Identification of Gain Mismatches in Control Electronics of Rate Integrating CVGs

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***Abstract*—In Coriolis Vibratory Rate Integrating Gyroscopes (CVRIG), accuracy of angle measurement is known to be coupled to symmetry of the mechanical structure. This paper provides a study on the effect of asymmetries in control electronics on operation and accuracy of direct angle measurements. We demonstrated that gain mismatch in detection electronics affects the estimation of the pendulum variables in the CVRIG mathematical model. An error in the pendulum variables was shown to adversely affect the estimated orientation of the orbital trajectory and the closed-loop control. In the case of gain mismatch in actuation electronics, the control forces were observed to interfere with free precession of the oscillation pattern causing additional errors in the angle measurement. We proposed a method to distinguish the angle errors due to mechanical asymmetries from the angle errors caused by imperfections in control electronics. Using the method, we identified gain mismatches in the control electronics and subsequently used the identified parameters for calibration of a micro-fabricated gyroscope. By applying the method of calibration to a Dual Foucault Pendulum (DFP) gyroscope, we were able to reduce the angle bias error by 10-times and reached a 0.06 degree of precession accuracy at the input angular rate of 500 dps, without any compensation for mechanical asymmetries.**

I. Introduction

Rate Integrating Gyroscopes (RIG), as compared to Rate Gyroscopes (RG), provide a direct measurement of the instantaneous orientation of an object. The orientation is utilized in an Inertial Navigation System (INS) for positioning and other dead-reckoning applications, [1]. By having a direct measurement of orientation in a strap-down navigation system, time-integration of error in angular rate measurements is avoided and accumulation of position error is reduced. To achieve high accuracy of measurements with Coriolis Vibratory Rate Integrating Gyroscopes (CVRIG), strict requirements exist on structural symmetry and quality factor of the mechanical structure, [2,3]. Recently, multiple micro-scale resonators have been realized that demonstrated high structural symmetry (on the order of mHz) and a high quality factor (on the order of several million), [4-9].

At the current level of maturity in development of micro-mechanical structures, the effect of imperfections in the CVRIG control electronics on the accuracy of angle measurement becomes highly relevant. In this paper, we study the effect of mismatch in actuation gains (i.e., $G_{Ax} \neq G_{Ay}$) and detection gains (i.e., $G_{Dx} \neq G_{Dy}$) on precession of a CVRIG, illustrated in Fig. 1.

This material is based on work supported by the Defense Advanced Research Projects Agency (DARPA) and U.S. Navy under Contract No. N66001-16-1-4021.

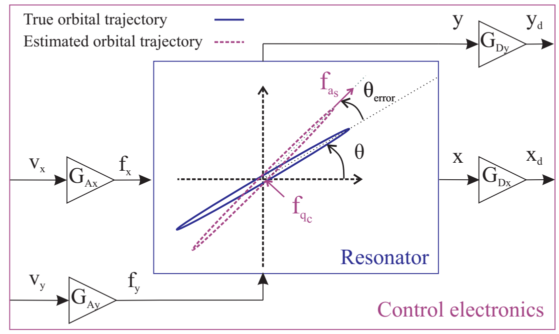

Fig. 1. Schematics of a Coriolis vibratory RIG illustrating the mechanical resonator and the control electronics. Parameters G_{Ax}, G_{Ay}, G_{Dx}, and G_{Dy} denote non-identical gains, along X and Y axes, for voltage-to-force and displacement-to-voltage conversion.

In a CVG instrumented for the Whole Angle (WA) mode of operation, the 2-dimensional oscillation pattern follows an elliptical orbit, [2,3]. By applying rotation to the sensing element, the elliptical oscillation pattern precesses relative to the sensor package, in a direction opposite to the applied rotation. Variations in the pattern angle have been shown to be proportional to the input angle of rotation, or equivalently, to integration of the input angular rate (Ω), represented as

$$\theta(t) = -k_e \int \Omega \; dt + \theta_b, \tag{1}$$

where k_e and θ_b denote the effective angular gain and bias error in the angle measurement mechanization. The effect of mechanical imperfections, such as anisodamping and anisoelastcity, on precession of the pattern angle has been studied to a great extend in [2,3,10]. However, limited literature exists on the effect of imperfections in control electronics of CVRIG on accuracy of angle measurements.

