

## MATH 2E REVIEW FOR FINAL

The final is in the usual classroom, Wed, December 12, 1:30pm – 3:30pm, 8–9 problems, covering Chapter 15 and 16 of Stewart calculus, no notes.

### Chapter 15.

- (1) Calculate  $\iint_R ye^{xy}dA$ , where  $R = \{(x, y) \mid 0 \leq x \leq 2, 0 \leq y \leq 3\}$ .
- (2) Calculate  $\int_0^1 \int_{\sqrt{y}}^1 \frac{ye^{x^2}}{x^3} dx dy$ .
- (3) Calculate  $\iiint_E z dV$ , where  $E$  is bounded by the planes  $y = 0$ ,  $z = 0$ ,  $x + y = 2$  and the cylinder  $y^2 + z^2 = 1$  in the first octant.
- (4) Calculate  $\iiint_E yz dV$  where  $E$  lies above the plane  $z = 0$ , below the plane  $z = y$ , and inside the cylinder  $x^2 + y^2 = 4$ .
- (5) Calculate  $\iiint_H z^3 \sqrt{x^2 + y^2 + z^2} dV$ , where  $H$  is the solid hemisphere that lies above the  $xy$ -plane and has center the origin and radius 1.
- (6) Evaluate  $\iint_R \frac{x-y}{x+y} dA$  where  $R$  is the square with vertices  $(0, 2)$ ,  $(1, 1)$ ,  $(2, 2)$  and  $(1, 3)$ .
- (7) Find the volume of the region bounded by the surface  $\sqrt{x} + \sqrt{y} + \sqrt{z} = 1$  and the coordinate planes. Consider the transformation  $x = u^2$ ,  $y = v^2$ , and  $z = w^2$ .
- (8) Evaluate  $\iint_R xy dA$ , where  $R$  is the square with vertices  $(0, 0)$ ,  $(1, 1)$ ,  $(2, 0)$ , and  $(1, -1)$ .
- (9) Given a curve  $r(t) = \langle 1 + t, t^2, t^3 \rangle$ , find the area of the triangle with vertices  $r(-1)$ ,  $r(1)$  and  $r(0)$ .

### Chapter 16.

- (1) Evaluate  $\int_C x ds$ , where  $C$  is the arc of the parabola  $y = x^2$  from  $(0, 0)$  to  $(1, 1)$ .
- (2) Evaluate  $\int_C y dx + (x + y^2) dy$ ,  $C$  is the ellipse  $4x^2 + 9y^2 = 36$  with counter clockwise orientation.
- (3) Evaluate  $\int_C F \cdot dr$ , where  $F = \langle \sqrt{xy}, e^y, xz \rangle$ ,  $C$  is given by  $r(t) = \langle t^4, t^2, t^3 \rangle$ ,  $0 \leq t \leq 1$ .
- (4) Compute  $\text{curl } F$  where  $F = \langle e^y, xe^y + e^z, ye^z \rangle$ . Then compute the line integral  $\int_C F \cdot dr$  where  $C$  is **any** curve from  $(0, 2, 0)$  to  $(4, 0, 3)$ . Hint: fundamental theorem of line integrals.
- (5) Verify Green's theorem is true for the line integral  $\int_C xy^2 dx - x^2 y dy$ , where  $C$  consists of the parabola  $y = x^2$  from  $(-1, 1)$  to  $(1, 1)$  and the line segment from  $(1, 1)$  to  $(-1, 1)$ .
- (6) Find the area of the part of the surface  $z = x^2 + 2y$  that lies above the triangle with vertices  $(0, 0)$ ,  $(1, 0)$  and  $(1, 2)$ .
- (7) Find an equation of the tangent plane at the point  $(4, -2, 1)$  to the parametric surface  $S$  given by  $r(u, v) = \langle v^2, -uv, u^2 \rangle$ ,  $0 \leq u \leq 3$ ,  $-3 \leq v \leq 3$ .

- (8) Evaluate  $\iint_S zdS$  and  $\iint_S xdS$  where  $S$  is the part of the paraboloid  $z = x^2 + y^2$  that lies under the plane  $z = 4$ .
- (9) Evaluate  $\iint_S x^2z + y^2zdS$ , where  $S$  is the part of the plane  $z = 4 + x + y$  that lies inside the cylinder  $x^2 + y^2 = 4$ .
- (10) Evaluate  $\iint_S F \cdot dS$  where  $F = \langle xz, -2y, 3x \rangle$  and  $S$  is the sphere  $x^2 + y^2 + z^2 = 4$  with outward orientation.
- (11) Verify Stokes' theorem is true for  $F = \langle x^2, y^2, z^2 \rangle$ , where  $S$  is the part of the paraboloid  $z = 1 - x^2 - y^2$  that lies above the  $xy$ -plane and  $S$  has upward orientation.
- (12) Evaluate  $\int_C F \cdot dr$  where  $F = \langle xy, yz, zx \rangle$  and  $C$  is the triangle with vertices  $(1, 0, 0)$ ,  $(0, 1, 0)$  and  $(0, 0, 1)$ , oriented counter clockwise as viewed from above.
- (13) Calculate  $\iint_S F \cdot dS$  where  $F = \langle x^3, y^3, z^3 \rangle$  and  $S$  is the surface of the solid bounded by the cylinder  $x^2 + y^2 = 1$  and the planes  $z = 0$  and  $z = 2$ .
- (14) Compute the outward flux of  $F = \left\langle \frac{x}{(x^2+y^2+z^2)^{\frac{3}{2}}}, \frac{y}{(x^2+y^2+z^2)^{\frac{3}{2}}}, \frac{z}{(x^2+y^2+z^2)^{\frac{3}{2}}} \right\rangle$  through the ellipsoid  $4x^2 + 9y^2 + 6z^2 = 36$ .
- (15) Compute  $\int_C F \cdot dr$  where  $F = \left\langle \frac{2x^3+2xy^2-2y}{x^2+y^2}, \frac{2y^3+2xy+2x}{x^2+y^2} \right\rangle$  around any simple closed curve containing the origin  $(0, 0)$ .
- (16) Find the positively oriented simple closed curve  $C$  for which the value of the line integral  $\int_C (y^3 - y)dx - 2x^3dy$  is a maximum.