1. (10 points) Suppose $S$ is a nonempty and bounded subset of $\mathbb{R}$. Show that

$$\inf(S) = -\sup(-S)$$
2. (10 points)

Suppose \((s_n)\) and \((t_n)\) are convergent sequences in \(\mathbb{R}\) with \(s_n \to s\) and \(t_n \to t\). Show using the definition of a limit that

\[
\lim_{n \to \infty} s_n - t_n = s - t
\]
3. \( (10 = 8+2 \text{ points}) \)

(a) Show that if \( (s_n) \) is a bounded sequence in \( \mathbb{R} \) and \( k \geq 0 \), then

\[
\limsup_{n \to \infty} ks_n = k \left( \limsup_{n \to \infty} s_n \right)
\]

(b) Is the same result true if you replace \( k \geq 0 \) by \( k \in \mathbb{R} \)?
4. (10 = 4 + 4 + 2 points) Let $(S,d)$ be a metric space

(a) Show that the union of any number of open subsets of $S$ is open

(b) Show that the intersection of finitely many open subsets of $S$ is open

(c) Is the intersection of infinitely many open subsets of $S$ always open?

**Suggestion:** I *highly* recommend also checking out the metric space problems on Mock Midterm 2.
5. (10 points) Prove the integral test in the case of

$$\sum_{n=1}^{\infty} \frac{1}{\sqrt{n}}$$
6. (10 points) Suppose $f$ is a continuous function on $(-1, 1)$ that satisfies $f(x^2) = f(x)$ for all $x$. Show that $f$ is constant.

**Hint:** Show that $f(x) = f(0)$ for all $x$. For this, calculate $f(x^4), f(x^8)$, and, more generally, $f(x^{2^n})$ for $n \geq 0$. 


7. (10 points) Suppose $y \geq 0$ is given. Show that there is a unique $x \geq 0$ such that $y = x^2$. 
8. (10 points) Show directly, using the definition of uniform continuity that \( f(x) = x^2 \) is not uniformly continuous on \([0, \infty)\)

**Hint:** This is similar to (but easier than) Example 3 in Lecture 27. Let \( \epsilon > 0 \) be TBA and fix \( \delta \). Let \( a = |x - y| \) and assume \( x < y \).