In the WA closed-loop control [2], three control loops are involved. A Phase Locked Loop (PLL) is used to lock a reference phase to the phase of oscillation for signal modulation and demodulation, and additional control loops are used to compensate for energy dissipation and anisoelasticity. Conventionally, a set of variables (pendulum variables) are estimated and utilized as inputs of the control loops. The pendulum variables are estimated through I/Q demodulation signals measured along the X and Y axes, as shown below

$$\begin{aligned} E_d &= x_{dc}^2 + x_{ds}^2 + y_{dc}^2 + y_{ds}^2 \\ Q_d &= 2(x_{dc}y_{ds} - y_{dc}x_{ds}) \\ R_d &= x_{dc}^2 + x_{ds}^2 - y_{dc}^2 - y_{ds}^2 \\ S_d &= 2(x_{dc}y_{dc} + y_{ds}x_{ds}) \end{aligned} \tag{2}$$

where x_{dc}, x_{ds}, y_{dc}, and y_{ds} are measurements of the in-phase and in-quadrature components of oscillation, with respect to the reference phase, along the X and Y axes, respectively.

Subsequently, the pendulum variables are used as inputs of the control loops in which the control outputs are estimated as

$$\begin{aligned} v_{a_s} &= +K_E[E_d - E_0 + \frac{1}{2\tau_E}\int (E_d - E_0)\, dt], \\ v_{q_c} &= -K_Q[Q_d + \frac{1}{2\tau_Q}\int Q_d\, dt], \end{aligned} \tag{3}$$

where variables v_{a_s} and v_{q_c} denote the output of control loops utilized for energy control and quadrature control, respectively. Control parameters K_E and K_Q are the proportional coefficients of the corresponding PI controllers, and τ_E and τ_Q are the integration times constants.

The two control voltages v_{a_s} and v_{q_c} are exerted as control forces (f_{a_s} and f_{q_c} in Fig. 1) along the major axis and minor axis of the elliptical orbit, to preserve energy in the system at a set value (i.e., $E_d = E_0$), and compensate for anisoelasticity (i.e., $Q_d = 0$). The subscript 'd' in the pendulum variables denotes that these values are estimates based on measurements of the in-phase and quadrature components of vibration along the X and Y axes. Assuming ideal electronics, the variables x_{dc}, x_{ds}, y_{dc}, and y_{ds} would be readout voltages that are scaled using a conversion gain G_D from their mechanical equivalents x_c, x_s, y_c, and y_s, measured in microns.

Based on the instantaneous orientation of the orbit (θ_d), the control voltages v_{a_s} and v_{q_c} are projected onto the X and Y axes and subsequently converted to electrostatic forces with a conversion gain of G_A and applied using the X and Y direction electrodes. The pendulum variables S_d and R_d are utilized, as shown in Eqn. (4), to estimate the orientation of the oscillation pattern, [2].

$$\theta_d = \frac{1}{2}arctan(S_d/R_d) \tag{4}$$

It is noted that in the WA closed-loop control architecture, a correct estimation of the pendulum variables and applying the correct control forces rely on having identical conversion gains in position-to-voltage for detection and voltage-to-force for actuation, along X and Y axes. However, due to imperfections in different components of the control electronics, including non-idealities in fabricated electrodes in the microstructure, gain mismatches cannot be avoided.

In section II, we study the effect of gain mismatches in detection electronics on estimation of the pendulum variables and the closed-loop control of the CVRIG. Also, we analyze the effect of gain mismatch in actuation electronics on the outcome of the WA closed-loop control. In section III, we discuss a methodology for identification of gain mismatches in control electronics of CVRIG. Experimental results on characterization of the angle bias error before and after compensation of mismatch in gains are reported.

