1.

**STEP 1:** First of all, if \( s \in A + B \), then \( s = a + b \) where \( a \in A \) and \( b \in B \), but by definition of \( \sup(A) \) we get \( a \leq \sup(A) \) and similarly \( b \leq \sup(B) \), hence

\[
s = a + b \leq \sup(A) + \sup(B)
\]

Since \( s \) was arbitrary, \( \sup(A) + \sup(B) \) is an upper bound for \( A + B \), so because \( \sup(A + B) \) is the *least* upper bound for \( A + B \), we get

\[
\sup(A + B) \leq \sup(A) + \sup(B) \checkmark
\]

**STEP 2:** Fix \( a \in A \), then for every \( b \in B \), since \( a + b \in A + B \) and by definition of \( \sup(A + B) \), we get:

\[
a + b \leq \sup(A + B)
\]

\[
a \leq \sup(A + B) - b
\]

But since \( a \in A \) is arbitrary, \( \sup(A + B) - b \) is an upper bound for \( A \), and hence since \( \sup(A) \) is the *least* upper bound:

\[
\sup(A) \leq \sup(A + B) - b
\]

\[
b \leq \sup(A + B) - \sup(A)
\]
But since \( b \in B \) is arbitrary, \( \sup(A + B) - \sup(A) \) is an upper bound for \( B \), so since \( \sup(B) \) is the \textit{least} upper bound:

\[
\sup(B) \leq \sup(A + B) - \sup(A) \\
\sup(A) + \sup(B) \leq \sup(A + B) \vee
\]

Therefore \( \sup(A + B) = \sup(A) + \sup(B) \). \( \square \)
2.

**STEP 1: Scratchwork**

Since \((s_n)\) converges, \((s_n)\) is bounded above, so there is \(M > 0\) such that \(|s_n| \leq M\) for all \(n\).

\[
\left| (s_n)^2 - s^2 \right| = |s_n - s| |s_n + s| \\
\leq |s_n - s| (|s_n| + |s|) \\
\leq |s_n - s| (M + |s|) \\
< \epsilon
\]

Which gives:

\[
|s_n - s| < \frac{\epsilon}{M + |s|}
\]

**STEP 2: Actual Proof**

First of all, since \((s_n)\) converges, \((s_n)\) is bounded, so there is \(M > 0\) such that \(|s_n| \leq M\) for all \(n\).

Let \(\epsilon > 0\) be given

Then since \(s_n \to s\) there is \(N\) such that for all \(n > N\), \(|s_n - s| < \frac{\epsilon}{M + |s|}\)

With that same \(N\), if \(n > N\), we get:
\[ |(s_n)^2 - s^2| = |s_n - s| |s_n + s| \]
\[ \leq |s_n - s| (|s_n| + |s|) \]
\[ \leq |s_n - s| (M + |s|) \]
\[ < \left( \frac{\epsilon}{M + |s|} \right) (M + |s|) \]
\[ = \epsilon \sqrt{ } \]

Therefore \((s_n)^2\) converges to \(s^2\)
3.

Suppose by contradiction that \( \text{sup}(B) = M \) where \( M < \infty \). Since \( B \) has at least one positive term, we may assume \( M > 0 \).

Now consider \( M_1 = \frac{M}{2} < M \) (since \( M > 0 \)). By definition of \( \text{sup} \) this means there is \( 2^n \in B \) such that \( 2^n > \frac{M}{2} \), which implies \( M < 2^{n+1} \).

But this contradicts the fact that \( M \) is an upper bound for \( B \), so all \( n \in \mathbb{N} \), \( 2^n \leq M \) \( \Rightarrow \Leftrightarrow \) \( \square \)
4. **Scratchwork:** Notice that $3 = 1 + 2$, so by the binomial theorem, we get:

\[
3^n = (1 + 2)^n = 1^n + n1^{n-1}2 + \text{POSITIVE JUNK} = 1 + 2n + \text{POSITIVE JUNK} > 2n > M
\]

Which suggests $N = \frac{M}{2}$.

**Actual Proof:** Let $M > 0$ be given and let $N = \frac{M}{2}$. Then if $n > N$, we have:

\[
3^n = (1 + 2)^n = 1 + 2n + \text{POSITIVE JUNK} > 2n > 2\left(\frac{M}{2}\right) = M \checkmark
\]

Therefore $\lim_{n \to \infty} 3^n = \infty$