<td>κ</td>
<td>1.61 (0.49, 2.76)</td>
<td>1.67 (0.56, 2.81)</td>
<td>1.89 (0.70, 2.97)</td>
<td>1.89 (0.70, 2.97)</td>
<td>4.20 (2.44, 5.86)</td>
<td>Customer Group</td>
</tr>
<tr>
<td>ζ</td>
<td>-0.10 (-0.21, 0.03)</td>
<td>-0.32 (0.14, 0.49)</td>
<td>0.32 (0.14, 0.49)</td>
<td>0.32 (0.14, 0.49)</td>
<td>0.60 (0.36, 0.86)</td>
<td>Post LP</td>
</tr>
<tr>
<td>ω</td>
<td>-0.83 (-1.06, -0.58)</td>
<td>-0.91 (-1.12, -0.71)</td>
<td>-0.91 (-1.12, -0.71)</td>
<td>-0.91 (-1.12, -0.71)</td>
<td>-0.91 (-1.12, -0.71)</td>
<td>Group x LP</td>
</tr>
<tr>
<td>δ</td>
<td>6.95 (5.10, 9.22)</td>
<td>6.97 (5.12, 9.19)</td>
<td>6.97 (5.15, 9.31)</td>
<td>6.97 (5.15, 9.31)</td>
<td>6.90 (5.16, 9.28)</td>
<td>regional SD</td>
</tr>
<tr>
<td>ρ</td>
<td>0.15 (0.05, 0.19)</td>
<td>0.15 (0.05, 0.19)</td>
<td>0.16 (0.05, 0.19)</td>
<td>0.16 (0.04, 0.19)</td>
<td>0.16 (0.04, 0.19)</td>
<td>reg. spillover</td>
</tr>
</tbody>
</table>