II. Effect of Gain Mismatches on Precession

A. Mismatch in Detection Gains

A mismatch in the detection gains affects the pendulum variable estimation. To study the correlation between the estimated pendulum variables and their mechanical equivalents, the definition of the pendulum variables as in (2) was used. For example, for the quadrature Q variable

$$Q_d = 2G_{Dx}G_{Dy}(x_c y_s - y_c x_s) = G_{Dx}G_{Dy}Q, \tag{5}$$

where parameters without subscript 'd' denote the mechanical equivalents as originally defined in [2]. It is noted that in presence of a mismatch in the detection gains, the estimated quadrature Q_d would still be linearly proportional to the mechanical quadrature Q. Therefore, the quadrature PI controller would null both the estimated quadrature and the mechanical quadrature. As a result, an oscillation in-quadrature with the reference phase along X and Y axes (i.e., x_s and y_s) was assumed to be negligible. An expression for the estimated energy E_d was derived as

$$E_d = G_{Dx}^2 E[1 + (\frac{1-\cos 2\theta}{2})(\frac{G_{Dy}^2}{G_{Dx}^2} - 1)] \tag{6}$$

The estimated energy E_d is shown to be proportional to the mechanical energy, however, with a scaling factor that changes as a function of the pattern angle. Due to an angle-dependent conversion gain between the estimated energy and the mechanical energy, the mechanical energy would change as a function of the pattern angle. It should be noted that this variation takes place while the energy control loop maintains the estimated energy at the set value E_0. Based on equations describing the evolution of the elliptical orbit [2], a variation in the mechanical energy represented using the pendulum variable E in the CVRIG model does not affect precession. However, a variation in the mechanical energy would result in angle-dependent variations in the energy control output, that would not correlate to anisodamping in the system.

In the case of the pattern angle estimation, by neglecting the quadrature in the system, Eqn. (4) is simplified to

$$\theta_d = atan2(\frac{G_{Dy}}{G_{Dx}} \times sin(\theta), cos(\theta)), \tag{7}$$

where it shows that, due to mismatch in detection gains, we would have an error in the estimated pattern angle, Fig 1. The error in the angle estimation ($\theta_{error} = \theta_d - \theta$) was calculated as a function of the true angle (θ) for different values of detection gain mismatch, shown in Fig. 2. The error in pattern angle estimation would be superposed to the angle error caused by mechanical imperfections and results in a higher bias error in angle measurements represented as θ_b in Eqn. (1).

An error in estimation of the pattern angle also affects the closed-loop WA control. Due to an error in the pattern angle estimation, the control forces would not be applied at the correct orientation relative to the true orbital trajectory. As shown in Fig. 1, the control forces would be exerted at the estimated orientation (θ_d), and if we project these forces on the

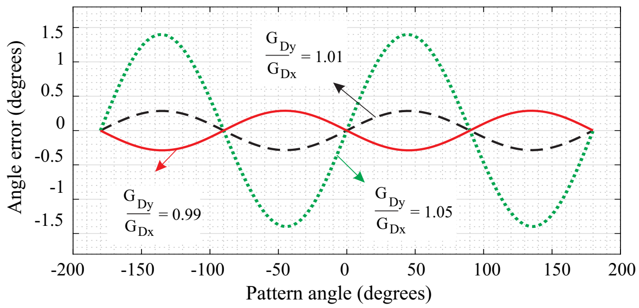

Fig. 2. Error in pattern angle estimation (θ_{error}) as a function of the pattern angle (θ) for different values of detection gain mismatch.

correct orientation of the orbit (θ), we find that the following force components affect the evolution of the pendulum.

$$\begin{aligned} f_{a_s} &= +G_A v_{a_s} cos(\theta_{error}) \\ f_{q_s} &= +G_A v_{a_s} sin(\theta_{error}) \\ f_{a_c} &= -G_A v_{q_c} sin(\theta_{error}) \\ f_{q_c} &= +G_A v_{q_c} cos(\theta_{error}) \end{aligned} \tag{8}$$

When compared to the control voltages v_{a_s} and v_{q_c}, the excitation forces f_{a_s}, f_{q_s}, f_{a_c}, and f_{q_c} are the actual effective forces acting along the major and minor axes of the orbital trajectory, in-phase and in-quadrature with the reference phase. The results indicate that due to a mismatch in detection gains, the output of the energy controller would have an unwanted component along the semi-minor axis (f_{q_s}) and the output of the quadrature controller would have a force component along the semi-major axis (f_{a_c}), in-quadrature and in-phase with the reference phase, respectively. In the frame of evolution of the pendulum [2], this denotes that the energy controller would affect the free precession of angle and the quadrature controller would affect the input-output phase relation. It is worth mentioning that the energy control output (v_{a_s}) is inversely proportional to the average energy decay time constant (τ). Therefore, in a high Q-factor gyroscope, the indirect effect of mismatch in detection gains on precession and accuracy of angle measurements is negligible as compared to the pattern angle estimation error (θ_e), illustrated in Fig. 2.

