1. (10 points)
Let \((s_n)\) be the sequence defined by \(s_1 = 2\) and, for all \(n \in \mathbb{N}:\)

\[ s_{n+1} = \sqrt{2 + s_n} \]

Use induction on \(n\) to show that \(s_n \leq 2\) for all \(n \in \mathbb{N}\)
2. (10 points)

Use contradiction to prove that for every $x \in \mathbb{R}$ there is an integer $k \in \mathbb{Z}$ such that $k > x$.

Do not use the Archimedean property!

**Hint:** At some point you might consider $M - 1$ (for some $M$).
3. (10 points)

Let $f$ be a function with the following property: There is $C > 0$ such that, for all $a$ and $b$, $|f(b) - f(a)| \leq C |b - a|$

Let $(s_n)$ be a sequence in $\mathbb{R}$ that converges to $s$. Show that $f(s_n)$ converges to $f(s)$. Include your scratchwork.

**Note:** $f(s_n)$ means “$f$ of $s_n$” and $f(s)$ means “$f$ of $s$”
4. (10 points)

Let \((s_n)\) be a sequence in \(\mathbb{R}\) that is bounded above by \(M\). Suppose moreover that \((s_n)\) converges to \(M\). Show that

\[
\sup \{s_n \mid n \in \mathbb{N}\} = M
\]