B. Mismatch in Actuation Gains

A similar analysis can be performed in the case of a mismatch in actuation gains. Assuming a mismatch in actuation gains, a direct projection of control voltages along the X and Y axes would not yield the appropriate forces for energy and quadrature control. The projected control voltages on the X and Y axes would yield the following electrostatic force components

$$\begin{aligned} f_{x_c} &= -G_{Ax} v_{q_c} sin(\theta) \\ f_{x_s} &= +G_{Ax} v_{a_s} cos(\theta) \\ f_{y_c} &= +G_{Ay} v_{q_c} cos(\theta) \\ f_{y_s} &= +G_{Ay} v_{a_s} sin(\theta) \end{aligned} \tag{9}$$

where f_{x_c}, f_{x_s}, f_{y_c}, and f_{y_s} denote the control forces in-phase and in-quadrature with the reference phase along the X and Y axes. If we project these forces back to the frame of the orbital trajectory we find

$$\begin{aligned} f_{a_s} &= \frac{1}{2}(G_{Ay} - G_{Ax}) v_{a_s} (1 - cos2\theta) + G_{Ax} v_{a_s} \\ f_{q_s} &= \frac{1}{2}(G_{Ay} - G_{Ax}) v_{a_s} sin2\theta \\ f_{a_c} &= \frac{1}{2}(G_{Ay} - G_{Ax}) v_{q_c} sin2\theta \\ f_{q_c} &= \frac{1}{2}(G_{Ay} - G_{Ax}) v_{q_c} (1 + cos2\theta) + G_{Ax} v_{q_c} \end{aligned} \tag{10}$$

These equations indicate that the control forces for energy and quadrature control would not be correctly aligned with the orientation of the orbital trajectory. As a result, the energy loop would interfere with the precession of the RIG. Similar to the previous case, it can be concluded that in a CVRIG with a high time constant, the effect of a mismatch in the actuation gains on precession (f_{q_s}) would be very small. Never the less, a mismatch in actuation gains would result in pattern angle dependent variations in the output of the energy control loop that would not correlate to anisodamping in the CVRIG.

III. Identification of Gain Mismatches

In a CVRIG, since the displacement of the resonator cannot be directly measured, an estimation of the detection gain (G_{Dx} or G_{Dy}) along a certain axis would only be possible by knowing the actuation gain along that axis (G_{Ax} or G_{Ay}) and vise-versa.

Here, we propose to utilize the measured Angle-Dependent Bias (ADB) error in a CVRIG, denoted as θ_b in Eqn. (1), for identification of mismatch in detection gains. The ADB due to mechanical imperfections, such as anisodamping and anisoelastcity, is inversely proportional to the input angular rate, [10,11]. Therefore, it can be assumed that at a high enough angular rate input, the ADB due to mechanical imperfections becomes negligible. Hence, the measured ADB would be mainly due to the error in estimation of the orientation of the oscillation pattern caused by a mismatch in detection gains. In this method, the CVRIG is physically rotated at an input angular rate, which is orders of magnitude above the minimum detectable rate threshold Ω_{th}, defined in [10], and the measured ADB is used to estimate the gain mismatch in detection gains. After compensation of the mismatch in detection gains, the output of the energy control loop is used for identification of mismatch in actuation gains.

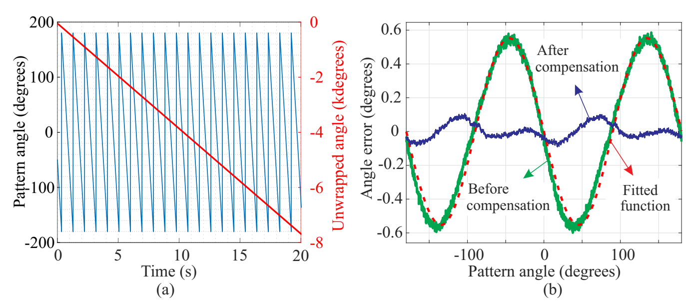

Fig. 3. (a) Measured pattern angle and the unwrapped angle of the DFP CVRIG to an input angular rate of 500dps. (b) Estimated angle bias error is shown before and after compensation of the detection gain mismatch.

To demonstrate the proposed method, the closed-loop WA control was implemented on a Dual Foucault Pendulum (DFP) gyroscope, [6]. Details on implementation of the WA control can be found in [5]. Design parameters of the DFP used in our study can be found in [7]. The as-fabricated frequency split of the DFP was electrostatically reduced to less than 50 mHz.

Using an Ideal Aerosmith 1291BR rate-table, input angular rate of 500 dps was applied to the DFP gyroscope and the angle output was measured, shown in Fig 3 (a). A linear fit to the unwrapped angle was used to estimate the effective angular gain and the angle bias error, defined in Eqn. (1). The angle bias as a function of the pattern angle (i.e., the ADB) is shown in Fig. 3 (b).

As a result of the quadrature control loop, the Q/E ratio was observed to be less than 100 ppm, making anisodamping the main mechanical source of the angle error, [2]. By measuring the energy decay time constant at different pattern angles, anisodamping of the DFP was determined to be on the order of 4.7 mHz. Using the analytical equations reported in [10], the angle error due to anisodamping for an input angular rate of 500 dps was calculated to be on the order of 10 mdegrees. Therefore, the two-orders of magnitude higher error in angle measurements shown in Fig. 3 (b) was concluded to have been originated from a mismatch in the detection gains.

The function shown in Eqn. (4) was fitted to the experimentally measured angle bias error using the non-linear least squares method and a 1.32% mismatch in detection gains was identified. The identified gain mismatch was included in the Digital Signal Processing (DSP) for correct estimation of the pendulum variables. After compensation, an order of magnitude reduction in angle bias error was observed, shown in Fig. 3 (b).

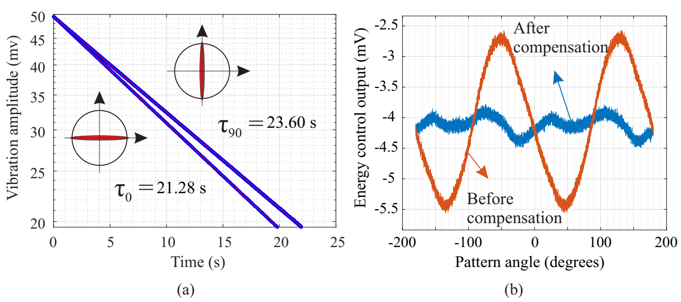

Fig. 4. Experimental results. (a) The decay of vibration amplitude was used for estimation of time constants along the X and Y axes. (b) The output of the energy control loop as a function of the pattern angle is shown before and after compensation of gain mismatches.

Subsequently, to identify the mismatch in actuation gains, a rate control loop [2] was utilized and the orientation of the oscillation pattern was maintained at 0 and 90 degrees. While the rate control loop was engaged, the energy loop was turned off. Based on the energy decay rate, illustrated in Fig. 4. (a), time constants along the X and Y axes were estimated. The estimated time constants corresponded to damping of the resonator precisely along X and Y axes. Therefore, the mismatch in the actuation gain was calculated based on

$$\frac{G_{Ax}}{G_{Ay}} = \frac{\tau_{90}\ v_{a_{s@90}}}{\tau_0\ v_{a_{s@0}}} \tag{11}$$

A 1.13% mismatch in actuation gains was identified and compensated as a part of the DSP. Variations in the output of the energy controller are shown before and after compensation of mismatches in control electronics, Fig. 4 (b). A higher variation in energy control output is mainly due to the error in estimation of energy in the system caused by the mismatch in detection electronics, Eqn. (6). After compensation of gain mismatches, the remaining angle bias error and variations in the energy control output were observed to have 4θ dependencies. The 4θ dependent variations are believed to have been caused by nonlinearity in parallel plate electrodes used for detection, previously reported in [12].

IV. Conclusion

In this paper, we studied the effect of conversion gain asymmetries in control electronics of CVRIG on accuracy of direct angle measurements. We demonstrated that a mismatch in detection gains results in an error in estimation of the pendulum variables and the pattern angle. Mismatches in detection and actuation gains were shown to adversely affect the free precession of the pattern angle. Based on the analytical equations, we developed a methodology to identify mismatches in control electronics gains. For example, by identification and compensation of the gain mismatches in a DFP CVRIG, the angle bias error at 500 dps angular rate input was reduced by 10-time and an accuracy of 0.06 degrees was demonstrated.

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